RAFFLES INSTITUTION
2021 YEAR 5 PROMOTION EXAMINATION

CANDIDATE NAME

CLASS 22

MATHEMATICS 9758

2.5 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper. **You may use the blank pages on page 21 and 22 if necessary and you are reminded to indicate the question number(s) clearly.**
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 85.

For examiner's use only					
Q1	**Q2**	**Q3**	**Q4**	**Q5**	
/ 4	**/ 5**	**/ 6**	**/ 8**	**/ 8**	
Q6	**Q7**	**Q8**	**Q9**	**Q10**	**TOTAL**
/ 10	**/ 10**	**/ 10**	**/ 12**	**/ 12**	**/ 85**

This document consists of **20** printed pages and **2** blank pages.

RAFFLES INSTITUTION
Mathematics Department

1 A curve C has equation $y = \dfrac{ax+b}{cx-2}$, where a, b and c are constants. It is given that C passes through the points with coordinates $(1,\ 5)$ and $(-8,\ 0.5)$. The curve C is translated 1 unit in the positive x-direction. The new curve passes through the point with coordinates $(0,\ -0.2)$. Find the values of a, b and c. [4]

2 The curve C has equation $y^3 = 4 - \frac{xy^2}{2}$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{y}{6y+2x}$. [2]

(ii) Find the equation of the normal to C at the point P where $y = 1$. [3]

3 The points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively.

(a) Show that the area of triangle ABC is $\frac{1}{2}\left|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}\right|$. Hence show that the shortest distance from B to AC is

$$\frac{\left|\mathbf{a}\times\mathbf{b}+\mathbf{b}\times\mathbf{c}+\mathbf{c}\times\mathbf{a}\right|}{\left|\mathbf{c}-\mathbf{a}\right|}.$$ [4]

(b) Given that $\mathbf{a}$ and $\mathbf{b}$ are non-zero vectors such that $|\mathbf{a}-\mathbf{b}|=|\mathbf{a}+\mathbf{b}|$, find the value of $\mathbf{a}\cdot\mathbf{b}$. [2]

4 A sequence $u_1, u_2, u_3, ...$ is defined by

$$u_n = \sum_{r=1}^{n} (2r + n + 1).$$

Another sequence $v_1, v_2, v_3, ...$ is given by $v_n = \frac{2}{u_n}$, where $n \in \mathbb{Z}^+$.

(i) Find u_n in terms of n. [2]

(ii) Show that $v_n = \frac{1}{n} - \frac{1}{n+1}$. [1]

(iii) Describe the behaviour of the sequence $v_1, v_2, v_3, ...$. [1]

(iv) Find the sum, S_N, of the first N terms of the sequence $v_1, v_2, v_3, \ldots$. [2]

(v) Give a reason why the series S_N converges, and write down the value of the sum to infinity. [2]

5 **(i)** Using standard series from the List of Formulae (MF26), expand $\frac{\cos 3x}{4-x}$ as far as the term in x^3. Give the coefficients as exact fractions in their simplest form. [4]

(ii) It is given that the third and fourth terms found in part **(i)** are equal to the third and fourth terms in the series expansion of $(a+bx)^5$, in ascending powers of x, respectively. Find the values of the constants a and b. [4]

6 **(i)** On the same axes, sketch the graphs of $y = x + 2 + \dfrac{1}{x-1}$ and $y = |2x + 2|$, stating the coordinates of any points of intersections with the axes, turning points and the equations of any asymptotes. [5]

(ii) Hence solve the inequality

$$x + 2 + \frac{1}{x-1} > |2x + 2|,$$

giving your answers in exact form. [5]

6 [Continued]

7 **(a)** An arithmetic sequence $a_1, a_2, a_3, \ldots$ has common difference d, where $d < 0$. The sum of the first n terms of the sequence is denoted by S_n. Given that $|a_8| = |a_{13}|$, find the value of n for which S_n is maximum. [4]

(b) The terms u_1, u_2 and u_3 are three consecutive terms of a geometric progression. It is given that

