O-LEVEL A-MATHS 2014 – PAPER 2

Question 1

(i) Given $T = 20 + Ae^{-kt}$
Initially, $T = 80$
$\therefore 20 + Ae^{0} = 80$
$\Rightarrow A = 80 - 20 = 60$

(ii) $T = 20 + 60e^{-kt}$

When $t = 1$,
$T = 65$
$20 + 60e^{-k} = 65$
$e^{-k} = \frac{65-20}{60} = \frac{3}{4}$
$-k = \ln\frac{3}{4}$
$k = -\ln\frac{3}{4}$
$k = 0.288$

(iii) $T = 20 + 60e^{t\ln\frac{3}{4}}$

When $t = 4$,
$T = 20 + 60e^{(4)\ln\frac{3}{4}} = 38.984℃ < 40℃$

$\therefore$ It is safe to give the food to the baby 4 minutes after removal from the microwave.

Question 2

(i) Remainder
$= f(2) = 2(2)^3 - 3(2)^2 - 11(2) + 6$
$= -12$

(ii) $f(-2)$
$= 2(-2)^3 - 3(-2)^2 - 11(-2) + 6 = 0$
$\therefore x - 2$ is a factor of $f(x)$.

$$
\begin{array}{r}
2x^2 - 7x + 3 \\
x + 2 \,|\, 2x^3 - 3x^2 - 11x + 6 \\
\underline{-(2x^3 + 4x^2)} \\
-7x^2 - 11x \\
\underline{-(-7x^2 - 14x)} \\
3x + 6 \\
\underline{-(3x + 6)} \\
0
\end{array}
$$

$\therefore f(x) = 0$
$(x + 2)(2x^2 - 7x + 3) = 0$
$(x + 2)(2x - 1)(x - 3) = 0$
$x = -2$ or $x = \frac{1}{2}$ or $x = 3$

Question 3

(i) Length

$$= \frac{13 - \sqrt{48}}{3 - \sqrt{3}}$$

$$= \frac{13 - 4\sqrt{3}}{3 - \sqrt{3}}\left(\frac{3 + \sqrt{3}}{3 + \sqrt{3}}\right)$$

$$= \frac{39 + 13\sqrt{3} - 12\sqrt{3} - 12}{3^2 - \left(\sqrt{3}\right)^2}$$

$$= \frac{27 + \sqrt{3}}{6}$$

$$= \frac{9}{2} + \frac{1}{6}\sqrt{3}$$

(ii) $\left(2\sqrt{3} + c\right)^2 = 13 - \sqrt{48}$

$\left(2\sqrt{3}\right)^2 + 4\sqrt{3}c + c^2 = 13 - 4\sqrt{3}$

$12 + 4\sqrt{3}c + c^2 = 13 - 4\sqrt{3}$

$c^2 + 4\sqrt{3}c + 4\sqrt{3} - 1 = 0$

$\left(c + 4\sqrt{3} - 1\right)(c + 1) = 0$

$c = 1 - 4\sqrt{3}$ or $c = -1$

When $c = -1$,

$2\sqrt{3} + c = 2\sqrt{3} - 1 > 0$

When $c = 1 - 4\sqrt{3}$,

$2\sqrt{3} + c = 2\sqrt{3} + 1 - 4\sqrt{3} = 1 - 2\sqrt{3} < 0$ (NA)

$\therefore c = -1$

Question 4

(i) $2x^2 + 5x + 4 = 0$

$\alpha + \beta = -\frac{5}{2}, \qquad \alpha\beta = \frac{4}{2} = 2$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= \left(-\frac{5}{2}\right)^2 - 2(2) = \frac{9}{4}$

(ii) $(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$

$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha^2\beta - 3\alpha\beta^2$

$= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= \left(-\frac{5}{2}\right)^3 - 3(2)\left(-\frac{5}{2}\right) = -\frac{5}{8}$

∴ Sum of new roots

$= \alpha^3 + \beta^3$

$= -\frac{5}{8}$

Product of new roots

$= \alpha^3\beta^3$

$= (\alpha\beta)^3$

$= (2)^3 = 8$

New equation:

$x^2 - \left(-\frac{5}{8}\right)x + 8 = 0$

$8x^2 + 5x + 64 = 0$

Question 5

(a) $2\log_2 x - \log_2(x - 4) = 3$

$\log_2 x^2 - \log_2(x - 4) = 3$

$\log_2 \frac{x^2}{x - 4} = 3$

$\frac{x^2}{x - 4} = 2^3$

$x^2 = 8x - 32$

$x^2 - 8x + 32 = 0$

Discriminant

$= (-8)^2 - 4(1)(32)$

$= -64 < 0$

∴ There is no real roots.

