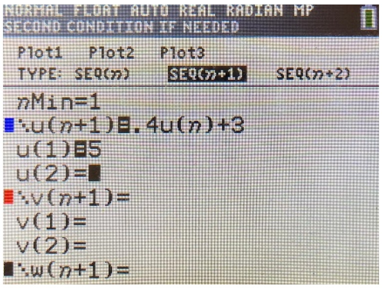
Q1	Suggested Solutions
(i)	$\dfrac{\sin\left[(n+1)\theta-n\theta\right]}{\cos n\theta\cos(n+1)\theta}$ $=\dfrac{\sin(n+1)\theta\cos n\theta-\cos(n+1)\theta\sin n\theta}{\cos n\theta\cos(n+1)\theta}$ $=\dfrac{\sin(n+1)\theta}{\cos(n+1)\theta}-\dfrac{\sin n\theta}{\cos n\theta}$ $=\tan(n+1)\theta-\tan n\theta$ (shown)
(ii)	$\sec\theta\sec 2\theta+\sec 2\theta\sec 3\theta+\sec 3\theta\sec 4\theta+...+\sec n\theta\sec(n+1)\theta$ $=\sum_{r=1}^{n}\sec r\theta\sec(r+1)\theta$ $=\sum_{r=1}^{n}\dfrac{1}{\cos r\theta\cos(r+1)\theta}$ $=\sum_{r=1}^{n}\left(\dfrac{\tan(r+1)\theta-\tan r\theta}{\sin\left[(r+1)\theta-r\theta\right]}\right)$ $=\dfrac{1}{\sin\theta}\sum_{r=1}^{n}\left(\tan(r+1)\theta-\tan r\theta\right)$ $=\dfrac{1}{\sin\theta}\left[\begin{array}{l}\cancel{\tan 2\theta}-\tan\theta\\+\cancel{\tan 3\theta}-\cancel{\tan 2\theta}\\+...\\+\cancel{\tan n\theta}-\cancel{\tan(n-1)\theta}\\+\tan(n+1)\theta-\cancel{\tan n\theta}\end{array}\right]$ $=\dfrac{\tan(n+1)\theta-\tan\theta}{\sin\theta}$

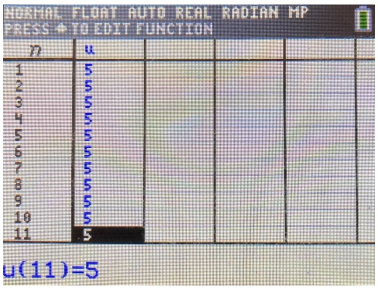
<table>
<tr><th>Q2</th><th>Suggested Solutions</th></tr>
<tr><td>(i)</td><td>

$u_{n+1} = 0.4u_n + 3$

</td></tr>
<tr><td>(ii)</td><td>

The number of bacteria in culture *A* remains constant at 5 million.





</td></tr>
<tr><td>(iii)</td><td>

$$v_n = \frac{pn}{n^2 + qn + r}$$

At the start of the 1st day, $n = 1$:

$$2 = \frac{p}{1+q+r} \Rightarrow 2 + 2q + 2r = p$$

$p - 2q - 2r = 2$

At the start of the 2nd day, $n = 2$:

$$2.4 = \frac{2p}{4+2q+r} \Rightarrow 9.6 + 4.8q + 2.4r = 2p$$

$2p - 4.8q - 2.4r = 9.6 \quad (\text{or } p - 2.4q - 1.2r = 4.8)$

At the start of the 4th day, $n = 4$:

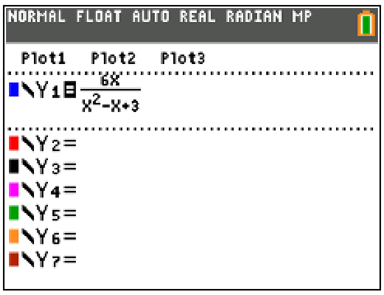
$$1.6 = \frac{4p}{16+4q+r} \Rightarrow 25.6 + 6.4q + 1.6r = 4p$$

