A-LEVEL H2 MATH 2023 - PAPER 1

Question 1

[Ans: $y=-10ex+21e$]

$$\frac{d}{dx}(\ln y)=\frac{d}{dx}(11-5x)^2$$

$$\frac{1}{y}\frac{dy}{dx}=2(11-5x)(-5)$$

$$\frac{dy}{dx}=-10y(11-5x)$$

When $x=2$,

$$\ln y=\left[11-5(2)\right]^2$$

$$\ln y=1 \Rightarrow y=e$$

$$\frac{dy}{dx}=-10e\left[11-5(2)\right]=-10e$$

Equation of tangent:

$$y-e=-10e(x-2)$$

$$y=-10ex+21e$$

Question 2

[Ans: (a) $u_n = 4n^3 + 23n^2 - 46n + 29$ (b) $\{n \in \mathbb{Z} : n \geq 17\}$]

(a) Let $u_n = an^3 + bn^2 + cn + d$

$u_1 = 10$

$a(1)^3 + b(1)^2 + c(1) + d = 10 \Rightarrow a + b + c + d = 10$ --- (1)

$u_2 = 61$

$a(2)^3 + b(2)^2 + c(2) + d = 61 \Rightarrow 8a + 4b + 2c + d = 61$ --- (2)

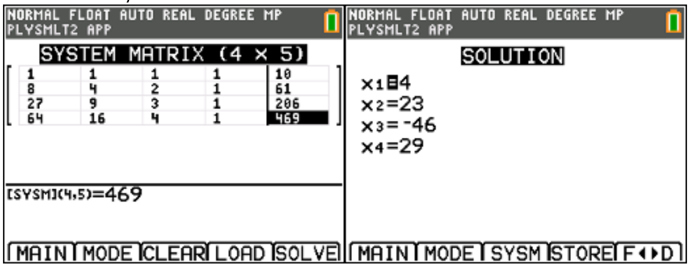
$u_3 = 206$

$a(3)^3 + b(3)^2 + c(3) + d = 206 \Rightarrow 27a + 9b + 3c + d = 206$ --- (3)

$u_4 = 469$

$a(4)^3 + b(4)^2 + c(4) + d = 469 \Rightarrow 64a + 16b + 4c + d = 469$ --- (4)

From GC,



$a = 4$, $b = 23$, $c = -46$, $d = 29$

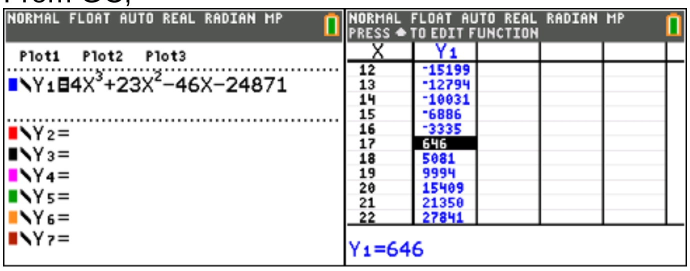
$u_n = 4n^3 + 23n^2 - 46n + 29$

(b) Let $u_n > 25000$

$4n^3 + 23n^2 - 46n + 29 > 25000$

$4n^3 + 23n^2 - 46n - 24871 > 0$

From GC,



$\{n \in \mathbb{Z} : n \geq 17\}$

Question 3

[Ans: show (b) $135°$]

(a) $(\mathbf{a}\times\mathbf{b}+\mathbf{a})\cdot(\mathbf{a}\times\mathbf{b}+\mathbf{b})=0$

$(\mathbf{a}\times\mathbf{b})\cdot(\mathbf{a}\times\mathbf{b})+(\mathbf{a}\times\mathbf{b})\cdot\mathbf{b}+\mathbf{a}\cdot(\mathbf{a}\times\mathbf{b})+\mathbf{a}\cdot\mathbf{b}=0$

$|\mathbf{a}\times\mathbf{b}|^2+0+0-1=0$

$|\mathbf{a}\times\mathbf{b}|^2=1$

$|\mathbf{a}\times\mathbf{b}|=1$ (shown)

(b) Let θ be the angle between the direction of $\mathbf{a}$ and the direction of $\mathbf{b}$.

