# O-LEVEL ADDITIONAL MATH 2025 – PAPER 2

Question 1

[ Ans: $a=10$ , $b=3$ ]

$$\frac{1}{2}\left(4-\sqrt{3}\right)\left[\left(6+\sqrt{3}\right)+l\right]=26$$

$$6+\sqrt{3}+l=\frac{52}{4-\sqrt{3}}$$

$$l=\frac{52\left(4+\sqrt{3}\right)}{\left(4-\sqrt{3}\right)\left(4+\sqrt{3}\right)}-6-\sqrt{3}$$

$$=\frac{208+52\sqrt{3}}{13}-6-\sqrt{3}$$

$$=16+4\sqrt{3}-6-\sqrt{3}$$

$$=10+3\sqrt{3}$$

$$\therefore a=10,\ b=3$$

Question 2

[ Ans: (a) $T=6-\frac{1}{2}(h-2)^2$ ; $h=2$ , $T_{\max}=6$ (b) $h=2$ , $T_{\max}=3$ ]

(a) $T=4+2h-\frac{1}{2}h^2$

$$T=-\frac{1}{2}\left(h^2-4h\right)+4$$

$$=-\frac{1}{2}\left[\left(h-2\right)^2-4\right]+4$$

$$=6-\frac{1}{2}\left(h-2\right)^2$$

$\therefore A=6$, $B=\frac{1}{2}$, $C=-2$

Greatest Temperature $T_{\max}=6$ when $h=2$.

(b) $T_{\text{new}}=\frac{1}{2}\left[6-\frac{1}{2}\left(h-2\right)^2\right]=3-\frac{1}{4}\left(h-2\right)^2$

Greatest Temperature $T_{\max}=3$ when $h=2$.

Question 3

[ Ans: $y = e^{3x} + \frac{1}{3}e^{-3x} - \frac{1}{2}\sin 2x - \frac{4}{3}$ ]

$$\frac{dy}{dx} = e^{x}\left(3e^{2x} - e^{-4x}\right) - \cos 2x$$

$$y = \int e^{x}\left(3e^{2x} - e^{-4x}\right) - \cos 2x dx$$

$$= \int 3e^{3x} - e^{-3x} - \cos 2x dx$$

$$= e^{3x} + \frac{1}{3}e^{-3x} - \frac{1}{2}\sin 2x + C$$

When $x = 0$, $y = 0$,

$$0 = e^{0} + \frac{1}{3}e^{0} - \frac{1}{2}\sin 0 + C$$

$$C = -\frac{4}{3}$$

$$\therefore y = e^{3x} + \frac{1}{3}e^{-3x} - \frac{1}{2}\sin 2x - \frac{4}{3}$$

Question 4

[ Ans: (a) $p = 2$, $q = -5$ (b) $g(x) = (3x-1)(3x+1)(x-2)$ ]

(a) $f(x) = x^{3} + px + qx - 6$

$$f(-1) = (-1)^{3} + p(-1)^{2} + q(-1) - 6 = 0$$

$p - q = 7$ --- (1)

$$f(-2) = (-2)^{3} + p(-2)^{2} + q(-2) - 6 = 4$$

$2p - q = 9$ --- (2)

(2) − (1) $p = 2$

From (1) $2 - q = 7 \Rightarrow q = -5$

(b) $g(x) = (3x-1)(3x^{2} + Bx - 2)$

$$9x^{3} - 18x^{2} - x + 2 = (3x-1)(3x^{2} + Bx - 2)$$

Coefficient of $x^{2}$: $-18 = 3B - 3 \Rightarrow B = -5$

$$g(x) = (3x-1)(3x^{2} - 5x - 2)$$

$$= (3x-1)(3x+1)(x-2)$$

Question 5

[ Ans: prove ]

$$y=\frac{1-e^{-x}}{1+e^{-x}}$$

$$\frac{dy}{dx}=\frac{\left(1+e^{-x}\right)\left(e^{-x}\right)-\left(1-e^{-x}\right)\left(-e^{-x}\right)}{\left(1+e^{-x}\right)^2}$$

$$=\frac{e^{-x}+e^{-2x}+e^{-x}-e^{-2x}}{\left(1+e^{-x}\right)^2}$$

$$=\frac{2e^{-x}}{\left(1+e^{-x}\right)^2}$$

Since $e^{-x}>0$ and $\left(1+e^{-x}\right)^2>0 \Rightarrow \frac{dy}{dx}>0$

$\therefore y$ increases for all real values of $x$. (proven)

Question 6

[ Ans: $\theta=45^\circ$, $116.6^\circ$, $225^\circ$, $296.6^\circ$ ]

$2\operatorname{cosec}^2\theta-\cot\theta-3=0$

$2\left(1+\cot^2\theta\right)-\cot\theta-1=0$

$2\cot^2\theta-\cot\theta-1=0$

$\left(2\cot\theta+1\right)\left(\cot\theta-1\right)=0$

$\cot\theta=-\frac{1}{2}$ or $\cot\theta=1$

$\tan\theta=-2$

Basic Angle $=63.435^\circ$

$\Rightarrow \theta=180^\circ-63.435^\circ=116.6^\circ$ (2nd Quad.)