(b) $\frac{(\log_x y)^2}{\log_y x} + 8 = 0$

$\frac{(\log_x y)^2}{\left(\frac{\log_x x}{\log_x y}\right)} = -8$

$(\log_x y)^3 = -8$

$\log_x y = -2$

$y = x^{-2}$

Question 6

(i) $\angle DEF = \angle EAC + \angle ACE$ (exterior $\angle$ of Δ)

$CE = AE$ (tangent to an exterior point)
$\angle EAC = \angle ACE$ (base $\angle$s of isosc. Δ)
$\therefore \angle DEF = 2\angle ACE$

$\angle ACE = \angle ABC$ (alternate segment theorem)

$\therefore \angle DEF = 2\angle ABC$ (shown)

(ii) $\angle DFE = 2\angle ACB$

(iii) $\angle EDF + \angle DEF + \angle DFE = 180°$ ($\angle$ sum of Δ)

$\angle EDF + 2\angle ABC + 2\angle ACB = 180°$ (from (i) and (ii))

$\angle EDF + 2(\angle ABC + \angle ACB) = 180°$
$\angle EDF + 2(180° - \angle BAC) = 180°$ ($\angle$ sum of Δ)
$\angle EDF + 360° - 2\angle BAC = 180°$
$\therefore 2\angle BAC = 180° + \angle EDF$ (proven)

Question 7

(i) $y = 2 - (3 - x)^4$

$\frac{dy}{dx} = -4(3 - x)^3(-1)$
$= 4(3 - x)^3$

Let $\frac{dy}{dx} = 0$,
$4(3 - x)^3 = 0 \Rightarrow x = 3$

$y = 2 - (3 - 3)^4 = 2$

$\therefore p = 3, q = 2$

(ii) (a) For $x < 3$,
$\frac{dy}{dx} = 4(3 - x)^3 > 0$
$\therefore y$ is increasing

(b) For $x > 3$,
$\frac{dy}{dx} = 4(3 - x)^3 < 0$
$\therefore y$ is decreasing

(iii) From (ii),

	$x < 3$	$x = 3$	$x > 3$
$\frac{dy}{dx}$	+	0	−
y	/	–	\

$\therefore$ The stationary point is a maximum.

(iv) $\frac{d^2y}{dx^2} = 4(3)(3-x)^2(-1) = -12(3-x)^2$

When $x = 3$,

$$\frac{d^2y}{dx^2} = -12(3-3)^2 = 0$$

Question 8

(i) $a = -e^{-0.1t}$

$$v = \int -e^{-0.1t}\,dt = -\frac{e^{-0.1t}}{-0.1} + c$$
$$= 10e^{-0.1t} + c$$

When $t = 0$,

$$v = 8$$
$$10e^0 + c = 8 \Rightarrow c = -2$$

$\therefore v = 10e^{-0.1t} - 2$

At P,

$$v = 0$$
$$10e^{-0.1t} - 2 = 0$$
$$e^{-0.1t} = \frac{2}{10} = \frac{1}{5}$$
$$-0.1t = \ln\frac{1}{5} = -\ln 5$$
$$t = \frac{-\ln 5}{-0.1} = 10\ln 5$$

(shown)

(ii) Displacement, s

$$= \int 10e^{-0.1t} - 2\,dt = 10\left(\frac{e^{-0.1t}}{-0.1}\right) - 2t + c = -100e^{-0.1t} - 2t + c$$

When $t = 0$,

$$s = 0$$
$$-100e^0 + c = 0 \Rightarrow c = 100$$

$\therefore s = -100e^{-0.1t} - 2t + 100$

When $t = 10\ln 5$,

$$s = -100e^{-0.1(10\ln 5)} - 2(10\ln 5) + 100$$
$$= 47.811$$

$\therefore OP = 47.8$m

(iii) When $t = 49$,

$$s = -100e^{-0.1(49)} - 2(49) + 100$$
$$= 1.2553$$

When $t = 50$,

$$s = -100e^{-0.1(50)} - 2(50) + 100$$
$$= -0.67380$$

$\therefore$ The particle is again at O at some instant during the fiftieth second after first passing through O.

Question 9

(i) $3\cos 2A + \sin A - 2 = 0$

$3(1 - 2\sin^2 A) + \sin A - 2 = 0$

$-6\sin^2 A + \sin A + 1 = 0$

$6\sin^2 A - \sin A - 1 = 0$

$(3\sin A + 1)(2\sin A - 1) = 0$

$\sin A = -\frac{1}{3}$ or $\sin A = \frac{1}{2}$

Basic ∠ $= \sin^{-1}\frac{1}{3} = 19.471°$

$A = 180° + 19.471°, 360° - 19.471°$

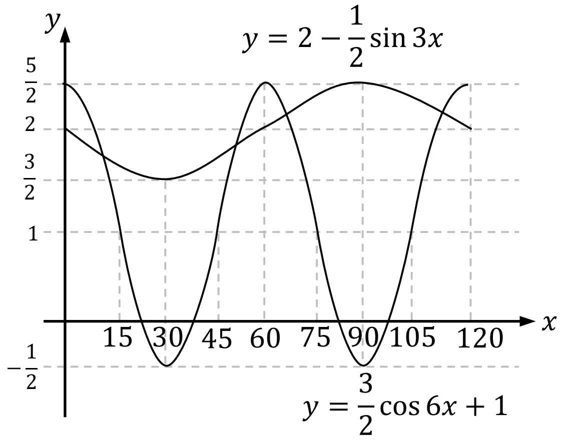
$A = 199.5°, 340.5°$

Basic ∠ $= \sin^{-1}\frac{1}{2} = 30°$

$A = 30°, 150°$

$\therefore A = 30°, 150°, 199.5°$ or $340.5°$

(ii)