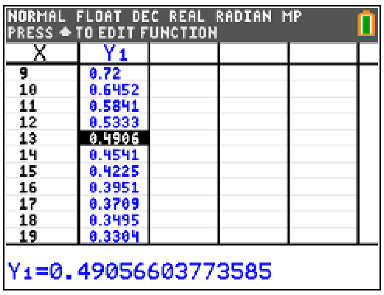
$4p - 6.4q - 1.6r = 25.6 \quad (\text{or } p - 1.6q - 0.4r = 6.4)$

By solving the system of linear equations using GC,
$p = 6,\ q = -1,\ r = 3$.

</td></tr>
<tr><td>(iv)</td><td>

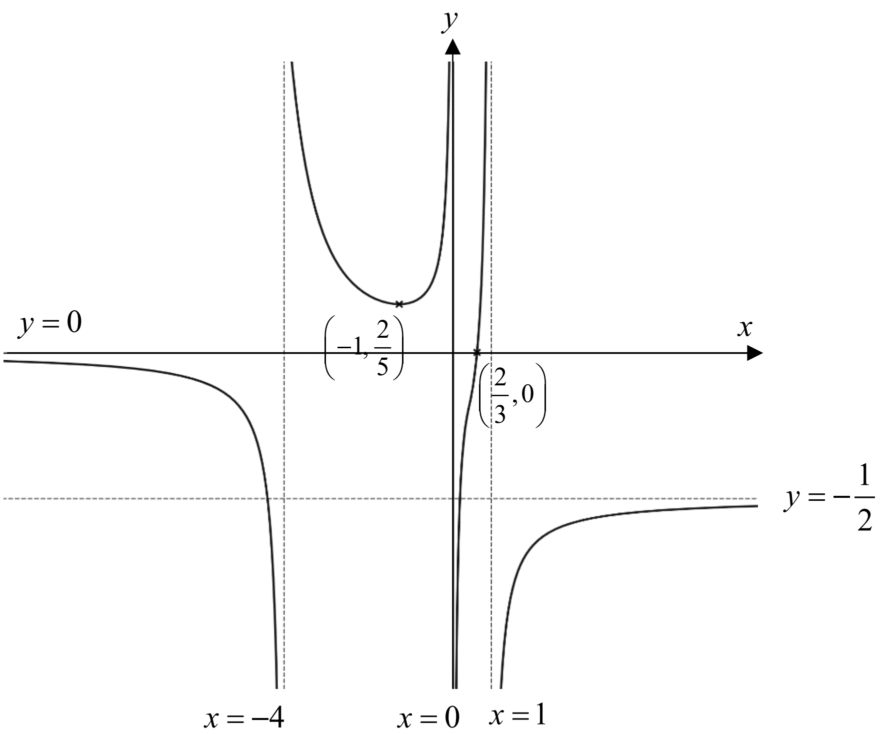
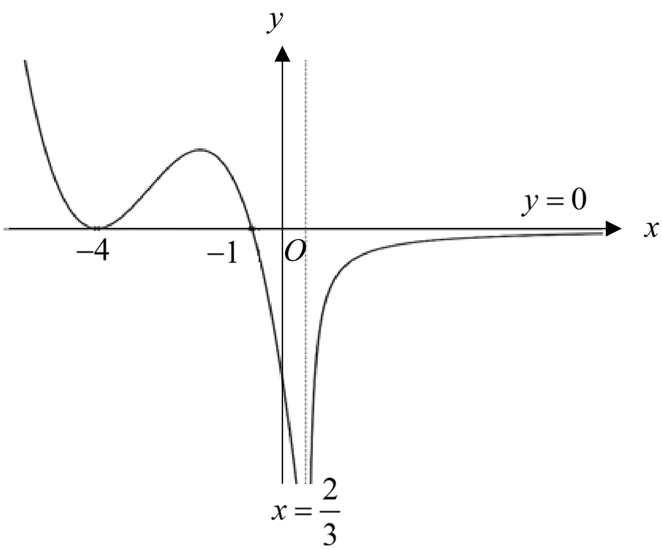
Find least *n* such that $v_n < 0.5$.