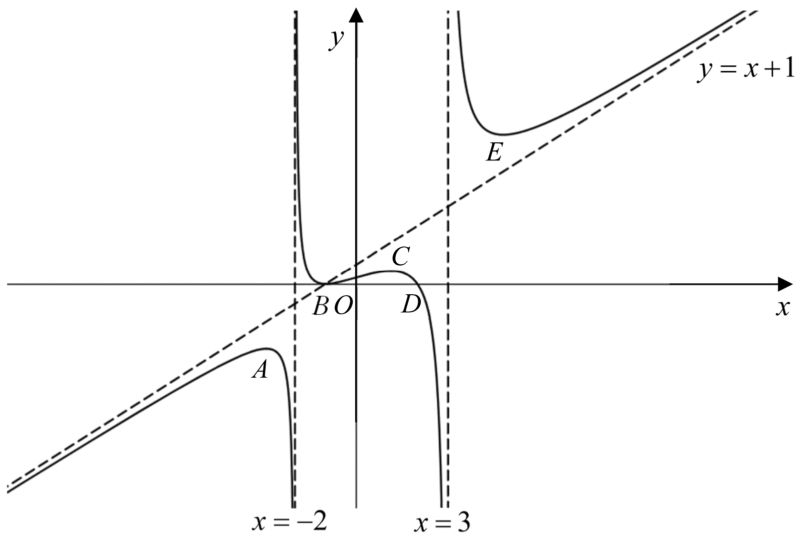
$$u_1,\ u_2 \text{ and } u_3-32$$

form an arithmetic progression, and that

$$u_1,\ u_2-4 \text{ and } u_3-32$$

form another geometric progression. Find the possible values of u_1, u_2 and u_3. [6]

8 The diagram below shows the graph of $y = \mathrm{f}(x)$ with asymptotes $x = -2$, $x = 3$ and $y = x + 1$. The curve intersects the x-axis at points B and D, and has turning points at points A, B, C and E. The coordinates of A, B, C, D and E are $(-3,-3)$, $(-1,0)$, $\left(\frac{3}{2},\frac{3}{4}\right)$, $(2,0)$ and $(5,8)$ respectively.



(i) By showing clearly the equations of asymptotes and the coordinates of any turning points and the points where the curve crosses the axes, where possible, sketch, on **separate diagrams**, the graphs of

(a) $y = \mathrm{f}(|x|)$, [4]

(b) $y = \frac{1}{\mathrm{f}(x)}$. [4]

(ii) By drawing another suitable graph on the same diagram in part **(i)(b)**, determine the number of solutions to the equation

$$\frac{x^2}{36} + \frac{1}{16\left[\mathrm{f}(x)\right]^2} = 1.$$ [2]

9 Distances in this question are in metres.

Harry and Tom's model airplanes are taking off from the horizontal ground, which is the *x*-*y* plane. Tom's airplane takes off after Harry's. The position of Harry's airplane t seconds after it takes off is given by $\mathbf{r} = (5\mathbf{i} + 6\mathbf{j}) + t(-4\mathbf{i} + 2\mathbf{j} + 4\mathbf{k})$. The position of Tom's airplane s seconds after it takes off is given by $\mathbf{r} = (-39\mathbf{i} + 44\mathbf{j}) + s(4\mathbf{i} - 6\mathbf{j} + 7\mathbf{k})$.

(i) State the height of Harry's airplane two seconds after it takes off and find its distance travelled in the two seconds. [3]

(ii) Find the acute angle between the path of Harry's airplane and the ground. [2]

(iii) Show that the paths of the airplanes are perpendicular. [1]

(iv) Given that the two airplanes collide, find the coordinates of the point of collision. How long after Harry's airplane takes off does Tom's airplane take off? [3]

(v) Find the cartesian equation of the plane in which both paths of the airplanes lie. [3]

10 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 4x - 2k \qquad \text{for } x \in \mathbb{R}, \text{ where } k \text{ is a constant,}$$
$$\mathrm{g} : x \mapsto \frac{9}{2-x} \qquad \text{for } x \in \mathbb{R},\ x \neq 2.$$

(i) Explain why gf does not exist. [1]

(ii) Find the range of values of k for which the equation $\mathrm{fg}(x) = x$ has real roots. [4]

For the rest of the question, let $k = 5$.

(iii) Sketch the graph of $y = \mathrm{fg}(x)$ for $x < 2$. Hence sketch the graph of $y = (\mathrm{fg})^{-1}(x)$ on the same diagram, showing clearly the relationship between the two graphs. [4]

10 [Continued]

The function h represents the height in metres of an object at time t seconds and is defined for the domain $0 \le t \le b$ by

$$\mathrm{h}(t) = \begin{cases} \dfrac{30}{7} + \mathrm{g}(t+9) & \text{for } 0 \le t \le a, \\ 2 - \mathrm{f}(t) & \text{for } a < t \le b, \end{cases}$$

where a and b are constants. At $t = 0$, the object was thrown up from 3 metres above the ground level. When $t = a$, the object started to descend and finally reached the ground at $t = b$.

(iv) Find the values of a and b. [2]

(v) Sketch the graph of $y = \mathrm{h}(t)$ for $0 \le t \le b$. [1]

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