O-LEVEL ADDITIONAL MATH 2025 – PAPER 1

Question 1

[Ans: 18]

Let P be the price of the item.

By the end of the 1st month, $P = 300(0.95)$

By the end of the 2nd month, $P = 300(0.95)^2$

By the end of the n th month, $P = 300(0.95)^n$

$$300(0.95)^n < 120$$
$$(0.95)^n < 0.4$$
$$\lg(0.95)^n < \lg 0.4$$
$$n \lg(0.95) < \lg 0.4$$
$$n > \frac{\lg 0.4}{\lg 0.95}$$
$$n > 17.864$$
$$\therefore n = 18$$

18 complete months have elapsed before the item is sold.

Question 2

[Ans: $k < -\frac{1}{2}$]

Let $kx^2 + 2x + k + 1 = x + 1$

$kx^2 + x + k = 0$

For $y = kx^2 + 2x + k + 1$ to be completely below $y = x + 1$,

$k < 0$ and $(kx^2 + x + k)$'s Discriminant < 0

$$(1)^2 - 4(k)(k) < 0$$
$$4k^2 - 1 > 0$$
$$(2k+1)(2k-1) > 0$$
$$\therefore k < -\frac{1}{2} \text{ or } k > \frac{1}{2}$$

$\therefore k < -\frac{1}{2}$ ($\because k < 0$)

Question 3

[Ans: (a)(i) $(a-b)(a^3+ab+b^2)$ (ii) 61 or 301447 (b) prove]

(a) (i) $a^3-b^3=(a-b)(a^2+ab+b^2)$

(ii) $347^3-286^3=(347-286)\left[347^2+(347)(286)+286^2\right]=(61)(301447)$

$\therefore$ One of the factors is 61.

(b) $\dfrac{(x+n)^3}{2}-\dfrac{(x-n)^3}{2}$

$=\dfrac{1}{2}\left[(x+n)^3-(x-n)^3\right]$

$=\dfrac{1}{2}\left[x+n-(x-n)\right]\left[(x+n)^2+(x+n)(x-n)+(x-n)^2\right]$

$=\dfrac{1}{2}(2n)\left[(x^2+2nx+n^2)+(x^2-n^2)+(x^2-2nx+n^2)\right]$

$=n(3x^2+n^2)$

Since n is a factor of $\dfrac{(x+n)^3}{2}-\dfrac{(x-n)^3}{2}$, it is divisible by n. (proven)

Question 4

[Ans: (a) $x^n+(n+1)x^n\ln x$ (b) $\dfrac{1}{n+1}x^{n+1}\ln x-\dfrac{1}{(n+1)^2}x^{n+1}+C$]

(a) $\dfrac{d}{dx}\left(x^{n+1}\ln x\right)=x^{n+1}\left(\dfrac{1}{x}\right)+(n+1)x^n\ln x=x^n+(n+1)x^n\ln x$

(b) From part (a),

$$\frac{d}{dx}\left(x^{n+1}\ln x\right)=x^n+x^n(n+1)\ln x$$

$$x^{n+1}\ln x=\int x^n+(n+1)x^n\ln x\,dx$$

$$x^{n+1}\ln x=\int x^n dx+\int(n+1)x^n\ln x\,dx$$

$$(n+1)\int x^n\ln x\,dx=x^{n+1}\ln x-\int x^n dx$$

$$(n+1)\int x^n\ln x\,dx=x^{n+1}\ln x-\frac{1}{n+1}x^{n+1}$$

$$\int x^n\ln x\,dx=\frac{1}{n+1}x^{n+1}\ln x-\frac{1}{(n+1)^2}x^{n+1}+C$$

Question 5

[Ans: (a) $\ln x$ (b) $A = 49.4$, $b = 2$]

(a) $y = Ax^b$

$\ln y = \ln\left(Ax^b\right)$

$\ln y = \ln A + \ln x^b$

$\ln y = b\ln x + \ln A$

$\therefore \ln x$ is plotted on the X-axis.