$\mathbf{a}\cdot\mathbf{b}=-1$

$|\mathbf{a}||\mathbf{b}|\cos\theta=-1$ --- (1)

$|\mathbf{a}\times\mathbf{b}|=1$

$|\mathbf{a}||\mathbf{b}|\sin\theta=1$ --- (2)

(2)/(1) $\dfrac{|\mathbf{a}||\mathbf{b}|\sin\theta}{|\mathbf{a}||\mathbf{b}|\cos\theta}=\dfrac{1}{-1}$

$\tan\theta=-1$

$\theta=180°-\tan^{-1}(1)=135°$

Question 4

[Ans: (a) $\frac{1}{2}\left[\frac{\sin(p+q)x}{p+q}+\frac{\sin(p-q)x}{p-q}\right]+c$ (b) show (c) show; $k=-2$ or 0 (d) $\frac{\pi}{4}$]

(a) $\int \cos px \cos qx dx$

$$=\frac{1}{2}\int 2\cos px \cos qx dx$$

$$=\frac{1}{2}\int \cos\left[(p+q)x\right]+\cos\left[(p-q)x\right]dx$$

$$=\frac{1}{2}\left[\frac{\sin(p+q)x}{p+q}+\frac{\sin(p-q)x}{p-q}\right]+c$$

(b) $\int x\cos nx dx$

Let $u=x$ and $\frac{dv}{dx}=\cos nx$

$\therefore \frac{du}{dx}=1$ and $v=\frac{\sin nx}{n}$

$$=\frac{x\sin nx}{n}-\int(1)\left(\frac{\sin nx}{n}\right)dx=\frac{x\sin nx}{n}-\frac{1}{n}\int \sin nx dx$$

$$=\frac{x\sin nx}{n}-\frac{1}{n}\left(\frac{-\cos nx}{n}\right)+c$$

$$=\frac{x\sin nx}{n}+\frac{\cos nx}{n^2}+c \text{ (shown)}$$

(c) $\int_0^{\pi} x\cos nx dx$

$$=\left[\frac{x\sin nx}{n}+\frac{\cos nx}{n^2}\right]_0^{\pi}$$

$$=\left(\frac{\pi\sin n\pi}{n}+\frac{\cos n\pi}{n^2}\right)-\left(\frac{x\sin 0}{n}+\frac{\cos 0}{n^2}\right)=\left(0+\frac{\cos n\pi}{n^2}\right)-\left(0+\frac{1}{n^2}\right)$$

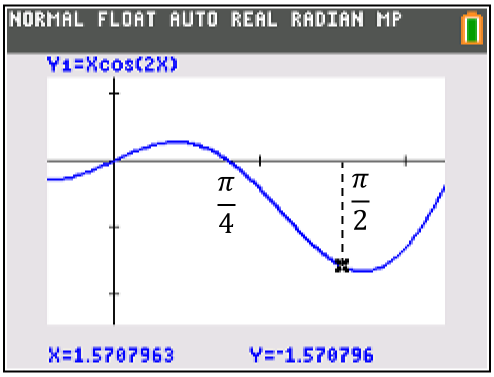
$$=\frac{1}{n^2}(\cos n\pi-1)$$

$$=\frac{1}{n^2}(\pm 1-1) \text{ for } n\in\mathbb{Z}$$

$$=\frac{-2}{n^2} \text{ or } \frac{0}{n^2}\text{, where } k=-2 \text{ or } 0$$

[continue on next page]

