

A-LEVEL H2 MATHS 2010 – PAPER 2

Question 1

[Ans: (i) $3 \pm 5i$ (ii) $a = -16, b = -20; -2 - i, -2$ and 2]

(i) $x^2 - 6x + 34 = 0$

$$x = \frac{6 \pm \sqrt{6^2 - 4(1)(34)}}{2}$$

$$x = \frac{6 \pm \sqrt{-100}}{2} = 3 \pm 5i$$

(ii) Let $f(x) = x^4 + 4x^3 + x^2 + ax + b$

$$f(-2 + i) = 0$$

$$(-2 + i)^4 + 4(-2 + i)^3 + (-2 + i)^2 + a(-2 + i) + b = 0$$

$$-12 + 16i - 2a + ia + b = 0$$

$$(-12 - 2a + b) + i(16 + a) = 0$$

$$\therefore 16 + a = 0$$

$$\Rightarrow a = -16$$

$$\therefore -12 - 2a + b = 0$$

$$-12 - 2(-16) + b = 0$$

$$\Rightarrow b = -20$$

$$\therefore x^4 + 4x^3 + x^2 - 16x - 20 = 0$$

From GC,

$a_4x^4 + \dots + a_1x + a_0 = 0$ $a_4 = 1$ $a_3 = 4$ $a_2 = 1$ $a_1 = -16$ $a_0 = -20$	$a_4x^4 + \dots + a_1x + a_0 = 0$ $x_1 = -2 + 1i$ $x_2 = -2 - 1i$ $x_3 = 2$ $x_4 = -2$
MAIN MODE CLR LOAD SOLVE	MAIN MODE COEFF STO

The other roots are $-2 - i, -2$ and 2 .

Question 2

[Ans: (i) prove (ii)(a) prove (b) explain; $\frac{3}{4}$]

(i) Let

$$p_n: \sum_{r=1}^n r(r+2) = \frac{1}{6}n(n+1)(2n+7), n \in \mathbb{Z}^+$$

$$P_1: LHS = \sum_{r=1}^1 r(r+2) = (1)(1+2) = 3$$

$$RHS = \frac{1}{6}(1)(1+1)(2+7) = 3 = LHS$$

 $\therefore P_1$ is true.Assume P_k is true. i.e.

$$\sum_{r=1}^k r(r+2) = \frac{1}{6}k(k+1)(2k+7).$$

[Aim: To prove P_{k+1} is true. i.e.

$$\sum_{r=1}^{k+1} r(r+2) = \frac{1}{6}(k+1)[(k+1)+1][2(k+1)+7] = \frac{1}{6}(k+1)(k+2)(2k+9)]$$

$$\begin{aligned} & \sum_{r=1}^{k+1} r(r+2) \\ &= \sum_{r=1}^k r(r+2) + (k+1)(k+3) \\ &= \frac{1}{6}k(k+1)(2k+7) + (k+1)(k+3) \\ &= \frac{1}{6}(k+1)[k(2k+7) + 6(k+3)] \\ &= \frac{1}{6}(k+1)(2k^2 + 13k + 18) \\ &= \frac{1}{6}(k+1)(k+2)(2k+9) \end{aligned}$$

 $\therefore P_{k+1}$ is true if P_k is true.Since P_1 is true, \therefore by mathematical induction, P_n is true for $n \in \mathbb{Z}^+$.

(ii) (a) Let $\frac{1}{r(r+2)} = \frac{A}{r} + \frac{B}{r+2}$

$$A = \frac{1}{2} \text{ and } B = -\frac{1}{2}$$

$$\begin{aligned} \therefore \frac{1}{r(r+2)} &= \frac{1}{2r} - \frac{1}{2(r+2)} \\ &= \frac{1}{2} \left(\frac{1}{r} - \frac{1}{r+2} \right) \end{aligned}$$

