A-LEVEL H2 MATH 2025 – PAPER 2

Question 1

[Ans: (a) $c=7$, $t=3$, $b=5$ (b) 6 cars, 7 trains, 10 boats]

(a) $8c+11t+5b=114$
$5c+14t+7b=112$
$9c+9t+4b=110$

Solving, $c=7$, $t=3$, $b=5$

(b) Sam score is 113 since he came in second.

Let the number of cars, trains and boats be $x+6$, $y+6$ and $z+6$ respectively, where $x\in\mathbb{Z}_0^+$, $y\in\mathbb{Z}_0^+$, $z\in\mathbb{Z}_0^+$, $x\neq y\neq z$ and $y<z$.

$7(x+6)+3(y+6)+5(z+6)=113$
$7x+3y+5z=23$

By observation, $x=0$, $y=1$ and $z=4$.

Sam has 6 cars, 7 trains, and 10 boats.

Question 2

[Ans: (a) $\begin{pmatrix}5\\10\\14\end{pmatrix}$ (b) $c=11$ (c) $52.7°$]

(a) $3\overrightarrow{QR}=4\overrightarrow{PQ}\Rightarrow 3\left(\overrightarrow{OR}-\overrightarrow{OQ}\right)=4\left(\overrightarrow{OQ}-\overrightarrow{OP}\right)$

$$3\left[\overrightarrow{OR}-\begin{pmatrix}1\\6\\6\end{pmatrix}\right]=4\left[\begin{pmatrix}1\\6\\6\end{pmatrix}-\begin{pmatrix}-2\\3\\0\end{pmatrix}\right]$$

$$3\overrightarrow{OR}-\begin{pmatrix}3\\18\\18\end{pmatrix}=4\begin{pmatrix}3\\3\\6\end{pmatrix}$$

$$3\overrightarrow{OR}=\begin{pmatrix}15\\30\\42\end{pmatrix}\Rightarrow\overrightarrow{OR}=\begin{pmatrix}5\\10\\14\end{pmatrix}$$

(b) $\overrightarrow{SQ}\cdot\overrightarrow{PQ}=0$

$$\left(\overrightarrow{OQ}-\overrightarrow{OS}\right)\cdot\left(\overrightarrow{OQ}-\overrightarrow{OP}\right)=0$$

$$\left[\begin{pmatrix}1\\6\\6\end{pmatrix}-\begin{pmatrix}-1\\-2\\c\end{pmatrix}\right]\cdot\left[\begin{pmatrix}1\\6\\6\end{pmatrix}-\begin{pmatrix}-2\\3\\0\end{pmatrix}\right]=0$$

$$\begin{pmatrix}2\\8\\6-c\end{pmatrix}\cdot\begin{pmatrix}3\\3\\6\end{pmatrix}=0$$

$$6+24+36-6c=0\Rightarrow c=11$$

(c) Let the required angle be θ.

$$\cos\theta=\frac{\overrightarrow{PS}\cdot\overrightarrow{PQ}}{\left|\overrightarrow{PS}\right|\left|\overrightarrow{PQ}\right|}$$

$$\overrightarrow{PS}=\begin{pmatrix}-1\\-2\\11\end{pmatrix}-\begin{pmatrix}-2\\3\\0\end{pmatrix}=\begin{pmatrix}1\\-5\\11\end{pmatrix}$$

$$\cos\theta=\frac{\begin{pmatrix}1\\-5\\11\end{pmatrix}\cdot\begin{pmatrix}3\\3\\6\end{pmatrix}}{\sqrt{1^2+5^2+11^2}\sqrt{3^2+3^2+6^2}}=\frac{54}{\sqrt{147}\sqrt{54}}$$

$$\therefore\theta=52.7°$$

Question 3

[Ans: (a) show, $A=3$, $B=4$ (b)(i) sketch, $c=\frac{3}{5}$, $c=1$ (ii) $x=\frac{k\pi}{3}$, $k\in\mathbb{Z}$

(iii) $\mathrm{f}\left(x+\pi\right)=\frac{3}{4-\cos 3x}$ (c) show, $P=3\cos 3x$, $Q=2\sin^2 3x$]