(b) Equation of straight line: $Y = bX + \ln A$

Gradient of the line, $b = \dfrac{12.1 - 6.3}{4.1 - 1.2} = 2$

$\therefore Y = 2X + \ln A$

When $X = 1.2$, $Y = 6.3$,

$6.3 = 2(1.2) + \ln A$

$\therefore A = e^{6.3 - 2(1.2)} = 49.4$

Question 6

[Ans: (a) show (b) minimum point]

(a) $y = \dfrac{1}{2}x^2 - 3\sqrt{x} + 1 = \dfrac{1}{2}x^2 - 3x^{\frac{1}{2}} + 1$

$\dfrac{dy}{dx} = \dfrac{1}{2}(2x) - 3\left(\dfrac{1}{2}\right)x^{-\frac{1}{2}} = x - \dfrac{3}{2\sqrt{x}}$

At stationary point, $\dfrac{dy}{dx} = 0$,

$x - \dfrac{3}{2\sqrt{x}} = 0 \Rightarrow x^{\frac{3}{2}} = \dfrac{3}{2}$

$\therefore x = \left(\dfrac{3}{2}\right)^{\frac{2}{3}}$ (shown)

(b) $\dfrac{dy}{dx} = x - \dfrac{3}{2\sqrt{x}} = x - \dfrac{3}{2}x^{-\frac{1}{2}}$

$\dfrac{d^2y}{dx^2} = 1 - \dfrac{3}{2}\left(-\dfrac{1}{2}x^{-\frac{3}{2}}\right) = 1 + \dfrac{3}{4\sqrt{x^3}}$

When $x = \left(\dfrac{3}{2}\right)^{\frac{2}{3}}$, $\dfrac{d^2y}{dx^2} = 1 + \dfrac{3}{4\sqrt{\left(\left(\dfrac{3}{2}\right)^{\frac{2}{3}}\right)^3}} = \dfrac{3}{2} > 0$

$\therefore$ It is a minimum point.

Question 7

[Ans: (a) $\frac{1}{32}-\frac{5a}{16}x+\frac{5a^2}{4}x^2-\frac{5a^3}{2}x^3+...$ (b) $a=-\frac{1}{3}$]

(a) $\left(\frac{1}{2}-ax\right)^5=\left(\frac{1}{2}\right)^5+\binom{5}{1}\left(\frac{1}{2}\right)^4(-ax)+\binom{5}{2}\left(\frac{1}{2}\right)^3(-ax)^2+\binom{5}{3}\left(\frac{1}{2}\right)^2(-ax)^3+...$

$=\frac{1}{32}-\frac{5a}{16}x+\frac{5a^2}{4}x^2-\frac{5a^3}{2}x^3+...$

(b) $\left(4+\frac{5}{ax}\right)\left(\frac{1}{2}-ax\right)^5=\left(4+\frac{5}{ax}\right)\left(\frac{1}{32}-\frac{5a}{16}x+\frac{5a^2}{4}x^2-\frac{5a^3}{2}x^3+...\right)$

Coefficient of $x:-\frac{20a}{16}+\frac{25a}{4}=5a$

Coefficient of $x^2:\frac{20a^2}{4}-\frac{25a^2}{2}=-\frac{15a^2}{2}$

$5a=2\left(-\frac{15}{2}a^2\right)$

$5a+15a^2=0\Rightarrow 5a(1+3a)=0$

$a=-\frac{1}{3}$ or $a=0$ (NA)

Question 8

[Ans: (a) $-\sqrt{3}<a<\sqrt{3}$ (b) $a=-\sqrt{3}$ or $a=\sqrt{3}$]

(a) $\frac{(x-a)^2}{2}+(y-1)^2=1$ --- (1) $\quad$ $y=x+1$ --- (2)

Sub. (2) into (1): $\frac{(x-a)^2}{2}+\left[(x+1)-1\right]^2=1$

$(x-a)^2+2x^2=2$

$(x^2-2ax+a^2)+2x^2=2$

$3x^2-2ax+a^2-2=0$

For 2 distinct intersection points,

$(3x^2-2ax+a^2-2)$'s Discriminant >0

$(-2a)^2-4(3)(a^2-2)>0$

$4a^2-12a^2+24>0$

$a^2-3<0$

$(a-\sqrt{3})(a+\sqrt{3})<0$

$\therefore -\sqrt{3}<a<\sqrt{3}$

(b) If $y=x+1$ is a tangent to the curve, $3x^2-2ax+a^2-2$ has Discriminant$=0$.