$$\begin{aligned}
\sum_{r=1}^n \frac{1}{r(r+2)} &= \frac{1}{2} \sum_{r=1}^n \left(\frac{1}{r} - \frac{1}{r+2} \right) \\
&= \frac{1}{2} \left[1 - \frac{1}{3} \right. \\
&\quad + \frac{1}{2} - \frac{1}{4} \\
&\quad + \frac{1}{3} - \frac{1}{5} \\
&\quad + \frac{1}{4} - \frac{1}{6} \\
&\quad \vdots \\
&\quad + \frac{1}{n-2} - \frac{1}{n} \\
&\quad \left. + \frac{1}{n-1} - \frac{1}{n+1} \right. \\
&\quad \left. + \frac{1}{n} - \frac{1}{n+2} \right] \\
&= \frac{1}{2} \left(1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2} \right) \\
&= \frac{3}{4} + \frac{1}{2(n+1)} - \frac{1}{2(n+2)} \\
&\quad \text{(shown)}
\end{aligned}$$

(b) When $n \rightarrow \infty$, $\frac{1}{2(n+1)} \rightarrow 0$ and $\frac{1}{2(n+2)} \rightarrow 0$, \therefore series is convergent.

Sum to infinity

$$\begin{aligned}
&\sum_{r=1}^{\infty} \frac{1}{r(r+2)} \\
&= \lim_{n \rightarrow \infty} \left[\frac{3}{4} + \frac{1}{2(n+1)} - \frac{1}{2(n+2)} \right] \\
&= \frac{3}{4}
\end{aligned}$$

Question 3

[Ans: (i) $\frac{dy}{dx} = \frac{3x+4}{2\sqrt{x+2}}$; show (ii)(a) $\pm\sqrt{2}$ (b) sketch (iii) sketch]

$$\begin{aligned} \text{(i)} \quad \frac{dy}{dx} &= x \left(\frac{1}{2}\right) (x+2)^{-\frac{1}{2}} + \sqrt{x+2} \\ &= \frac{x}{2\sqrt{x+2}} + \sqrt{x+2} \\ &= \frac{x+2x+4}{2\sqrt{x+2}} = \frac{3x+4}{2\sqrt{x+2}} \end{aligned}$$

Let $\frac{dy}{dx} = 0$

$$\therefore \frac{3x+4}{2\sqrt{x+2}} = 0 \Rightarrow x = -\frac{4}{3}$$

\therefore There is only one turning point at $x = -\frac{4}{3}$.

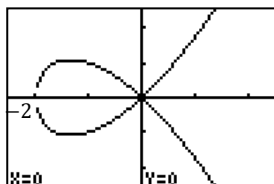
$$\begin{aligned} \text{(ii) (a)} \quad y^2 &= x^2(x+2) \Rightarrow y = \pm x\sqrt{x+2} \\ \therefore \frac{dy}{dx} &= \pm \frac{3x+4}{2\sqrt{x+2}} \end{aligned}$$

When $x = 0$,

$$\frac{dy}{dx} = \pm \frac{4}{2\sqrt{2}} = \pm\sqrt{2}$$

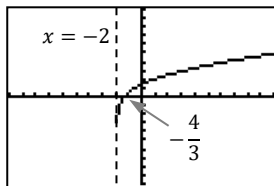
(b) $y^2 = x^2(x+2)$

Plot1	Plot2	Plot3
$\sqrt{Y_1} = \sqrt{X^2(X+2)}$		
$\sqrt{Y_2} = -\sqrt{Y_1}$		
$\sqrt{Y_3} =$		
$\sqrt{Y_4} =$		
$\sqrt{Y_5} =$		



(iii) $y = f'(x)$

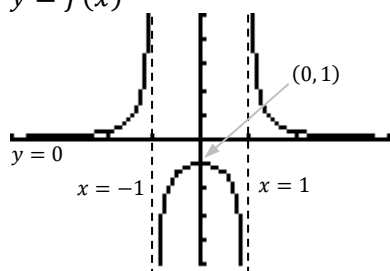
Plot1	Plot2	Plot3
$\sqrt{Y_1} = \sqrt{X(X+2)}$		
$\sqrt{Y_2} = \frac{d}{dx}(\sqrt{Y_1}) _{X=X}$		
$\sqrt{Y_3} =$		
$\sqrt{Y_4} =$		



Question 4

[Ans: (i) sketch (ii) state; $k = 0$ (iii) show (iv) $2 < x < 3$ or $3 < x < 4$ (v) $(-\infty, -1) \cup (0, \infty)$]