(d) From GC, $y = x\cos 2x$:



$$\int_0^{\frac{\pi}{2}} |x\cos 2x|\,dx$$

$$= \int_0^{\frac{\pi}{4}} x\cos 2x dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x\cos 2x dx$$

$$= \left[\frac{x\sin 2x}{2} + \frac{\cos 2x}{2^2}\right]_0^{\frac{\pi}{4}} - \left[\frac{x\sin 2x}{2} + \frac{\cos 2x}{2^2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left[\left(\frac{\frac{\pi}{4}\sin\frac{\pi}{2}}{2} + \frac{\cos\frac{\pi}{2}}{2^2}\right) - \left(\frac{0\sin 0}{2} + \frac{\cos 0}{2^2}\right)\right] - \left[\left(\frac{\frac{\pi}{2}\sin\pi}{2} + \frac{\cos\pi}{2^2}\right) - \left(\frac{\frac{\pi}{4}\sin\frac{\pi}{2}}{2} + \frac{\cos\frac{\pi}{2}}{2^2}\right)\right]$$

$$= \left[\left(\frac{\pi}{8} + 0\right) - \left(0 + \frac{1}{4}\right)\right] - \left[\left(\frac{0}{2} + \frac{-1}{4}\right) - \left(\frac{\pi}{8} + 0\right)\right]$$

$$= \frac{\pi}{4}$$

Question 5

[Ans: (a) show (b) show; $-\ln 2$ (c) show; $a=189$ and $b=200$]

(a) $\sum_{r=2}^{n} \ln\left[\frac{(r-1)(r+1)}{r^2}\right]$

$= \sum_{r=2}^{n}\left[\ln(r-1)+\ln(r+1)-2\ln r\right]$

$= \ln 1 + \ln 3 - 2\ln 2$
$+ \ln 2 + \ln 4 - 2\ln 3$
$+ \ln 3 + \ln 5 - 2\ln 4$
$+ \ln 4 + \ln 6 - 2\ln 5$
$\vdots$
$+ \ln(n-3) + \ln(n-1) - 2\ln(n-2)$
$+ \ln(n-2) + \ln n - 2\ln(n-1)$
$+ \ln(n-1) + \ln(n+1) - 2\ln n$

$= \ln 1 - 2\ln 2 + \ln 2 + \ln n + \ln(n+1) - 2\ln n$

$= -\ln 2 + \ln(n+1) - \ln n = \ln\left(\frac{n+1}{n}\right) - \ln 2$ (shown)

(b) When $n \to \infty$, $\ln\left(\frac{n+1}{n}\right) = \ln\left(1+\frac{1}{n}\right) \to \ln(1+0) = 0$. $\therefore$ the corresponding infinite series of $\sum_{r=2}^{n} \ln\left[\frac{(r-1)(r+1)}{r^2}\right]$ is convergent.

$\sum_{r=2}^{\infty} \ln\left[\frac{(r-1)(r+1)}{r^2}\right] = -\ln 2$

(c) $\sum_{r=10}^{20} \ln\left[\frac{(r-1)(r+1)}{r^2}\right]$

$= \sum_{r=2}^{20} \ln\left[\frac{(r-1)(r+1)}{r^2}\right] - \sum_{r=2}^{9} \ln\left[\frac{(r-1)(r+1)}{r^2}\right]$

$= \left[\ln\left(\frac{20+1}{20}\right) - \ln 2\right] - \left[\ln\left(\frac{9+1}{9}\right) - \ln 2\right]$

$= \ln\left(\frac{21}{20}\right) - \ln\left(\frac{10}{9}\right) = \ln\frac{189}{200}$ (shown), where $a=189$ and $b=200$

Question 6

[Ans: (a) show (b) $\frac{81}{8}\pi\left(\pi-\frac{9\sqrt{3}}{8}\right)$ units³]

