O-LEVEL ADDITIONAL MATH 2023 – PAPER 2

Question 1

[Ans: $k = 6$]

$$(k+2x)\left[2^6 + \binom{6}{1}(2)^5\left(-\frac{1}{2}x\right) + \binom{6}{2}(2)^4\left(-\frac{1}{2}x\right)^2 + \binom{6}{3}(2)^3\left(-\frac{1}{2}x\right)^3 ...\right]$$

$$(k+2x)\left[64 - 96x + 60x^2 - 20x^3 ...\right]$$

The term in x^3:

$$k\left(-20x^3\right) + (2x)\left(60x^2\right) = 0x^3$$

$$-20k + 120 = 0$$

$$k = \frac{-120}{-20}$$

$$k = 6$$

Question 2

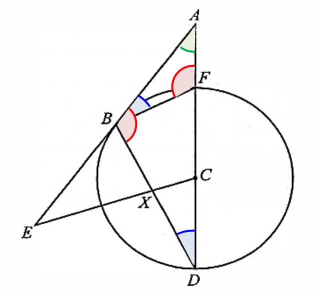
[Ans: (a) Prove (b) Trapezium ; Reasons]

(a) $\angle BAD = \angle FAB$ (Common angle)

$\angle BDA = \angle ABF$ (Alternate segment theorem)

$\angle ABD = \angle AFB$ (Sum of angles in a $\triangle$ is 180°)

$\Delta ABD \sim \Delta AFB$ (AAA)



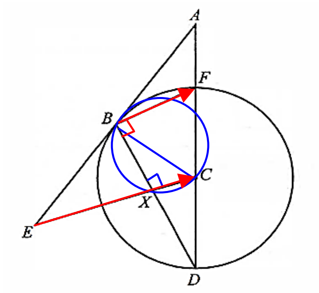
(b) $\angle FBD = \angle BXC = 90°$ (Angle in a semicircle)

$\angle FBX + \angle BXC = 180°$

$\Rightarrow BF \parallel EC$ (Co-interior angles add up to 180°)

$BF \parallel EC$ and $BF \neq EC$, EB is not parallel to CF.

$EBFC$ is a trapezium (1 pair of parallel sides).



Question 3

[Ans: (a) $86°C$ (b) 14.1 minutes (c)(i) 0.00794 (ii) 11.7 minutes]

(a) $t=0, \quad T_c = 86\,e^{-0.06(0)} = 86$

(b) $86\,e^{-0.06t} = 37$

$$e^{-0.06t} = \frac{37}{86}$$

$$-0.06t = \ln\frac{37}{86}$$

$$t = \ln\frac{37}{86} \div (-0.06)$$

$$= 14.1$$

(c) (i) 1 hour, $t = 60$ (minutes)

$$86\,e^{-\lambda(60)} = 82$$

$$e^{-60\lambda} = \frac{82}{86}$$

$$-60\lambda = \ln\frac{82}{86}$$

$$\lambda = \ln\frac{82}{86} \div (-60)$$

$$= 0.000794$$

(c) (ii) $T_c = \frac{1}{2}T_f$

$$86\,e^{-0.06t} = \frac{1}{2}\left(86\,e^{-0.000794t}\right)$$

$$\frac{e^{-0.06t}}{e^{0.000794t}} = \frac{1}{2}$$

$$e^{-0.06t-(-0.000794t)} = \frac{1}{2}$$

$$e^{-0.059206t} = \frac{1}{2}$$

$$-0.059206t = \ln\frac{1}{2}$$

$$t = \ln\frac{1}{2} \div (-0.059206)$$

$$= 11.7$$

Question 4

[Ans: (a) Show (b) Prove]

(a) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

Let $A = 15°$,

$$\tan 30° = \frac{2\tan 15°}{1-\tan^2 15°}$$

$$\frac{1}{\sqrt{3}} = \frac{2\tan 15°}{1-\tan^2 15°}$$

$$1-\tan^2 15° = 2\sqrt{3}\tan 15°$$

$$0 = \tan^2 15° + 2\sqrt{3}\tan 15° - 1$$

$$\tan 15° = \frac{-2\sqrt{3} \pm \sqrt{\left(2\sqrt{3}\right)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-2\sqrt{3} \pm 4}{2}$$

$$= \pm 2 - \sqrt{3}$$

$$\because \tan 15° > 0, \quad \tan 15° = 2 - \sqrt{3}$$

(b) $\tan 105° = \tan(60^\circ + 45^\circ)$

$$= \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ}$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{3+2\sqrt{3}+1}{1^2 - \sqrt{3}^2}$$

$$= \frac{4+2\sqrt{3}}{-2}$$

$$= -2 - \sqrt{3}$$

$$\tan 15° - \tan 105°$$

$$= \left(2-\sqrt{3}\right) - \left(-2-\sqrt{3}\right)$$

$$= 2 - \sqrt{3} + 2 + \sqrt{3}$$

$$= 4$$

Queston 5

[Ans: (a) Show ; $B=-36+45p-9q$ (b) $p=1, q=1$]