$(-2a)^2-4(3)(a^2-2)=0$

$\therefore a=-\sqrt{3}$ or $a=\sqrt{3}$

Question 9

[Ans: (a) prove (b) $\theta = \frac{5\pi}{4}$]

(a) LHS $= \frac{1+\sin 2\theta}{\sin\theta} - \frac{1+\cos 2\theta}{\cos\theta}$

$= \frac{\cos\theta + \sin 2\theta\cos\theta - \sin\theta - \sin\theta\cos 2\theta}{\sin\theta\cos\theta}$

$= \frac{\cos\theta + 2\sin\theta\cos^2\theta - \sin\theta - \sin\theta\left(2\cos^2\theta - 1\right)}{\sin\theta\cos\theta}$

$= \frac{\cos\theta + 2\sin\theta\cos^2\theta - \sin\theta - 2\sin\theta\cos^2\theta + \sin\theta}{\sin\theta\cos\theta} = \frac{\cos\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta} = \operatorname{cosec}\theta =$ RHS (proven)

(b) $\frac{1+\sin\theta}{\sin\theta} - \frac{1+\cos 2\theta}{\cos\theta} = 2\sin\theta$

$\operatorname{cosec}\theta = 2\sin\theta$

$\sin^2\theta = \frac{1}{2}$

$\sin\theta = -\frac{1}{\sqrt{2}}$ or $\sin\theta = \frac{1}{\sqrt{2}}$ (NA $\because \pi < \theta < \frac{3}{2}\pi$)

Basic angle $= \frac{\pi}{4}$

$\therefore \theta = \pi + \frac{\pi}{4} = \frac{5}{4}\pi$ ($\because \pi < \theta < \frac{3}{2}\pi$)

Question 10

[Ans: (a) $y = -\frac{1}{2a}x - 3$ (b) $a = 3$]

(a) $y = 2\sin ax - 3\cos 2x$

$\frac{dy}{dx} = 2a\cos ax + 6\sin 2x$

When $x = 0$, $y = 2\sin(0) - 3\cos(0) = -3$, $\frac{dy}{dx} = 2a\cos(0) + 6\sin(0) = 2a$

Equation of normal: $y - (-3) = -\frac{1}{2a}(x - 0) \Rightarrow y = -\frac{1}{2a}x - 3$

(b) $P(0, -3)$

At Q, $y = 0$,

$-\frac{1}{2a}x - 3 = 0 \Rightarrow x = -6a$ $\therefore Q(-6a, 0)$

$\frac{1}{2}(6a)(3) = 27$

$\therefore a = 3$

Question 11

[Ans: (a) 1.6 units/s (b) $x = 20$]

(a) Let $AB = y$

$$y^2 = x^2 + 3^2$$

$$y = \sqrt{x^2 + 9} = \left(x^2 + 9\right)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(x^2 + 9\right)^{-\frac{1}{2}}\left(2x\right) = \frac{x}{\sqrt{x^2 + 9}}$$

When $x = 4$,

$$\frac{dx}{dt} = 2 \text{ (given)}$$

$$\frac{dy}{dx} = \frac{4}{\sqrt{4^2 + 9}} = 0.8$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = 0.8 \times 2 = 1.6$$

(b) Let the area of the square be A.

$$A = y^2 = x^2 + 9$$

$$\frac{dA}{dx} = 2x$$

$$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt} \leq 80$$

$$2x\left(2\right) \leq 80$$

$$x \leq 20$$

$\therefore$ Maximum possible value of $x = 20$.