From table, least $n = 13$
Therefore, the researcher first record the number of bacteria in culture *B* to be below half a million on 13 April.

</td></tr>
</table>

Q3	Suggested Solutions
(i)	
(ii)	

Q4	**Suggested Solutions**
(i)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{\cos 2x}\right)=-2\mathrm{e}^{\cos 2x}\sin 2x$
(ii)	$\begin{aligned}\int \mathrm{e}^{\cos 2x}\sin 4x\ \mathrm{d}x &= -\int\left(-2\mathrm{e}^{\cos 2x}\sin 2x\right)\cos 2x\ \mathrm{d}x \\ &= -\mathrm{e}^{\cos 2x}\cos 2x+\int \mathrm{e}^{\cos 2x}\left(-2\sin 2x\right)\mathrm{d}x \\ &= -\mathrm{e}^{\cos 2x}\cos 2x+\mathrm{e}^{\cos 2x}+c \\ &= \mathrm{e}^{\cos 2x}\left(1-\cos 2x\right)+c\end{aligned}$
(iii)	$\int \mathrm{e}^{\cos 2x}\left(\cos 3x\sin x\right)\ \mathrm{d}x$ $=\dfrac{1}{2}\int \mathrm{e}^{\cos 2x}\left(\sin 4x-\sin 2x\right)\ \mathrm{d}x$ $=\dfrac{1}{2}\int \mathrm{e}^{\cos 2x}\left(\sin 4x\right)\ \mathrm{d}x+\dfrac{1}{2}\int \mathrm{e}^{\cos 2x}\left(-\sin 2x\right)\ \mathrm{d}x$ $=\dfrac{1}{2}\mathrm{e}^{\cos 2x}\left(1-\cos 2x\right)+\dfrac{1}{4}\mathrm{e}^{\cos 2x}+c$ $=\mathrm{e}^{\cos 2x}\left(\dfrac{3}{4}-\dfrac{1}{2}\cos 2x\right)+c$

Q5	**Suggested Solutions**
(i)	$\sin\left(\frac{\pi}{4}+\theta\right)=\frac{2}{AB} \Rightarrow AB=\frac{2}{\sin\left(\frac{\pi}{4}+\theta\right)}$ $\cos\left(\frac{\pi}{4}+\theta\right)=\frac{2}{BC} \Rightarrow BC=\frac{2}{\cos\left(\frac{\pi}{4}+\theta\right)}$ OR $\sin\left(\frac{\pi}{4}-\theta\right)=\frac{2}{BC}$ $BC=\frac{2}{\sin\left(\frac{\pi}{4}-\theta\right)}=\frac{2}{\cos\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-\theta\right)\right)}=\frac{2}{\cos\left(\frac{\pi}{4}+\theta\right)}$ $AC=AB+BC$ $=\frac{2}{\sin\left(\frac{\pi}{4}+\theta\right)}+\frac{2}{\cos\left(\frac{\pi}{4}+\theta\right)}$ $=2\left(\frac{1}{\sin\left(\frac{\pi}{4}+\theta\right)}+\frac{1}{\cos\left(\frac{\pi}{4}+\theta\right)}\right)$