(a) $\frac{\mathrm{d}y}{\mathrm{d}x}=-9\sin 3x-15\cos 3x$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-27\cos 3x+45\sin 3x$

$p\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+q\frac{\mathrm{d}y}{\mathrm{d}x}+14y+34\sin 3x$

$=p(-27\cos 3x+45\sin 3x)+q(-9\sin 3x-15\cos 3x)+14(3\cos 3x-5\sin 3x)+34\sin 3x$

$=-27p\cos 3x+45p\sin 3x-9q\sin 3x-15q\cos 3x+42\cos 3x-70\sin 3x++34\sin 3x$

$=(-27p-15q+42)\cos 3x+(45p-9q-36)\sin 3x$

$=A\cos 3x+B\sin 3x$

$\therefore A=42-27p-15q, \quad B=-36+45p-9q$

(b) $A=0, B=0$

$A=42-27p-15q=0.....(1)$

$B=-36+45p-9q=0.....(2)$

$(2)\div 3: \quad -12+15p-3q=0.....(3)$

$(3)\times 5: \quad -60+75p-15q=0.....(4)$

$(1)-(4): \quad 102-102p=0 \quad \Rightarrow p=1$

Sub into $(3): -12+15-3q=0 \quad \Rightarrow q=1$

Question 6

[Ans: (a) $(2x+1)(3x-2)(x-1)$ (b) $y=-0.585$ or $y=0$ (c) $1-\log_2 3$]

(a) Sub $x=-\frac{1}{2}$ into $6x^3-7x^2-x+2$,

$$6\left(-\frac{1}{2}\right)^3-7\left(-\frac{1}{2}\right)^2-\left(-\frac{1}{2}\right)+2=0$$

$\therefore (2x+1)$ is a factor of $6x^3-7x^2-x+2$

Factorising by comparing coefficient of x^3 and constant,

$$6x^3-7x^2-x+2=(2x+1)(3x^2+Bx+2)$$

By comparing coefficient of x,

$$6x^3-7x^2-x+2=(2x+1)(3x^2+Bx+2)$$

$B+4=-1$ Bx

$B=-5$ $4x$

$$6x^3-7x^2-x+2$$
$$=(2x+1)(3x^2-5x+2)$$
$$=(2x+1)(3x-2)(x-1)$$

(b) $6\left(4^y\right)+2\left(2^{-y}\right)=7\left(2^y\right)+1$

$$6\left(2^y\right)^2+\frac{2}{2^y}-7\left(2^y\right)-1=0$$

Let $x=2^y$,

$$6x^2+\frac{2}{x}-7x-1=0$$
$$6x^3+2-7x^2-x=0$$
$$6x^3-7x^2-x+2=0$$
$$(2x+1)(3x-2)(x-1)=0$$

$x=-\frac{1}{2}$ or $x=\frac{2}{3}$ or $x=1$

$2^y=-\frac{1}{2}$ $2^y=\frac{2}{3}$ $2^y=1$

(Undefined) $y\log 2=\log\frac{2}{3}$ $2^y=2^0$

$y=-0.585$ $y=0$

(c) $2^y=\frac{2}{3}$

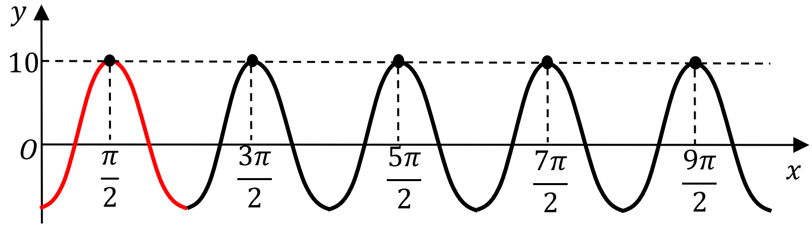
$$y=\log_2\frac{2}{3}$$
$$=\log_2 2-\log_2 3$$
$$=1-\log_2 3$$

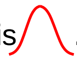
Question 7

[Ans: (a) $a=-10, b=2$ (b) $c=5$ (c) $10.4\,\text{m}$]

(a) Period $=\dfrac{9\pi}{2}-\dfrac{7\pi}{2}=\pi$

$\dfrac{2\pi}{b}=\pi \quad \Rightarrow b=2$



The amplitude of the curve is 10. The first shape of the curve on right of y-axis is .