Question 12

[Ans: (a)(i) $x=\frac{\pi}{2k}$, $x=\frac{3\pi}{2k}$ (a)(ii) prove (b) amplitude $=3^{\frac{1}{4}}$, period $=\frac{4}{3^{\frac{1}{4}}}$]

(a) (i) $y=a\cos kx$

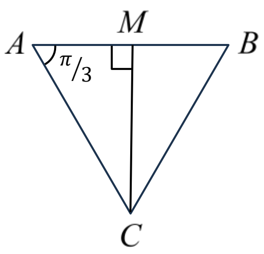
When $y=0$,

$$a\cos kx=0\Rightarrow\cos kx=0$$

$$kx=\frac{\pi}{2}\Rightarrow x=\frac{\pi}{2k} \text{ or } kx=\frac{3\pi}{2}\Rightarrow x=\frac{3\pi}{2k}$$

$\therefore$ x-coordinates of A and B are $\frac{\pi}{2k}$ and $\frac{3\pi}{2k}$ respectively.

(ii) Let M be the mid-point of A and B.



$$AM=\frac{\frac{3\pi}{2k}-\frac{\pi}{2k}}{2}=\frac{\pi}{2k},\ CM=a$$

$$\tan\frac{\pi}{3}=\frac{a}{\pi/2k}$$

$$a=\sqrt{3}\left(\frac{\pi}{2k}\right)=\frac{\pi\sqrt{3}}{2k} \text{ (shown)}$$

(b) Area of triangle $ABC=1$

$$\frac{1}{2}\left(\frac{\pi}{k}\right)\left(\frac{\pi\sqrt{3}}{2k}\right)=1$$

$$\frac{\pi^2\sqrt{3}}{4k^2}=1$$

$$k^2=\frac{\pi^2 3^{\frac{1}{2}}}{4}$$

$$k=-\frac{3^{\frac{1}{4}}\pi}{2} \text{ (NA } \because k>0\text{) or } k=\frac{3^{\frac{1}{4}}\pi}{2} \text{ (shown)}$$

$$\text{Amplitude }=a=\frac{\pi\sqrt{3}}{2\left(\frac{3^{\frac{1}{4}}\pi}{2}\right)}=3^{\frac{1}{4}}$$

$$\text{Period }=\frac{2\pi}{k}=\frac{2\pi}{\left(\frac{3^{\frac{1}{4}}\pi}{2}\right)}=\frac{4}{3^{\frac{1}{4}}}$$

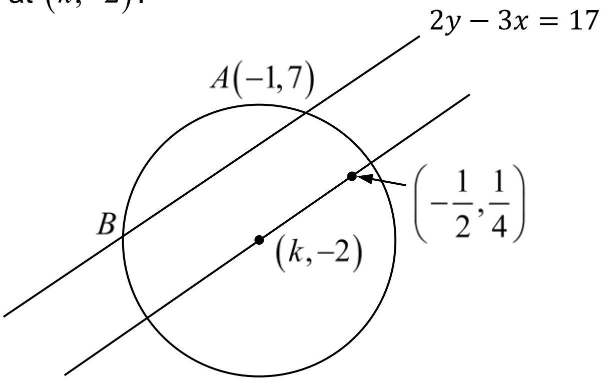
Question 13

[Ans: (a) $(x+2)^2+(y+2)^2=82$ (b) $-3<\alpha<-1$]

(a) $2y-3x=17$

$y=\frac{3}{2}x+\frac{17}{2}$

Let the center of the circle be at $(k,-2)$.



$$\frac{\frac{1}{4}-(-2)}{-\frac{1}{2}-k}=\frac{3}{2}$$

$$\frac{\frac{9}{4}}{-\frac{1}{2}-k}=\frac{3}{2}$$

$$-\frac{1}{2}-k=\frac{3}{2}$$

$k=-2$

$\Rightarrow$ Center of the circle is at $(-2,-2)$.

Radius of the circle $=\sqrt{(-2-(-1))^2+(-2-7)^2}=\sqrt{82}$

$\therefore$ Equation of the circle: $[x-(-2)]^2+[y-(-2)]^2=(\sqrt{82})^2 \Rightarrow (x+2)^2+(y+2)^2=82$

(b) When $x=7$, $9^2+(y+2)^2=82 \Rightarrow y=-3$ or $y=-1$

Since $(7,\alpha)$ lies within the circle, $-3<\alpha<-1$.