(i) $y = f(x)$



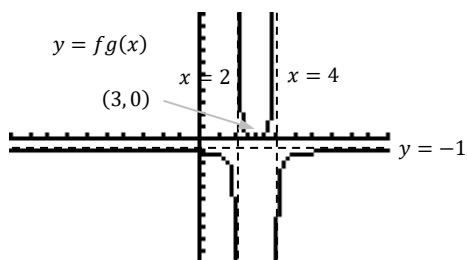
(ii) For f^{-1} to exist, the f must be a one-to-one function. Therefore least value of $k = 0$.

(iii) $fg(x)$

$$\begin{aligned} &= \frac{1}{\left(\frac{1}{x-3}\right)^2 - 1} \\ &= \frac{1}{\frac{1 - (x-3)^2}{(x-3)^2}} \\ &= \frac{(x-3)^2}{(1-x+3)(1+x-3)} \\ &= \frac{(x-3)^2}{(4-x)(x-2)} \end{aligned}$$

(shown)

(iv) $fg(x) > 0, x \in D_{fg} = D_g$



From GC,

$$2 < x < 3 \text{ or } 3 < x < 4$$

(v) From graph of $y = fg(x)$,

$$\therefore R_{fg} = (-\infty, -1) \cup (0, \infty)$$

Question 5

[Ans: (i) reason (ii) explain]

- (i) Opinions of international spectators in a stratified sample may not be representative of the population across the various strata due to cultural and regional influences.
- (ii) To perform a systematic sampling, we can first determine the size of the sample, $0.01N$, where N is the total number of spectators. Then we will randomly interview a spectator coming out of the catering location, and follow by every interviewing every $\frac{N}{0.01N} = 100^{\text{th}}$ spectators leaving the location.

Question 6

[Ans: $\mu \approx 41.3$, $s^2 = 1.584$; p -value = 0.0949, sufficient evidence to reject H_0]

$$\begin{aligned}\mu &\approx \bar{t} = \frac{\sum t}{n} \\ &= \frac{454.3}{11} = 41.3\end{aligned}$$

$$\begin{aligned}\sigma^2 &\approx s^2 = \frac{1}{n-1} \left[\sum t^2 - \frac{(\sum t)^2}{n} \right] \\ &= \frac{1}{10} \left[18778.43 - \frac{(454.3)^2}{11} \right] \\ &= 1.584\end{aligned}$$

$$H_0: \mu = 42.0$$

$$H_1: \mu \neq 42.0$$

$$n = 11$$

$$\bar{t} = 41.3$$

$$s^2 = 1.584$$

$$\text{Test statistics, } T = \frac{\bar{T} - 42.0}{\sqrt{1.584/11}} \sim t(10)$$

From GC,

T-Test	T-Test
Inpt:Data State	$\mu \neq 42$
$\mu_0: 42$	$t = -1.844661968$
$\bar{x}: 41.3$	$p = .0948714485$
$S_x: 1.258570617...$	$\bar{x} = 41.3$
$n: 11$	$S_x = 1.258570618$
$\mu: \mu_0 < \mu_0 > \mu_0$	$n = 11$
Calculate Draw	

$$p\text{-value} = 0.094871$$

Since $p\text{-value} < 0.10$, \therefore there is sufficient evidence to reject H_0 , i.e. there has been a change in the mean time required by an employee to complete the task, at 10% level of significance

Question 7

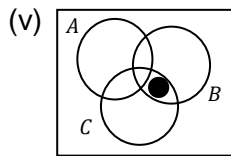
[Ans: (i) 0.32 (ii) 0.92 (iii) 0.457 (iv) 0.15 (v) $0.07 \leq P(A' \cap B \cap C) \leq 0.15$]

$$(i) \begin{aligned} P(A|B') &= 0.8 \\ \frac{P(A \cap B')}{P(B')} &= 0.8 \\ P(A \cap B') &= 0.8P(B') \\ P(A \cap B') &= 0.8[1 - P(B)] = 0.32 \end{aligned}$$

$$(ii) \begin{aligned} P(A \cup B) &= P(A \cap B') + P(B) \\ &= 0.32 + 0.6 = 0.92 \end{aligned}$$