(a) RHS

$$=\frac{1}{8}(\cos 4\theta+4\cos 2\theta+3)$$

$$=\frac{1}{8}\left[\left(2\cos^2 2\theta-1\right)+4\cos 2\theta+3\right]$$

$$=\frac{1}{4}\left(\cos^2 2\theta+2\cos 2\theta+1\right)$$

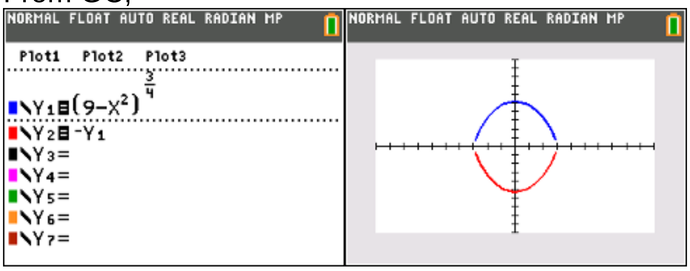
$$=\frac{1}{4}(\cos 2\theta+1)^2$$

$$=\frac{1}{4}\left(2\cos^2\theta\right)^2=\cos^4\theta=\text{LHS (shown)}$$

(b) $y^4=\left(9-x^2\right)^3$

$$y=\pm\left(9-x^2\right)^{\frac{3}{4}}$$

From GC,



Volume required

$$=\pi\int_{1.5}^{3}\left[\left(9-x^2\right)^{\frac{3}{4}}\right]^2 dx=\pi\int_{1.5}^{3}\left(9-x^2\right)^{\frac{3}{2}} dx$$

When $x=3\sin\theta\Rightarrow\frac{dx}{d\theta}=3\cos\theta$

When $x=1.5$, $3\sin\theta=1.5\Rightarrow\sin\theta=\frac{1}{2}\Rightarrow\theta=\frac{\pi}{6}$

When $x=3$, $3\sin\theta=3\Rightarrow\sin\theta=1\Rightarrow\theta=\frac{\pi}{2}$

$$=\pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(9-9\sin^2\theta\right)^{\frac{3}{2}}(3\cos\theta)\,d\theta=\pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}9^{\frac{3}{2}}\left(1-\sin^2\theta\right)^{\frac{3}{2}}(3\cos\theta)\,d\theta$$

$$=81\pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(\cos^2\theta\right)^{\frac{3}{2}}(\cos\theta)\,d\theta=81\pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\cos^4\theta\,d\theta$$

$$=81\pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{1}{8}(\cos 4\theta+4\cos 2\theta+3)\,d\theta=\frac{81}{8}\pi\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\cos 4\theta+4\cos 2\theta+3\,d\theta$$

$$=\frac{81}{8}\pi\left[\frac{\sin 4\theta}{4}+4\left(\frac{\sin 2\theta}{2}\right)+3\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

[continue on next page]

$$=\frac{81}{8}\pi\left[\left(\frac{\sin 2\pi}{4}+2\sin\pi+\frac{3\pi}{2}\right)-\left(\frac{\sin\frac{2\pi}{3}}{4}+2\sin\frac{\pi}{3}+\frac{\pi}{2}\right)\right]$$

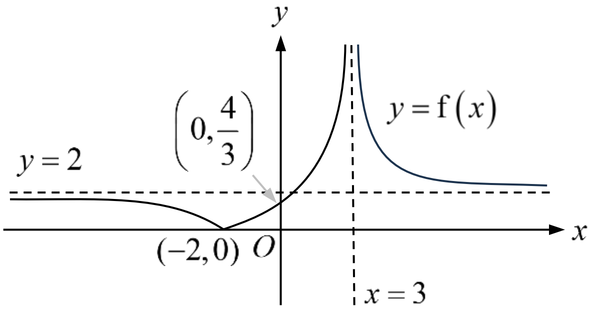
$$=\frac{81}{8}\pi\left[\frac{3\pi}{2}-\left(\frac{\sqrt{3}}{8}+\sqrt{3}+\frac{\pi}{2}\right)\right]$$

$$=\frac{81}{8}\pi\left(\pi-\frac{9\sqrt{3}}{8}\right)$$

Question 7

[Ans: (a) Sketch (b) $R_{\mathrm{f}}=[0,\infty)$ (c) Explain (d) $a=-2$ (e) $\mathrm{f}^{-1}(x)=\dfrac{3x+4}{x-2}$; $D_{\mathrm{f}^{-1}}=[0,2)$]