(a) $\frac{dy}{dx}=y^2\sin 3x$

$\int\frac{1}{y^2}dy=\int\sin 3x dx$

$-\frac{1}{y}=-\frac{1}{3}\cos 3x+C$

When $x=\pi$, $y=1$,

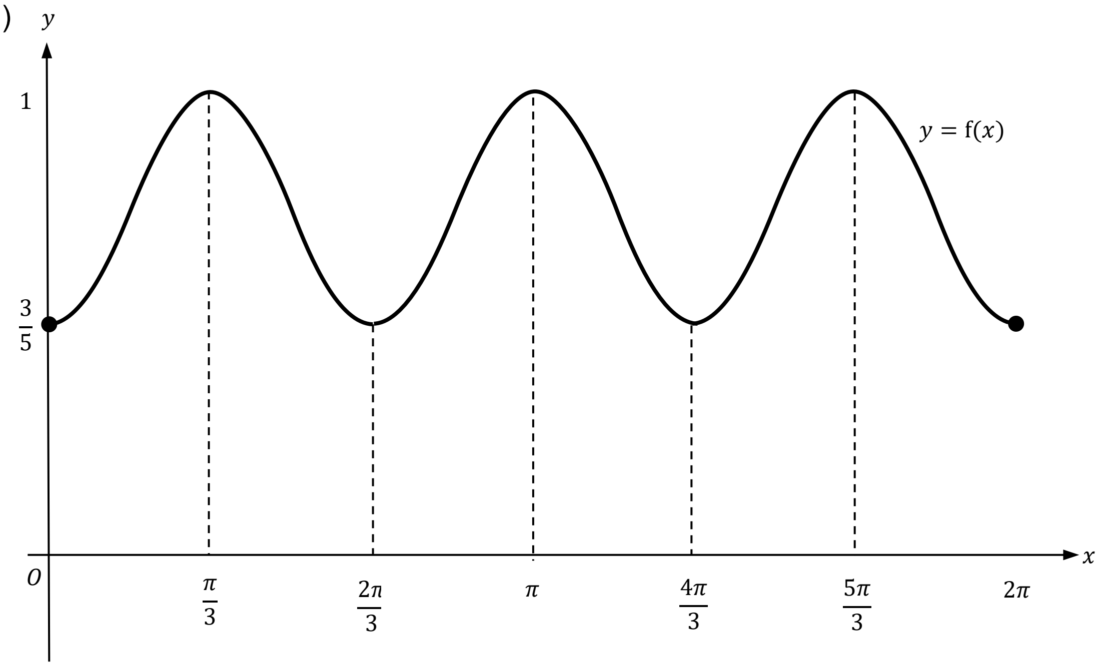
$-1=-\frac{1}{3}\left(-1\right)+C\Rightarrow C=-\frac{4}{3}$

$-\frac{1}{y}=-\frac{1}{3}\cos 3x-\frac{4}{3}$

$\frac{1}{y}=\frac{1}{3}\cos 3x+\frac{4}{3}$

$\therefore y=\frac{3}{4+\cos 3x}$, $A=3$, $B=4$

(b) (i)



From the graph if $y=c$ is a tangent to the curve, then $c=\frac{3}{5}$ or $c=1$.

(ii) When considering $y=\mathrm{f}\left(x\right)$, we are taking $x\in\mathbb{R}$ as the domain. The axis of symmetry will be at ..., $-\frac{3\pi}{3}$, $-\frac{2\pi}{3}$, $\frac{\pi}{3}$, 0, $\frac{\pi}{3}$, $\frac{2\pi}{3}$, $\frac{3\pi}{3}$, ... $\Rightarrow x=\frac{k\pi}{3}$, $k\in\mathbb{Z}$.

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(iii) $\mathrm{f}(x+\pi)=\dfrac{3}{4+\cos\left[3(x+\pi)\right]}$

$=\dfrac{3}{4+\cos(3x+3\pi)}$

$=\dfrac{3}{4+\cos 3x\cos 3\pi-\sin 3x\sin 3\pi}$

$=\dfrac{3}{4-\cos 3x}$

(c) $\dfrac{d^2y}{dx^2}=y^2(3\cos 3x)+\sin 3x\left(2y\dfrac{dy}{dx}\right)$

$\dfrac{d^2y}{dx^2}=(3\cos 3x)y^2+\sin 3x\left[2y\left(y^2\sin 3x\right)\right]$

$\dfrac{d^2y}{dx^2}=(3\cos 3x)y^2+\left(2\sin^2 3x\right)y^3$, where $P=3\cos 3x$, $Q=2\sin^2 3x$