Q5	Suggested Solutions
(ii)	$AC = 2\left(\dfrac{1}{\sin\left(\dfrac{\pi}{4}+\theta\right)}+\dfrac{1}{\cos\left(\dfrac{\pi}{4}+\theta\right)}\right)$ $= 2\left(\dfrac{1}{\sin\dfrac{\pi}{4}\cos\theta+\cos\dfrac{\pi}{4}\sin\theta}+\dfrac{1}{\cos\dfrac{\pi}{4}\cos\theta-\sin\dfrac{\pi}{4}\sin\theta}\right)$ $= 2\sqrt{2}\left(\dfrac{1}{\cos\theta+\sin\theta}+\dfrac{1}{\cos\theta-\sin\theta}\right)$ $\approx 2\sqrt{2}\left(\dfrac{1}{1-\dfrac{\theta^2}{2}+\theta}+\dfrac{1}{1-\dfrac{\theta^2}{2}-\theta}\right)$ $= 2\sqrt{2}\left[\left(1+\theta-\dfrac{\theta^2}{2}\right)^{-1}+\left(1-\theta-\dfrac{\theta^2}{2}\right)^{-1}\right]$ $= 2\sqrt{2}\left\{\left[1+(-1)\left(\theta-\dfrac{\theta^2}{2}\right)+\dfrac{(-1)(-2)}{2}\left(\theta-\dfrac{\theta^2}{2}\right)^2+\ldots\right]\right.$ $\left.+\left[1+(-1)\left(-\theta-\dfrac{\theta^2}{2}\right)+\dfrac{(-1)(-2)}{2}\left(-\theta-\dfrac{\theta^2}{2}\right)^2+\ldots\right]\right\}$ $= 2\sqrt{2}\left[\left(1-\theta+\dfrac{\theta^2}{2}+\theta^2+\ldots\right)+\left(1+\theta+\dfrac{\theta^2}{2}+\theta^2+\ldots\right)\right]$ $\approx 4\sqrt{2}+6\sqrt{2}\theta^2$

Q5	Suggested Solutions
	Alternative Method (Cosine double angle formula) $AC=2\left(\frac{1}{\sin\left(\frac{\pi}{4}+\theta\right)}+\frac{1}{\cos\left(\frac{\pi}{4}+\theta\right)}\right)$ $=2\left(\frac{1}{\sin\frac{\pi}{4}\cos\theta+\cos\frac{\pi}{4}\sin\theta}+\frac{1}{\cos\frac{\pi}{4}\cos\theta-\sin\frac{\pi}{4}\sin\theta}\right)$ $=2\sqrt{2}\left(\frac{1}{\cos\theta+\sin\theta}+\frac{1}{\cos\theta-\sin\theta}\right)$ $=2\sqrt{2}\left(\frac{2\cos\theta}{\cos^2\theta+\sin^2\theta}\right)$ $=2\sqrt{2}\left(\frac{2\cos\theta}{\cos 2\theta}\right)$ $\approx 4\sqrt{2}\left(\frac{1-\frac{\theta^2}{2}}{1-\frac{(2\theta)^2}{2}}\right)$ $=4\sqrt{2}\left(1-\frac{\theta^2}{2}\right)\left(1-2\theta^2\right)^{-1}$ $=4\sqrt{2}\left(1-\frac{\theta^2}{2}\right)\left(1+2\theta^2+\ldots\right)$ $=4\sqrt{2}\left(1+\frac{3}{2}\theta^2+\ldots\right)$ $\approx 4\sqrt{2}+6\sqrt{2}\theta^2$