(a)



(b) $R_{\mathrm{f}}=[0,\infty)$

(c) $\mathrm{f}^2(x)=\mathrm{ff}(x)$

$R_{\mathrm{f}}=[0,\infty)$

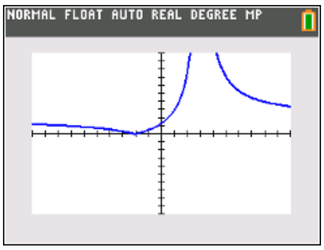
$D_{\mathrm{f}}=(-\infty,3)\cup(3,\infty)$

$\because R_{\mathrm{f}}\not\subseteq D_{\mathrm{f}}$, f^2 does not exist.

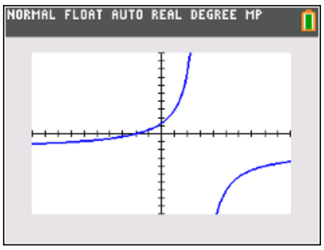
(d) $a=-2$

(e) From GC,

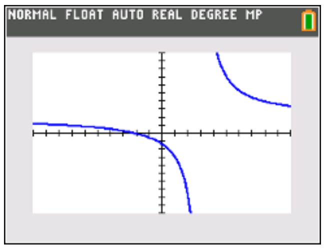
$y=\mathrm{f}(x)$



$y=\dfrac{2x+4}{3-x}$



$y=-\left(\dfrac{2x+4}{3-x}\right)$



For $x\le -2$,

$$\mathrm{f}(x)=-\left(\frac{2x+4}{3-x}\right)$$

$$y=-\frac{2x+4}{3-x}=\frac{2x+4}{x-3}$$

$$xy-3y=2x+4$$

$$xy-2x=3y+4$$

$$x(y-2)=3y+4$$

$$x=\frac{3y+4}{y-2}$$

$\therefore \mathrm{f}^{-1}(x)=\dfrac{3x+4}{x-2}$ and $D_{\mathrm{f}^{-1}}=R_{\mathrm{f}}=[0,2)$

Question 8

[Ans: (a)(i) $z=2e^{i\frac{2\pi}{3}}$ (ii) 2 (b) $v=-\frac{12}{5}$ and $w=-4+\frac{21}{5}i$, or $v=-2$ and $w=-4+3i$]

(a) (i) $|z|=\sqrt{1^2+\left(\sqrt{3}\right)^2}=2$

$\arg z=\pi-\tan^{-1}\frac{\sqrt{3}}{1}=\frac{2\pi}{3}$

$\therefore z=2e^{i\frac{2\pi}{3}}$

(ii) $\frac{z^n}{iz^*}=\frac{\left(2e^{i\frac{2\pi}{3}}\right)^n}{e^{i\frac{\pi}{2}}2e^{-i\frac{2\pi}{3}}}=2^{n-1}e^{i\left(\frac{2n\pi}{3}+\frac{\pi}{6}\right)}=2^{n-1}\left[\cos\left(\frac{2n\pi}{3}+\frac{\pi}{6}\right)-i\sin\left(\frac{2n\pi}{3}+\frac{\pi}{6}\right)\right]$

For $\frac{z^n}{iz^*}$ to be purely imaginary,

$\text{Re}\left(\frac{z^n}{iz^*}\right)=0$

$2^{n-1}\cos\left(\frac{2n\pi}{3}+\frac{\pi}{6}\right)=0$

$\frac{2n\pi}{3}+\frac{\pi}{6}=(2k+1)\frac{\pi}{2},\ k\in\mathbb{Z}$

$n=\frac{3}{2}\left(k+\frac{1}{3}\right)=\ldots,-1,2,5,8,\ldots$

$\therefore$ smallest positive integer value of $n=2$

(b) $2v+|w|=1\Rightarrow v=\frac{1}{2}-\frac{1}{2}|w|$ --- (1)