Question 4

[Ans: (a) 225 s (b) $\frac{5}{108}$ cm/s (c) 13.9 s (d) 0.738 cm/s]

(a) Volume $= \frac{1}{3}\pi\left(15^2\right)(45-15) = 2250\pi$

Time taken $= \frac{2250\pi}{10\pi} = 225$

(b) $V = \frac{1}{3}\pi h^2(45-h) = \frac{1}{3}\pi\left(45h^2 - h^3\right)$

$\frac{dV}{dt} = \frac{1}{3}\pi\left(90h - 3h^2\right)\frac{dh}{dt}$

$\frac{dV}{dt} = \pi\left(30h - h^2\right)\frac{dh}{dt}$

$10\pi = \pi\left[30(12) - 12^2\right]\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{5}{108}$

(c) $\frac{dV}{dt} = 10\pi t$

$V = 5\pi t^2 + C$

When $t = 0, V = 0 \Rightarrow C = 0$

$\Rightarrow V = 5\pi t^2$

When $V = 972\pi$,

$972\pi = 5\pi t^2$

$t = 13.943 \approx 13.9$

(d) $\frac{dV}{dt} = \pi\left(30h - h^2\right)\frac{dh}{dt}$ (from Part (b))

$10\pi t = \pi\left(30h - h^2\right)\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{10t}{30h - h^2}$

When $V = 972\pi$,

$\frac{1}{3}\pi h^2(45-h) = 972\pi$

From GC, $h = 9$ or $h = -7.46$ (NA) or $h = 43.5$ (NA)

$\therefore \frac{dh}{dt} = \frac{10(13.943)}{30(9)-(9)^2} = 0.738$

Question 5

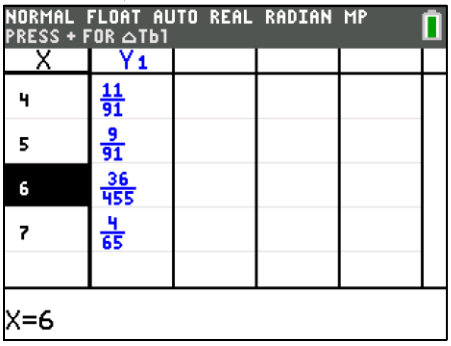
[Ans: (a) $\frac{71}{105}$ (b) 6]

(a) Probability $= \dfrac{{}^{3}C_1\,{}^{5}C_1 + {}^{3}C_1\,{}^{7}C_1 + {}^{5}C_1\,{}^{7}C_1}{{}^{15}C_2} = \dfrac{71}{105}$

(b) Probability that his first red counter is the n^{th} counter

$$= \frac{36}{455} = \frac{{}^{12}C_{n-1}}{{}^{15}C_{n-1}}\,\frac{3}{15-(n-1)}$$

From GC,



$\therefore n = 6$

Question 6

[Ans: (a) 0.05 (b) 9.96 (c) 6.29 (d) 3.93]

(a) $P(X<7)=0.5-0.45=0.05$

(b) $X \sim N(\mu, 1.8^2)$

$P(X<7)=0.05$

$P\left(Z<\frac{7-\mu}{1.8}\right)=0.05$

$\frac{7-\mu}{1.8}=-1.6449$

$\mu=9.96$

(c) $Y \sim N(\lambda, 2.8^2)$

Since X and Y are independent,

$P(X>7)P(Y>7)=0.38$

$(1-0.05)P(Y>7)=0.38$

$P(Y>7)=0.4$

$P\left(Z>\frac{7-\lambda}{2.8}\right)=0.4$

$\frac{7-\lambda}{2.8}=0.25335$

$\lambda=6.2906 \approx 6.29$

(d) $Y \sim N(6.2906, 2.8^2)$

$P(Y>a)=2P(Y>7)$

$P(Y>a)=2(0.4)=0.8$

From GC, $a=3.93$

Question 7

[Ans: (a) 10080 (b) 720 (c) 1080 (d) $\frac{1}{3}$]