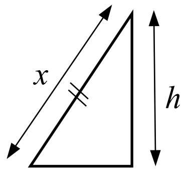
Q6	**Suggested Solutions**
(i)	$x=\tan\theta \Rightarrow \dfrac{dx}{d\theta}=\sec^2\theta$ When $x=0$, $\tan\theta=0 \quad\Rightarrow\quad \theta=0$ When $x=1$, $\tan\theta=1 \quad\Rightarrow\quad \theta=\dfrac{\pi}{4}$ $\displaystyle\int_0^1 \frac{1}{\sqrt{1+x^2}}\,dx=\int_0^{\frac{\pi}{4}}\frac{1}{\sqrt{1+\tan^2\theta}}\sec^2\theta\,d\theta$ $\displaystyle=\int_0^{\frac{\pi}{4}}\sec\theta\,d\theta$ $=\left[\ln\lvert\sec\theta+\tan\theta\rvert\right]_0^{\frac{\pi}{4}}$ $=\ln\left(\sec\dfrac{\pi}{4}+\tan\dfrac{\pi}{4}\right)-\ln\left(\sec 0+\tan 0\right)$ $=\ln\left(\sqrt{2}+1\right)-\ln\left(1+0\right)$ $=\ln\left(\sqrt{2}+1\right)$
(ii)	$A=\dfrac{1}{n}\left[\dfrac{1}{\sqrt{1+\left(\frac{1}{n}\right)^2}}+\dfrac{1}{\sqrt{1+\left(\frac{2}{n}\right)^2}}+\ldots+\dfrac{1}{\sqrt{1+\left(\frac{n-1}{n}\right)^2}}+\dfrac{1}{\sqrt{1+(1)^2}}\right]$ $=\dfrac{1}{\sqrt{n^2}}\left[\dfrac{1}{\sqrt{1+\left(\frac{1}{n}\right)^2}}+\dfrac{1}{\sqrt{1+\left(\frac{2}{n}\right)^2}}+\ldots+\dfrac{1}{\sqrt{1+\left(\frac{n-1}{n}\right)^2}}+\dfrac{1}{\sqrt{1+(1)^2}}\right]$ $=\dfrac{1}{\sqrt{n^2\left(1+\left(\frac{1}{n}\right)^2\right)}}+\dfrac{1}{\sqrt{n^2\left(1+\left(\frac{2}{n}\right)^2\right)}}+\ldots+\dfrac{1}{\sqrt{n^2\left(1+\left(\frac{n-1}{n}\right)^2\right)}}+\dfrac{1}{\sqrt{n^2\left(1+(1)^2\right)}}$ $=\dfrac{1}{\sqrt{n^2+1^2}}+\dfrac{1}{\sqrt{n^2+2^2}}+\ldots+\dfrac{1}{\sqrt{n^2+(n-1)^2}}+\dfrac{1}{\sqrt{n^2+n^2}}$ As $n\to\infty$, $A\to\ln\left(\sqrt{2}+1\right)$.

Q7	**Suggested Solutions**
(i)	Let d be the common difference. $a+(9-1)d=b$ $d=\dfrac{b-a}{8}$ $u_3=a+\dfrac{2(b-a)}{8}=\dfrac{3a+b}{4}$ $u_3+u_5+u_7$ $=\left(a+\dfrac{2(b-a)}{8}\right)+\left(a+\dfrac{4(b-a)}{8}\right)+\left(a+\dfrac{6(b-a)}{8}\right)$ $=3a+\dfrac{12(b-a)}{8}$ $=\dfrac{3}{2}(b+a)$
(ii)	$\dfrac{u_3}{a}=\dfrac{b}{u_3}$ $ab=(u_3)^2$ $=(a+2d)^2$ $=\left(a+\dfrac{b-a}{4}\right)^2$ $=\left(\dfrac{3a+b}{4}\right)^2$ $=\dfrac{9a^2+6ab+b^2}{16}$ $\dfrac{9a^2+6ab+b^2}{16}-ab=0$ $\dfrac{9a^2+6ab+b^2-16ab}{16}=0$ $9a^2-10ab+b^2=0$ $(9a-b)(a-b)=0$ Since the arithmetic progression is strictly increasing, $b\neq a$. Hence $b=9a$.

Q7	Suggested Solutions
(iii)	$\ln(u_5)-\ln(u_3)=\ln\left(\dfrac{a+\dfrac{4(9a-a)}{8}}{a+\dfrac{2(9a-a)}{8}}\right)$ $=\ln\left(\dfrac{a+4a}{a+2a}\right)$ $=\ln\left(\dfrac{5}{3}\right)$ $\ln(u_7)-\ln(u_5)=\ln\left(\dfrac{a+\dfrac{6(9a-a)}{8}}{a+\dfrac{4(9a-a)}{8}}\right)$ $=\ln\left(\dfrac{a+6a}{a+4a}\right)$ $=\ln\left(\dfrac{7}{5}\right)$ Since $\ln(u_7)-\ln(u_5)\neq\ln(u_5)-\ln(u_3)$, the terms are not consecutive terms of an arithmetic progression.