$3v-iw=-3+4i$ --- (2)

Sub. (1) into (2) $\frac{3}{2}-\frac{3}{2}|w|-iw=-3+4i$

Let $w=a+ib$

$\frac{3}{2}-\frac{3}{2}|a+ib|-i(a+ib)=-3+4i$

$\frac{3}{2}-\frac{3}{2}\sqrt{a^2+b^2}-ia+b=-3+4i$

$\left(\frac{3}{2}-\frac{3}{2}\sqrt{a^2+b^2}+b\right)-ia=-3+4i$

[continue on next page]

$\therefore a = -4$ & $\frac{3}{2} - \frac{3}{2}\sqrt{a^2 + b^2} + b = -3$

$$-\frac{3}{2}\sqrt{16 + b^2} = -\frac{9}{2} - b$$

$$3\sqrt{16 + b^2} = 9 + 2b$$

$$9\left(16 + b^2\right) = 81 + 36b + 4b^2$$

$$5b^2 - 36b + 63 = 0$$

$b = \frac{21}{5}$ or $b = 3$

$\therefore w = -4 + \frac{21}{5}i$ or $w = -4 + 3i$

Sub. $w = -4 + \frac{21}{5}i$ into (1) $v = \frac{1}{2} - \frac{1}{2}\left|-4 + \frac{21}{5}i\right| = \frac{1}{2} - \frac{1}{2}\sqrt{4^2 + \left(\frac{21}{5}\right)^2} = -\frac{12}{5}$

Sub. $w = -4 + 3i$ into (1) $v = \frac{1}{2} - \frac{1}{2}\left|-4 + 3i\right| = \frac{1}{2} - \frac{1}{2}\sqrt{4^2 + 3^2} = -2$

Question 9

[Ans: (a) $a=2$, $B(1,3,6)$ (b)(i) $\frac{20}{\sqrt{14}}$ (ii) $63.0°$ (c) $3x-4y-z=-15$]

(a) $l_1:\mathbf{r}=\begin{pmatrix}-3\\1\\2\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\a\end{pmatrix}$; $\quad l_2:\mathbf{r}=\begin{pmatrix}-2\\1\\5\end{pmatrix}+\mu\begin{pmatrix}3\\2\\1\end{pmatrix}$

At B,

$$\begin{pmatrix}-3\\1\\2\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\a\end{pmatrix}=\begin{pmatrix}-2\\1\\5\end{pmatrix}+\mu\begin{pmatrix}3\\2\\1\end{pmatrix}$$

$-3+2\lambda=-2+3\mu\Rightarrow 2\lambda-3\mu=1$ --- (1)

$1+\lambda=1+2\mu\Rightarrow \lambda-2\mu=0$ --- (2)

$2+a\lambda=5+\mu\Rightarrow a=\frac{3+\mu}{\lambda}$ --- (3)

Solving (1) & (2) using GC, $\lambda=2$, $\mu=1$

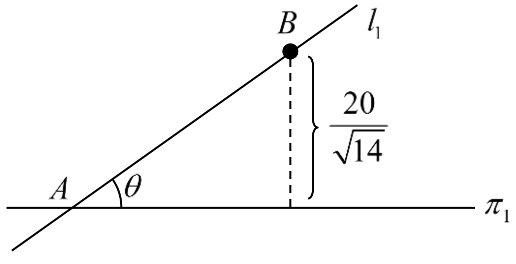
$\therefore a=\frac{3+1}{2}=2$

$$OB=\begin{pmatrix}-2\\1\\5\end{pmatrix}+(1)\begin{pmatrix}3\\2\\1\end{pmatrix}=\begin{pmatrix}1\\3\\6\end{pmatrix}\Rightarrow B(1,3,6)$$

(b) (i) $\pi_1:\mathbf{r}\cdot\begin{pmatrix}3\\2\\1\end{pmatrix}=\begin{pmatrix}-3\\1\\2\end{pmatrix}\cdot\begin{pmatrix}3\\2\\1\end{pmatrix}=-5$