$$(iii) \begin{aligned} P(B'|A) &= \frac{P(B' \cap A)}{P(A)} = \frac{P(A \cap B')}{P(A)} \\ &= \frac{0.32}{0.7} = 0.457 \end{aligned}$$

$$(iv) \begin{aligned} P(C) &= P(A \cap C) + P(A' \cap C) \\ P(A' \cap C) &= P(C) - P(A \cap C) \\ P(A' \cap C) &= P(C) - P(A)P(C) \\ P(A' \cap C) &= 0.5 - (0.7)(0.5) \\ &= 0.15 \end{aligned}$$



From the Venn diagram,

$$P(A' \cap B' \cap C) \leq 1 - P(A \cup B) = 1 - 0.92 = 0.08$$

$$P(A' \cap C) - P(A' \cap B \cap C) \leq 0.08$$

$$0.15 - P(A' \cap B \cap C) \leq 0.08$$

$$\Rightarrow P(A' \cap B \cap C) \geq 0.07$$

$$P(A' \cap B \cap C) \leq P(A' \cap C) = 0.15$$

$$\therefore 0.07 \leq P(A' \cap B \cap C) \leq 0.15$$

Question 8

[Ans: (i) $\frac{3}{5}$ (ii) $\frac{1}{10}$ (iii) $\frac{7}{20}$]

(i) Required probability

$$= \frac{3}{5}$$

(ii) _ _ _ E E

Required probability

$$= \frac{3!2!}{5!} = \frac{1}{10}$$

(iii) Case (1): 3 _ _ _ {1 / 5}

$$\text{No of ways} = {}^2C_1 3! = 12$$

Case (2): 4 _ _ _ {1 / 3 / 5}

$$\text{No of ways} = {}^3C_1 3! = 18$$

Case (3): 5 _ _ _ {1 / 3}

$$\text{No of ways} = {}^2C_1 3! = 12$$

Required probability

$$= \frac{12 + 18 + 12}{5!}$$

$$= \frac{7}{20}$$

Question 9

[Ans: (i) 0.681 (ii) 0.234 (iii) 0.362]

(i) Given $X \sim N(180, 30^2)$ and $Y \sim N(400, 60^2)$

$$\begin{aligned} E(Y - 2X) &= E(Y) - 2E(X) \\ &= 400 - 2(180) = 40 \end{aligned}$$

$$\begin{aligned} \text{Var}(Y - 2X) &= \text{Var}(Y) + 2^2\text{Var}(X) \\ &= 60^2 + 4(30^2) = 7200 \end{aligned}$$

$$\therefore Y - 2X \sim N(40, 7200)$$

$$\begin{aligned} \text{Required probability} \\ &= P(Y > 2X) = P(Y - 2X > 0) \\ &= 0.681 \end{aligned}$$

(ii) $E(0.12X + 0.05Y)$
 $= 0.12E(X) + 0.05E(Y)$
 $= 0.12(180) + 0.05(400) = 41.6$

$$\begin{aligned} \text{Var}(0.12X + 0.05Y) \\ &= 0.12^2\text{Var}(X) + 0.05^2\text{Var}(Y) \\ &= 0.12^2(30^2) + 0.05^2(60^2) \\ &= 21.96 \end{aligned}$$

$$\therefore 0.12X + 0.05Y \sim N(41.6, 21.96)$$

$$\begin{aligned} \text{Required probability} \\ &= P(0.12X + 0.05Y > 45) = 0.234 \end{aligned}$$

(iii) Let $C = 0.12X$
 $E(C) = 0.12E(X)$
 $= 0.12(180) = 21.6$
 $\text{Var}(C) = 0.12^2\text{Var}(X)$
 $= 0.12^2(30^2) = 12.96$

$$\therefore C \sim N(21.6, 12.96)$$

$$\begin{aligned} E(C_1 + C_2) &= 2E(C) \\ &= 2(21.6) = 43.2 \\ \text{Var}(C_1 + C_2) &= 2\text{Var}(C) \\ &= 2(12.96) = 25.92 \end{aligned}$$