$\Rightarrow \theta=360^\circ-63.435^\circ=296.6^\circ$ (4th Quad.)

or

$\tan\theta=1$

Basic Angle $=45^\circ$

$\Rightarrow \theta=45^\circ$ (1st Quad.)

$\Rightarrow \theta=180^\circ+45^\circ=225^\circ$ (4th Quad.)

$\therefore \theta=45$, $116.6^\circ$, $225^\circ$, $296.6^\circ$

Question 7

[ Ans: $v = 2.95$ m/s ]

$$a = (2t+1)^{-1} + 0.3t + 0.2$$
$$v = \int \frac{1}{2t+1} + 0.3t + 0.2 dt$$
$$= \frac{1}{2}\ln(2t+1) + \frac{1}{2}(0.3)t^2 + 0.2t + C$$

When $t = 0$, $v = -3$,
$$\Rightarrow -3 = C$$

$$\therefore v = \frac{1}{2}\ln(2t+1) + \frac{1}{2}(0.3)t^2 + 0.2t - 3$$

When $t = 5$,
$$v = \frac{1}{2}\ln(2(5)+1) + \frac{1}{2}(0.3)(5)^2 + 0.2(5) - 3$$
$$= 2.95$$

Question 8

[ Ans: (a) $p = 2.12$ (b) $x = 2$, $y = -1$ ]

(a) $2^p = 5(3^{2-p}) = \frac{5(3^2)}{3^p}$

$$6^p = 45 \Rightarrow \lg 6^p = \lg 45 \Rightarrow p = \frac{\lg 45}{\lg 6} = 2.12$$

(b) $9(2^x) + 2(3^{y+1}) = 38$

$$9(2^x) + 2(3^y 3^1) = 38 \Rightarrow 9(2^x) + 6(3^y) = 38 \text{ --- (1)}$$

$$15(2^{x-1}) - 21(3^y) = 23$$
$$15\left(\frac{2^x}{2}\right) - 21(3^y) = 23 \Rightarrow \frac{15}{2}(2^x) - 21(3^y) = 23 \text{ --- (2)}$$

From (1) $\quad 3^y = \frac{38 - 9(2^x)}{6}$ --- (3)

Sub. (3) into (2) $\quad \frac{15}{2}(2^x) - 21\left(\frac{38 - 9(2^x)}{6}\right) = 23$

$$\frac{15}{2}(2^x) - \frac{7}{2}(38 - 9(2^x)) = 23$$
$$\frac{15}{2}(2^x) - 133 + \frac{63}{2}(2^x) = 23$$
$$2^x = 4 \Rightarrow x = 2$$

From (3) $\quad 3^y = \frac{38 - 9(2^2)}{6} = \frac{1}{3} \Rightarrow y = -1$

Question 9

[ Ans: (a) $k=10-\alpha$, show (b) $\frac{5}{2}$ ]

(a) The line $y=kx$ and curve $y=x(10-x)$ will intersect when $x=\alpha$.

$\Rightarrow k\alpha=\alpha(10-\alpha)$

$\therefore k=10-\alpha \quad (\because \alpha \neq 0)$

$$R_1=\int_0^\alpha x(10-x)-kx\,dx$$
$$=\int_0^\alpha 10x-x^2-kx\,dx$$
$$=\left[5x^2-\frac{x^3}{3}-\frac{kx^2}{2}\right]_0^\alpha$$
$$=5\alpha^2-\frac{\alpha^3}{3}-\frac{k\alpha^2}{2}-0$$
$$=5\alpha^2-\frac{\alpha^3}{3}-\frac{(10-\alpha)\alpha^2}{2}$$
$$=5\alpha^2-\frac{\alpha^3}{3}-5\alpha^2+\frac{\alpha^3}{2}$$
$$=\frac{\alpha^3}{6} \quad \text{(shown)}$$