(a) No. of ways $=\frac{8!}{2!2!}=10080$

(b) No. of ways $={}^{3}C_{2}\,2!\,{}^{4}C_{1}\,\frac{5!}{2!2!}=720$

(c) No. of ways $=\frac{6!}{2!2!}3!=1080$

(d) Probabilty $=P(\text{odd digits together | even digits together})$

$$=\frac{P(\text{odd digits together and even digits together})}{P(\text{even digits together})}$$

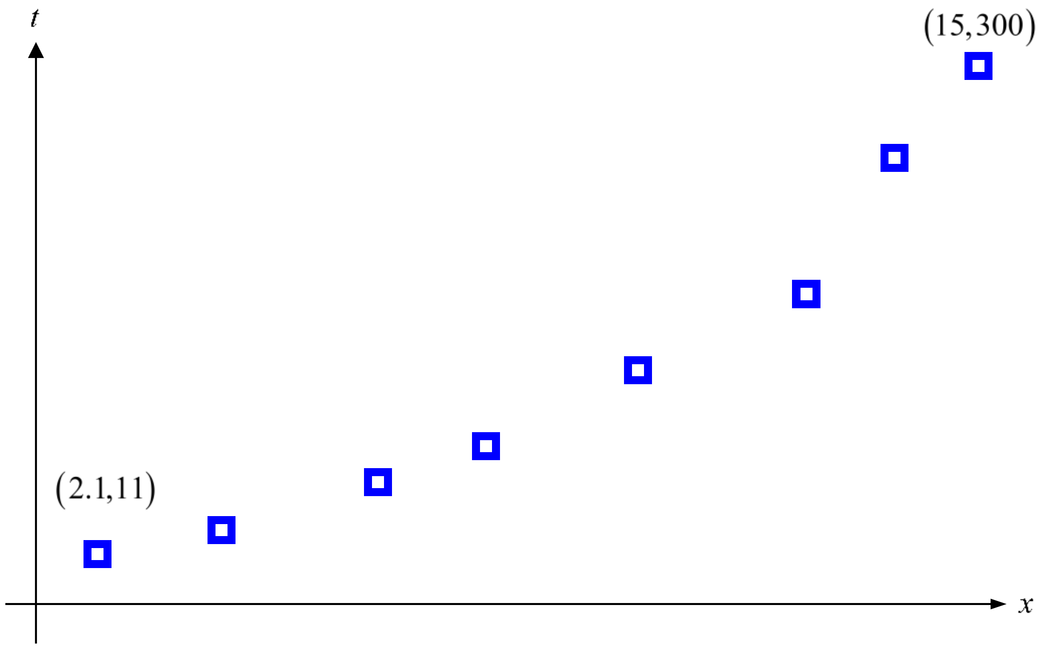
$$=\frac{\left(\dfrac{2!\frac{5!}{2!2!}3!}{10080}\right)}{\left(\dfrac{1080}{10080}\right)}=\frac{1}{3}$$

Question 8

[Ans: (a) justify (b)(i) sketch; state (ii)(A) $t = cx^2 + d$ (ii)(B) $t = 1.25x^2 + 0.876$ (iii)(A) 152 mins (iii)(B) explain]

(a) Since $r = 0.39$, it suggests a weak positive linear correlation between the best performances of high jumpers and long jumpers. Also the scattar diagram only shows some form of positive correlation but with only little linear correlation between the two.

(b) (i)



The scattar diagram exhibit a positive non-linear correlation between x and t. As x increases, t increases at an increasing rate.

(ii) (A) By using GC, for the model $t = ax + b$, the value of $r = 0.958$. For the model $t = cx^2 + d$, $r = 0.990$. Since the value of $|r|$ for the model $t = cx^2 + d$ is closer to 1, it will be a better fit.

(B) From GC, $t = 1.2497x^2 + 0.87580 \Rightarrow t = 1.25x^2 + 0.876$

(iii) (A) When $x = 11$,

$$t = 1.2497(11)^2 + 0.87580$$
$$\therefore t = 152$$

(B)

- The value of $|r|$ between x^2 and t is 0.990 which is close to 1 which implies a strong positive linear correlation.
- The value of $x = 11$ which is used for the estimation falls within the data range of $2.1 \le x \le 15$. Therefore the estimation is reliable.