Shortest distance from B to $\pi_1=\left|\frac{\begin{pmatrix}1\\3\\6\end{pmatrix}\cdot\begin{pmatrix}3\\2\\1\end{pmatrix}}{\left|\begin{pmatrix}3\\2\\1\end{pmatrix}\right|}-\frac{-5}{\left|\begin{pmatrix}3\\2\\1\end{pmatrix}\right|}\right|=\left|\frac{15}{\sqrt{14}}+\frac{5}{\sqrt{14}}\right|=\frac{20}{\sqrt{14}}$

(ii)



$\overrightarrow{AB}=\begin{pmatrix}1\\3\\6\end{pmatrix}-\begin{pmatrix}-3\\1\\2\end{pmatrix}=\begin{pmatrix}4\\2\\4\end{pmatrix}$

$\left|\overrightarrow{AB}\right|=\sqrt{4^2+2^2+4^2}=6$

$$\sin\theta=\frac{\frac{20}{\sqrt{14}}}{6}\Rightarrow\theta=\sin^{-1}\left(\frac{\frac{20}{\sqrt{14}}}{6}\right)=63.0°$$

[continue on next page]

(c) Vector normal to π_2

$$=\begin{pmatrix}3\\2\\1\end{pmatrix}\times\begin{pmatrix}2\\1\\2\end{pmatrix}=\begin{pmatrix}3\\-4\\-1\end{pmatrix}$$

$$\pi_2:\mathbf{r}\cdot\begin{pmatrix}3\\-4\\-1\end{pmatrix}=\begin{pmatrix}-3\\1\\2\end{pmatrix}\cdot\begin{pmatrix}3\\-4\\-1\end{pmatrix}=-15$$

Cartesian equation of π_2 is $3x-4y-z=-15$.

Question 10

[Ans: (a) show (b) $C = 3300$ (c) show (d) 51 (e)(i) sketch; explain (ii) $0 < C < 2400$]

(a) Energy expenditure per day $= 30M$

$$\frac{dM}{dt} \propto (C - 30M) \Rightarrow \frac{dM}{dt} = k(C - 30M) \text{ (shown)}$$

(b) To maintain his mass at $M = 110$,

$$\frac{dM}{dt} = 0$$

$$k\left[C - 30(110)\right] = 0 \Rightarrow C = 3300$$

(c) Let $C = 0.8(3300) = 2640$

$$\frac{dM}{dt} = k(2640 - 30M)$$

$$\frac{1}{2640 - 30M}\frac{dM}{dt} = k$$

$$\int \frac{1}{2640 - 30M}\, dM = k\int dt$$

$$-\frac{1}{30}\ln|2640 - 30M| = kt + A$$

$$|2640 - 30M| = e^{-30kt - 30A}$$

$$2640 - 30M = \pm e^{-30A}e^{-30kt} = Be^{-30kt}, \text{ where } B = \pm e^{-30A}$$

When $t = 0$, $M = 110$,

$$2640 - 30(110) = Be^{0} \Rightarrow B = -660$$

$$2640 - 30M = -660e^{-30kt} \Rightarrow M = 88 + 22e^{-30kt} \text{ (shown)}$$

(d) When $t = 75$,

$$M = 100$$

$$88 + 22e^{-30k(75)} = 100$$

$$88 + 22e^{-30k(75)} = 100 \Rightarrow e^{-30k(75)} = \frac{6}{11} \Rightarrow e^{-30k} = \left(\frac{6}{11}\right)^{\frac{1}{75}}$$

$$M = 88 + 22\left(e^{-30k}\right)^{t} = 88 + 22\left(\frac{6}{11}\right)^{\frac{t}{75}}$$