(b) $R_2=\frac{1}{2}\alpha(k\alpha)=\frac{1}{2}k\alpha^2$

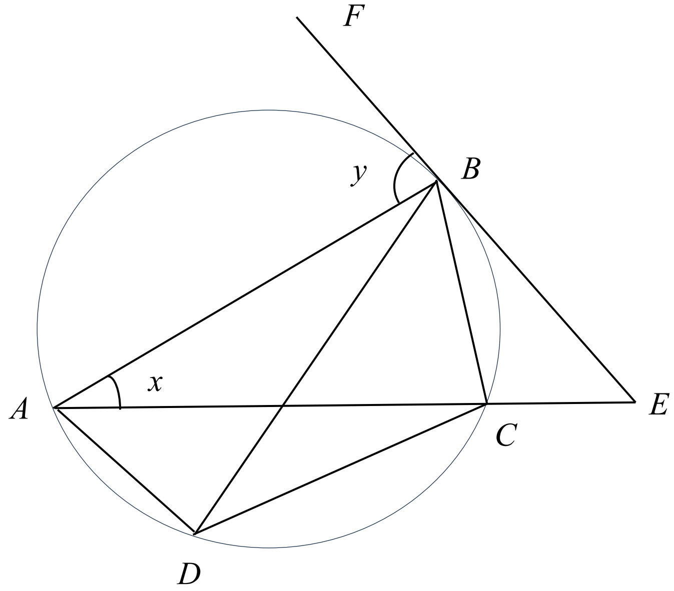
If $R_1=R_2$,

$$\frac{\alpha^3}{6}=\frac{1}{2}k\alpha^2$$
$$\frac{\alpha}{6}=\frac{1}{2}k \quad (\because \alpha \neq 0)$$
$$\frac{10-k}{6}=\frac{1}{2}k$$
$$10-k=3k$$
$$\therefore k=\frac{5}{2}$$

Question 10

[ Ans: (a) prove (b)(i) prove (ii) prove ]

(a)




(b) $\measuredangle CBE = \measuredangle BAC = x$ (Tangent Chord Theorem)

$\measuredangle BEC$ is the common angle $= y - x$ (ext $\measuredangle =$ sum of int. opp. $\measuredangle$ in $\Delta ABE$ )

Since 2 pairs of angles in $\Delta ABE$ and $\Delta BCE$ are the same, by AA Similarity, $\Delta ABE$ is similar to $\Delta BCE$.

(i) $\measuredangle ABC = 180° - x - y$ $(\measuredangle \text{ in a straight line})$

$\measuredangle ADC = 180° - (180° - x - y)$

$= x + y$ (opp $\measuredangle$ in cyclic quad add up to $180°$)

(proven)

(ii) To prove $AD = BC$, we will show that $\Delta ADC$ is congruent with $\Delta BCD$.

$\measuredangle BDC = \measuredangle BAC = x$ ($\measuredangle$ in the same segment)

$\measuredangle ACD = \measuredangle BAC$ (alt $\measuredangle$)

$CD$ is the common side for both $\Delta ABE$ and $\Delta BCE$.

$\measuredangle BCA = y$ (ext $\measuredangle =$ sum of int opp $\measuredangle$ in $\Delta BCE$)

$\measuredangle BCD = y + x = \measuredangle ADC$

$\therefore \Delta ADC$ is congruent with $\Delta BCD$, by ASA property. $\Rightarrow AD = BC$ (proven)

Question 11

[ Ans: (a) explain (b) $3.13$ s (c)(i) show (ii) speed $A = 5.87$ cm/s, speed $B = 1.63$ cm/s ]

(a) As both the speed of $A$ and $B$ must be positive, $\theta$ must be acute.

(b) If $A$ and $B$ collide at the midpoint it will mean that $A$ and $B$ each would have travelled $15$ cm each.

$\Rightarrow \dfrac{15}{6\cos\theta} = \dfrac{15}{8\sin\theta}$

$120\sin\theta = 90\cos\theta$

$\tan\theta = \dfrac{3}{4}$

$\theta = 36.870°$

$\therefore$ Time taken $= \dfrac{15}{6\cos(36.870°)} = 3.13$

(c) (i) $4(6\cos\theta) + 4(8\sin\theta) = 30$

$24\cos\theta + 32\sin\theta = 30$

$3\cos\theta + 4\sin\theta = 3.75$ (shown)

(ii) $3\cos\theta + 4\sin\theta = R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$R\cos\alpha = 3$ --- (1)

$R\sin\alpha = 4$ --- (2)

$(1)^2 + (2)^2$ $\quad R^2 = 3^2 + 4^2$

$\Rightarrow R = 5$

$\dfrac{(2)}{(1)}$ $\quad \tan\alpha = \dfrac{4}{3} \Rightarrow \alpha = 53.130°$

$\therefore 5\cos(\theta - 53.130°) = 3.75$

$\cos(\theta - 53.130°) = \dfrac{3.75}{5}$

Basic Angle $= 41.410°$

$\theta - 53.130° = 41.410°$ or $\theta - 53.130° = -41.410°$

$\therefore \theta = 94.540°$ (rejected) or $\theta = 11.720°$

Speed $A = 6\cos(11.720°) = 5.87$ cm/s

Speed $B = 8\sin(11.720°) = 1.63$ cm/s