Question 9

[Ans: (a) state (b) 0.540 (c) 0.142 (d) 0.587 (e) 0.177]

(a) The probability that a refrigerator is faulty is constant for every refrigerator.
The event that a refrigerator is faulty or not faulty is independent from each other.

(b) $X \sim B(90, 0.02)$

$$\begin{aligned} P(X > 1) &= 1 - P(X \le 1) \\ &= 0.53957 \\ &\approx 0.540 \end{aligned}$$

(c) Let W be the number of days with more than 1 faulty refrigerator out of 5 days.

$W \sim B(5, 0.53957)$

$$\begin{aligned} P(W < 2) &= P(W \le 1) \\ &= 0.14194 \\ &\approx 0.142 \end{aligned}$$

(d) Let T be the number of faulty refrigerators out of 450 days.

$T \sim B(450, 0.02)$

$$\begin{aligned} P(T < 10) &= P(T \le 9) \\ &= 0.58741 \\ &\approx 0.587 \end{aligned}$$

(e) $Y \sim B(60, 0.03)$

$$\begin{aligned} P(\text{required}) &= P(X = 2, Y = 0) + P(X = 0, Y = 2) + P(X = 1, Y = 1) \\ &= P(X = 2)P(Y = 0) + P(X = 0)P(Y = 2) + P(X = 1)P(Y = 2) \\ &= 0.177 \end{aligned}$$

Question 10

[Ans: (a) explain (b) 0.814 (c) 0.686 (d) state (e) $\bar{x}=2.15$, $s^2=0.015$
(f) $p=0.0253$, Reject H_0 (g) explain (h) explain]

(a) The production of the packets of flour are from 2 different machines P and Q which are not related to each other. So it is reasonable to assume that the 2 distributions are independent of each other.

(b) Let P and Q be the mass of a packet of flour produced by machine P and machine Q respectively.

$P \sim N\left(2.2, 0.1^2\right)$; $Q \sim N\left(2.1, 0.05^2\right)$

$P - Q \sim N\left(2.2 - 2.1, 0.1^2 + 0.05^2\right) \Rightarrow P - Q \sim N\left(0.1, 0.0125\right)$

$$P\left(P > Q\right) = P\left(P - Q > 0\right)$$
$$= 0.81445 \approx 0.814$$

(c) Let T be the total mass of 3 randomly packets from P and 5 randomly packets from Q.

$T = P_1 + P_2 + P_3 + Q_1 + Q_2 + Q_3 + Q_4 + Q_5$

$T \sim N\left(3(2.2) + 5(2.1), 3(0.1)^2 + 5(0.05)^2\right) \Rightarrow T \sim N\left(17.1, 0.0425\right)$

$P\left(T > 17\right) = 0.68619 \approx 0.686$

(d) Let μ be the population mean mass of the packets of flour produced by machine P after the adjustment.

$H_0 : \mu = 2.2$
$H_1 : \mu \neq 2.2$

(e) $\bar{x} = \dfrac{\sum(x-2)}{30} + 2$

$= \dfrac{4.5}{30} + 2$

$= 2.15$

$$s^2 = \frac{1}{n-1}\left[\sum(x-2)^2 - \frac{\left[\sum(x-2)\right]^2}{30}\right]$$

$$= \frac{1}{29}\left[1.11 - \frac{4.5^2}{30}\right]$$

$$= 0.015$$

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(f) Under H_0 and since $n = 30$ is large, by Central Limit Theorem

$$\overline{X} \sim N\left(2.2, \frac{0.015}{30}\right)$$ approximately.

Test statistics: $Z = \dfrac{\overline{x} - 2.2}{\sqrt{\dfrac{0.015}{30}}} \sim N(0,1)$

Using GC, p - value $= 0.025347 \approx 0.0253 < 0.05$

We reject H_0 at 5% level of significance. There is sufficient evidence to conclude that the population mean mass of packets of flour produced by machine P differs from 2.2 kg after the adjustment.

(g) The manager would have less confidence if fewer than 30 packets of flour had been used as $\overline{X}$ may not be a good approximation to a normal distribution as the criteria for Central Limt Theorem is only applicable when $n \geq 30$.

(h) In order to use Central Limit Theorem, one of the criteria is that the sample chosen must be random. If the sample is not random then the application of Central Limit Theorem will no longer be accurate as the sample will not be a good representation of the population of the packets of flour produced by machine P after the ajustment.