Q8	Suggested Solutions
(i)	$x^2+xy+ay^2=36$ $2x+x\frac{\mathrm{d}y}{\mathrm{d}x}+y+2ay\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $\left(x+2ay\right)\frac{\mathrm{d}y}{\mathrm{d}x}=-y-2x$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{y+2x}{x+2ay}$ For the normal to be parallel to y-axis, the tangent will be parallel to the x-axis. Hence $\frac{\mathrm{d}y}{\mathrm{d}x}=0$. Therefore, $y=-2x$. Substituting $y=-2x$: $x^2+xy+ay^2=36$ $x^2+x\left(-2x\right)+a\left(-2x\right)^2=36$ $x^2\left(4a-1\right)=36$ $x^2=\frac{36}{4a-1}$ $x=\frac{6}{\sqrt{4a-1}}$ or $-\frac{6}{\sqrt{4a-1}}$
(ii)	When $x=-2$, $\left(-2\right)^2-2y+\frac{5}{2}y^2=36$ $\frac{5}{2}y^2-2y-32=0$ $y=4$ or -3.2 by GC $x^2+xy+\frac{5}{2}y^2=36$ $x^2+xy+\left(\frac{5}{2}y^2-36\right)=0$ $x=\frac{-y\pm\sqrt{y^2-4(1)\left(2.5y^2-36\right)}}{2}$ $=\frac{-y\pm\sqrt{144-9y^2}}{2}$ Since R is in the region where $x\le-\frac{y}{2}$, $x=\frac{-y-\sqrt{144-9y^2}}{2}$

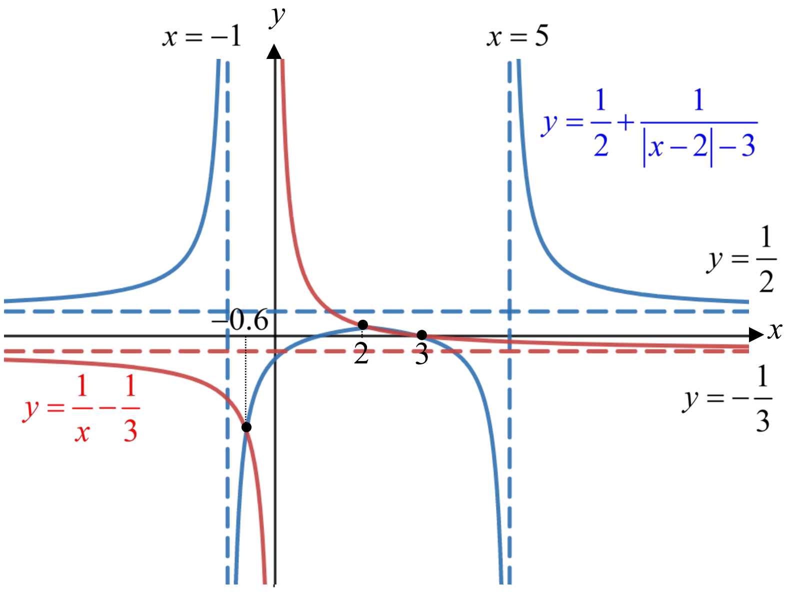
Q8	**Suggested Solutions**
	Required volume $= \pi \int_{-3.2}^{4} \left[\left(\frac{-y - \sqrt{144 - 9y^2}}{2} \right)^2 - (-2)^2 \right] \mathrm{d}y$ $= 542.8672117$ $= 542.87$ (2 d.p.)