$\therefore a=-10$

Alternative:

Sub $b=2$ and $\left(\dfrac{7\pi}{2},10\right)$ into $y=a\cos bx$

$10=a\cos\left(2\times\dfrac{7\pi}{2}\right)$

$10=a(-1)$

$a=-10$

(b) (i) In order that particle will never change its direction,

$v\geq 0 \quad \Rightarrow$ Particle is always traveling in the positive direction/at rest **or**

$v\leq 0 \quad \Rightarrow$ Particle is always traveling in the negative direction/at rest.

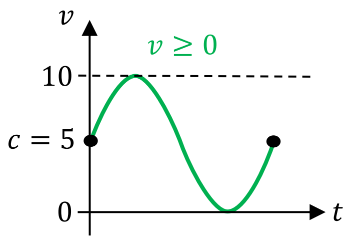
$\because c$ is a positive constant, $v\geq 0$.

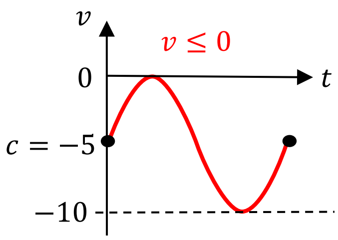
The value of c is smallest when the minimum point is on the t-axis.

Minimum value of $\sin 0.2t=-1$, $\quad v_{\text{min}}=c-5$.

Solving $c-5=0$, smallest value of $c=5$.

Alternative:





If c is a negative constant, the particle will never change direction when $v\leq 0$. $\Rightarrow c\leq -5$

The greatest value of $c=-5$.

From graph $v=5\sin 0.2t+5$, $\because c$ is a positive constant, smallest value of $c=-5$.

(b)(ii) $s=\int_2^3 (5\sin 0.2t+8)\,\mathrm{d}t$

$=\left[\dfrac{-5\cos 0.2t}{0.2}+8t\right]_2^3$

$=\left[-25\cos 0.2t+8t\right]_2^3$

$=\left[-25\cos 0.6+24\right]-\left[-25\cos 0.4+16\right]$

$=10.4$

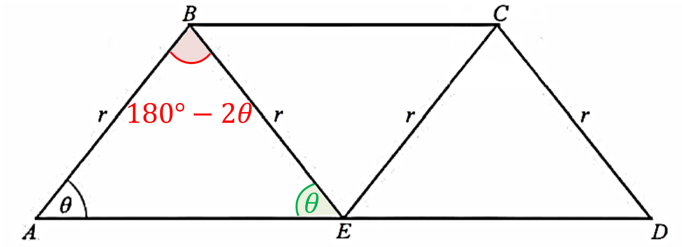
Question 8

[Ans: (a) $\frac{3}{2}r^2\sin 2\theta$ (b) $A=\frac{3}{2}r^2$; $\theta = 45°$]

(a) Area,

$$A = 3\times\frac{1}{2}r^2\sin(180°-2\theta)$$

$$=\frac{3}{2}r^2\sin 2\theta$$



Given that $0<\theta<90°$,

$0<2\theta<180°$

2θ is an acute/obtuse angle.

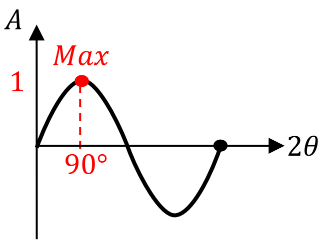
$\Rightarrow \sin(180°-2\theta)=\sin 2\theta$

(b) $A=\frac{3}{2}r^2\sin 2\theta$

For maximum possible area, $\sin 2\theta = 1$.

$2\theta = 90°$

$\therefore \theta = 45°$



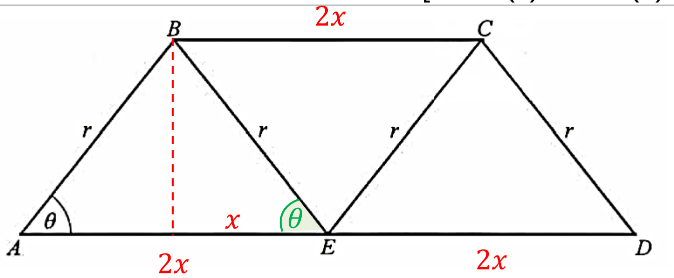
$$A=\frac{3}{2}r^2\sin 90°$$

$$\therefore A=\frac{3}{2}r^2$$

Question 8 (Continuation)

[Ans: (c) Show (d) $\theta = 60.4°$]