(iii) Let

$\frac{3}{2}\cos 6x + 1 = 2 - \frac{1}{2}\sin 3x$

$3\cos 6x + 2 = 4 - \sin 3x$

$3\cos 6x + \sin 3x - 2 = 0$

$3\cos 2(3x) + \sin 3x - 2 = 0$

$\therefore$ By replacing A from part (i) by $3x$ and subsequently making x the subject could be used to find the x-coordinates of the points of intersection of the graphs in part (ii).

Question 10

(i) $x^2 + y^2 + 4x - 6y = 36$
$(x+2)^2 - 2^2 + (y-3)^2 - 3^2 = 36$
$(x+2)^2 + (y-3)^2 = 49$
$(x+2)^2 + (y-3)^2 = 7^2$
∴ Radius = 7, Centre is at $(-2, 3)$

(ii) Tangent: $3y = 4x - 15$
$\Rightarrow y = \frac{4}{3}x - 5 \quad (1)$
∴ Equation of AB:
$y - 5 = -\frac{3}{4}(x+5)$
$y = -\frac{3}{4}x + \frac{5}{4} \quad (2)$

$(1) = (2)$
$\frac{4}{3}x - 5 = -\frac{3}{4}x + \frac{5}{4}$
$\frac{25}{12}x = \frac{25}{4} \Rightarrow x = 3$
Sub. $x = 3$ into (1)
$y = \frac{4}{3}(3) - 5 = -1$
∴ $B(3, -1)$

(iii) Centre of C_2
$= \left(\frac{-5+3}{2}, \frac{5-1}{2}\right) = (-1, 2)$

Radius of C_2
$= \sqrt{(3+1)^2 + (-1-2)^2} = 5$

(iv) For C_1,
distance from $(4, 6)$ to centre
$= \sqrt{(4+2)^2 + (6-3)^2}$
$= \sqrt{45} = 6.7082 <$ Radius of C_1
∴ $(4, 6)$ lie within C_1.

For C_2,
distance from $(4, 6)$ to centre
$= \sqrt{(4+1)^2 + (6-2)^2}$
$= \sqrt{41} = 6.4031 >$ Radius of C_2
∴ $(4, 6)$ does not lie within C_2.

Question 11

(a) $\frac{d}{dx}\left(\frac{x}{\sqrt{2x-1}}\right)=\frac{d}{dx}\left[\frac{x}{(2x-1)^{1/2}}\right]$

$$=\frac{(2x-1)^{\frac{1}{2}}(1)-\frac{1}{2}(2x-1)^{-\frac{1}{2}}(2)(x)}{2x-1}$$

$$=\frac{\sqrt{2x-1}-\frac{x}{\sqrt{2x-1}}}{2x-1}$$

$$=\frac{\left(\frac{2x-1-x}{\sqrt{2x-1}}\right)}{2x-1}=\frac{x-1}{\sqrt{(2x-1)^3}}$$

(shown)

(b) (i) For the curve, let $y=0$,

$$\frac{8(x-1)}{\sqrt{(2x-1)^3}}=0$$

$8(x-1)=0$

$x=1$

$\therefore A(1,0)$

Equation of AB:

$y-0=(1)(x-1)$

$y=x-1 \quad (1)$

$$y=\frac{8(x-1)}{\sqrt{(2x-1)^3}} \quad (2)$$

$(1)=(2)$

$$x-1=\frac{8(x-1)}{\sqrt{(2x-1)^3}}$$

$\sqrt{(2x-1)^3}=8$

$2x-1=8^{\frac{2}{3}}$

$2x-1=4 \Rightarrow x=\frac{5}{2}$

Sub. $x=\frac{5}{2}$ into (1),

$$y=\frac{5}{2}-1=\frac{3}{2}$$

$\therefore$ y-coordinate of B is $\frac{3}{2}$. (verified)

(ii) Area

$$= \frac{1}{2}\left(\frac{5}{2} - 1\right)\left(\frac{3}{2}\right) + \int_{\frac{5}{2}}^{5} \frac{8(x-1)}{\sqrt{(2x-1)^3}}\,dx$$

$$= \frac{9}{8} + 8\int_{\frac{5}{2}}^{5} \frac{x-1}{\sqrt{(2x-1)^3}}\,dx$$

$$= \frac{9}{8} + 8\left[\frac{x}{\sqrt{2x-1}}\right]_{\frac{5}{2}}^{5}$$

$$= \frac{9}{8} + 8\left(\frac{5}{3} - \frac{5}{4}\right)$$

$$= \frac{107}{24} \text{ units}^2$$