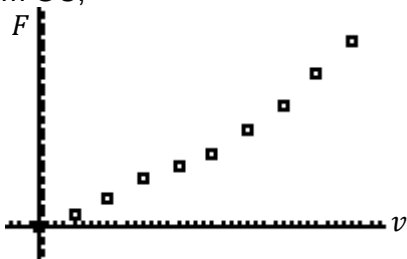
$$\therefore C_1 + C_2 \sim N(43.2, 25.92)$$

$$\begin{aligned} \text{Required probability} \\ &= P(C_1 + C_2 > 45) = 0.362 \end{aligned}$$

Question 10

[Ans: (i) draw (ii)(a) 0.9860 (b) 0.9907 (iii) $F = c + dv^2$ is the better model; explain (iv) $F = 3.1957 + 0.024242v^2$; 30.7; explain]

(i) From GC,



(ii) (a) From GC,

L1	L2	L3	2	EDIT	LINK	TESTS	LinReg(a+bx)	Xlist:L1	Ylist:L2	FreqList:	Store RegEQ:	Calculate	Y=a+bx	a=-1.990909091	b=.9022727273	r ² =.9722440983	r=.9860243903
16	11.2			LinReg													
20	13.6			ExpReg													
24	17.6			PwrReg													
28	22			Logistic													
32	27.8			SinReg													
36	33.9			Manual-Fit													
L2(10)=33.9																	

$r = 0.9860$

(b) From GC,

L1	L2	L3	3	EDIT	LINK	TESTS	LinReg(a+bx)	Xlist:L3	Ylist:L2	FreqList:	Store RegEQ:	Calculate	Y=a+bx	a=3.195652174	b=.0242419908	r ² =.9814487724	r=.990680964
16	11.2	256		LinReg													
20	13.6	400		ExpReg													
24	17.6	576		PwrReg													
28	22	784		Logistic													
32	27.8	1024		SinReg													
36	33.9	1296		Manual-Fit													
L3(10)=1296																	

$r = 0.9907$

(iii) From values of r in part (ii), $F = c + dv^2$ is the better model as the value of $|r|$ is closer to 1.

(iv) From GC,

$$F = 3.1957 + 0.024242v^2$$

When $F = 26.0$,

$$3.1957 + 0.024242v^2 = 26.0$$

$$v = 30.7$$

As F is the dependent variable and v is the independent variable in this experiment, neither the regression lines should be used as both lines are generated assuming F to be the independent variable and v the dependent variable.

Question 11

[Ans: (i) 0.0655 (ii) 32 s (iii) 0.192 (iv) 0.235 (v) 0.959]

- (i) Let X be the number of calls received in 4 minutes.

$$X \sim Po(3 \times 4) \Rightarrow X \sim Po(12)$$

$$P(X = 8) = 0.0655$$

- (ii) Let n be the length of time for which there is no calls, and Y be the number of calls received in n minutes.

$$Y \sim Po(3n)$$

$$P(Y = 0) = 0.2$$

From GC,



$$n = 0.53648 \text{ min} = 32 \text{ s}$$

- (iii) Let A be the number of calls received in 12 hours.

$$A \sim Po(3 \times 60 \times 12) \\ \Rightarrow A \sim Po(2160)$$

$$\lambda = 2160 > 10$$

$$\therefore A \sim N(2160, 2160) \text{ approx.}$$

$$P(A > 2200) = P(A \geq 2201) \\ = P(A > 2200.5) \text{ (c.c)} \\ = 0.192$$

- (iv) Let C be the number of working days out of 6 that are busy.

$$C \sim B(6, 0.19176)$$

$$P(C = 2) = 0.235$$

- (v) Let D be the number of working days out of 30 that are busy.

$$D \sim B(30, 0.19176)$$

$$n = 30 \text{ (large); } np = 5.75 > 5; n(1 - p) = 24.2 > 5$$

$$E(D) = np = 5.75, \quad Var(D) = np(1 - p) = 4.6496$$

$$\therefore D \sim N(5.75, 4.6496) \text{ approx.}$$

$$P(D < 10) = P(X \leq 9) \\ = P(X < 9.5) \text{ (c.c)} \\ = 0.959$$