(c) Let $AE = 2x$



$$\cos\theta = \frac{x}{r}$$

$$x = r\cos\theta$$

Total length,

$$6x + 4r = 8r\sin\theta$$

$$6r\cos\theta + 4r = 8r\sin\theta$$

$$6\cos\theta + 4 = 8\sin\theta$$

$$4 = 8\sin\theta - 6\cos\theta$$

$$2 = 4\sin\theta - 3\cos\theta$$

$4\sin\theta - 3\cos\theta = 2$ (Shown)

(d) $4\sin\theta - 3\cos\theta = R\sin(\theta - \alpha)$

$$4\sin\theta - 3\cos\theta = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$$

$$4 = R\cos\alpha .....(1)$$

$$3 = R\sin\alpha .....(2)$$

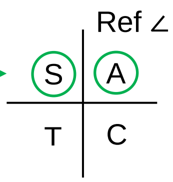
$$(1)^2 + (2)^2 : R^2 = 4^2 + 3^2 \quad \Rightarrow R = \sqrt{4^2 + 3^2} = 5$$

$$\frac{(2)}{(1)} : \frac{\sin\alpha}{\cos\alpha} = \frac{3}{4} \quad \Rightarrow \tan\alpha = \frac{3}{4} \quad \alpha = \tan^{-1}\frac{3}{4} = 36.87°$$

$$4\sin\theta - 3\cos\theta = 5\sin(\theta - 36.87°)$$

$$5\sin(\theta - 36.87°) = 2$$

$$\sin(\theta - 36.87°) = \frac{2}{5}$$



Ref $\angle = \sin^{-1}\frac{2}{5} = 23.578°$

$$0° < \theta < 90°$$

$$-36.87° < \theta - 36.87° < 53.13°$$

$$\theta - 36.87° = 23.578°$$

$$\theta = 60.4°$$

Question 9

[Ans: (a) Show (b) $\frac{1}{5}+\frac{x-15}{3\sqrt{x^2-30x+289}}$; Show (c) 5 hours 8 minutes]

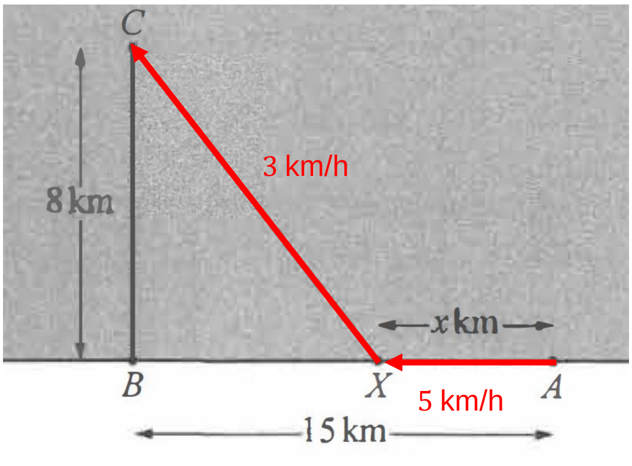
(a) Distance,

$$CX=\sqrt{8^2+(15-x)^2}$$
$$=\sqrt{64+225-30x+x^2}$$
$$=\sqrt{x^2-30x+289}$$

Total time, $T=T_{AX}+T_{XC}$

$$\text{Time}=\frac{\text{Distance}}{\text{Speed}}$$

$$T=\frac{x}{5}+\frac{\sqrt{x^2-30x+289}}{3} \text{ (Shown)}$$



(b) $T=\frac{1}{5}x+\frac{1}{3}(x^2-30x+289)^{\frac{1}{2}}$

$$\frac{dT}{dx}=\frac{1}{5}+\frac{1}{3}\times\frac{1}{2}(x^2-30x+289)^{-\frac{1}{2}}(2x-30)$$
$$=\frac{1}{5}+\frac{x-15}{3\sqrt{x^2-30x+289}}$$

For time taken to be minimum, $\frac{dT}{dx}=0$.

$$\frac{1}{5}+\frac{x-15}{3\sqrt{x^2-30x+289}}=0$$
$$\frac{x-15}{3\sqrt{x^2-30x+289}}=-\frac{1}{5}$$
$$-5x+75=3\sqrt{x^2-30x+289}$$
$$(-5x+75)^2=9(x^2-30x+289)$$
$$25x^2-750x+5625=9x^2-270x+2601$$
$$16x^2-480x+3024=0$$
$$x^2-30x+189=0 \text{ (Shown)}$$

(c) $x^2-30x+189=0$

$$(x-15)^2-225+189=0$$
$$(x-15)^2=36$$

$x-15=6$ or $x-15=-6$

$x=21$ (Rejected) $\quad x=9$

$\because$ Distance $x<15$

$$T=\frac{1}{5}(9)+\frac{1}{3}(9^2-30(9)+289)^{\frac{1}{2}}=5\frac{2}{15}=5\frac{8}{60}$$

5 hours 8 minutes