Let $M < 96$

$$88 + 22\left(\frac{6}{11}\right)^{\frac{t}{75}} < 96$$

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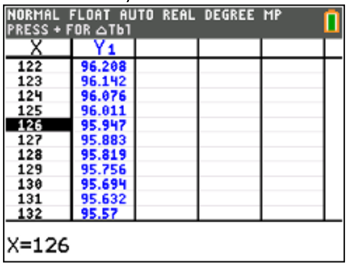
From GC,



NORMAL FLOAT AUTO REAL DEGREE MP
PRESS + FOR △Tbl

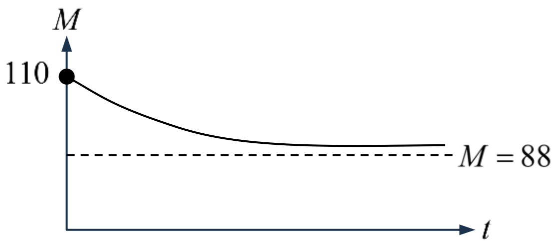
X	Y_1
122	96.208
123	96.142
124	96.076
125	96.011
126	95.947
127	95.883
128	95.819
129	95.756
130	95.694
131	95.632
132	95.57

X=126

Andrew's mass will fall below 96 kg after 126 days.

$\therefore$ additional days $= 126 - 75 = 51$

(e) (i)



As $t \to \infty$, $M \to 88$ and will not be falling below 88 before that. $\therefore$ Andrew cannot achieve a mass of 80 kg using this plan.

(ii) Let $0 < \dfrac{C}{30} < 80$

Required range: $0 < C < 2400$

Question 11

[Ans: (a) $\$513.89$ (b) show (c)(i) $\$1323.63$ (ii) $\$76506.80$ (d)(i) $k=287$ and $y=\$1321.54$ (ii) $\$15990$]

(a) Let the amount of money Wei deposit on n th month be u_n.

$u_n = a+(n-1)(50)$

Let $u_1+u_2+u_3+\ldots+u_{36} \geq 50000$

$\frac{36}{2}\left[2a+(36-1)(50)\right] \geq 50000$

$a \geq 513.89$

$\therefore$ smallest value of a is $\$513.89$.

(b)

Month	Total amount Wei owes at the end of the month
1	$400000\times1.001-x$
2	$\left[400000\times1.001-x\right](1.001)-x$ $=400000\times1.001^2-(x+1.001x)$
3	$\left[=400000\times1.001^2-(x+1.001x)\right](1.001)-x$ $=400000\times1.001^3-(x+1.001x+1.001^2x)$
n	$400000\times1.001^n-\frac{x(1.001^n-1)}{1.001-1}$ $400000\times1.001^n-1000x(1.001^n-1)$ (shown)

(c) (i) Let $400000\times1.001^{360}-1000x(1.001^{360}-1)\leq 0$

$1000x(1.001^{360}-1)\geq 400000\times1.001^{360}$

$x\geq 1323.634776$

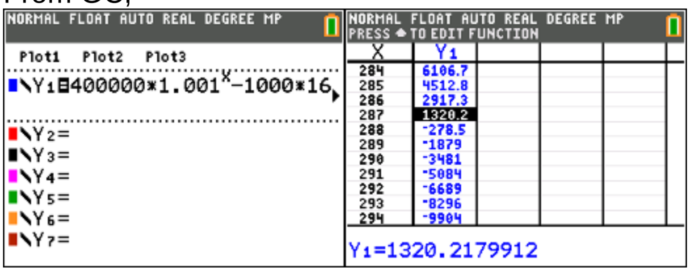
$\therefore$ the monthly repayment is $\$1323.63$

(ii) Total interested paid

$=1323.63\times360-400000=76508.51936\approx\$76506.80$

(d) (i) Let $400000\times1.001^n-1000\times1600(1.001^n-1)\leq 0$

From GC,



$k=287$ and $y=1320.2179912\times1.001=\$1321.54$

(ii) Total interested paid with the 2nd repayment method

$=1600\times287+1320.2179912\times1.001-400000=60521.53821$

Total savings

$=76508.51936-60521.53821=\$15990$ (to 4 s.f.)