Q9	Suggested Solutions
(i)	In Figure 1, considering the breadth of the rectangle, Base of the isosceles triangle $=a-2x$ and consider the triangle on the right and half the breadth of the rectangle, we have $x^2-h^2=\left(\frac{a}{2}-x\right)^2$ $=\frac{a^2}{4}-ax+x^2$ $x=\frac{a}{4}+\frac{h^2}{a}$ 
	Consider the length of the rectangle. We have $2a=y+2h \quad \Rightarrow \quad y=2a-2h$ $V=(\text{Area of isosceles triangle})\times y$ $=\frac{1}{2}(h)(a-2x)(y)$ $=\frac{1}{2}(h)\left[a-2\left(\frac{a}{4}+\frac{h^2}{a}\right)\right](2a-2h)$ $=\left(\frac{ah}{2}-\frac{2h^3}{a}\right)(a-h)$ $=\frac{a^2}{2}h-\frac{a}{2}h^2-2h^3+\frac{2}{a}h^4$ $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{a^2}{2}-ah-6h^2+\frac{8}{a}h^3$ For stationary V, $\frac{\mathrm{d}V}{\mathrm{d}h}=0$. $\frac{a^2}{2}-ah-6h^2+\frac{8}{a}h^3=0$ $16h^3-12ah^2-2a^2h+a^3=0$ $-16h^3+12ah^2+2a^2h-a^3=0$
(ii)	If $a=5$ $-16h^3+12ah^2+2a^2h-a^3=0$ $h=4.0451,-1.5451$ or 1.25 Since $h>0$, we reject $h=-1.5451$. Also if $h=4.0451$, $x\approx\frac{5}{4}+\frac{4.0451^2}{5}\approx4.522566$ and $2x\approx9.04513>5=a$. Thus the only possible value of h is 1.25.

Q9	**Suggested Solutions**
	$\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{a^2}{2}-ah-6h^2+\frac{8}{a}h^3$ $\frac{\mathrm{d}^2V}{\mathrm{d}h^2}=-a-12h+\frac{24}{a}h^2$ When $a=5$ and $h=1.25$, $\frac{\mathrm{d}^2V}{\mathrm{d}h^2}=-5-12(1.25)+\frac{24}{5}(1.25)^2$ $=-12.5<0$ Thus, V is maximum at $h=1.25$

Q10	Suggested Solutions
(i)	$\frac{\mathrm{d}x}{\mathrm{d}t}=-kx$, where k is a positive constant
(ii)	$\frac{\mathrm{d}x}{\mathrm{d}t}=-kx$ $\frac{1}{x}\frac{\mathrm{d}x}{\mathrm{d}t}=-k$ $\int\frac{1}{x}\,\mathrm{d}x=\int -k\,\mathrm{d}t$ $\ln\lvert x\rvert=-kt+C$ $\lvert x\rvert=\mathrm{e}^{-kt+C}$ $x=\pm\mathrm{e}^{-kt}.\mathrm{e}^{C}$ $x=A\mathrm{e}^{-kt}$, where $A=\pm\mathrm{e}^{C}$ When $t=0$, $x=x_0$, $A=x_0$. When $t=6$, $x=\frac{x_0}{1000}$. $\frac{x_0}{1000}=x_0\mathrm{e}^{-6k}$ $\mathrm{e}^{-6k}=\frac{1}{1000}$ $k=\frac{\ln 1000}{6}=\frac{1}{2}\ln 10=\ln\sqrt{10}$ $x=x_0\mathrm{e}^{\left(-\ln\sqrt{10}\right)t}$ $=x_0\mathrm{e}^{\left(\ln\frac{1}{\sqrt{10}}\right)t}$ $=\frac{x_0}{\left(\sqrt{10}\right)^t}$
	$\frac{x_0}{\left(\sqrt{10}\right)^t}=\frac{1}{4}x_0$ $\left(\sqrt{10}\right)^t=4$ $\ln\left(\sqrt{10}\right)^t=\ln 4$ $t\left(\ln\sqrt{10}\right)=\ln 4$ $t=\frac{1}{\left(\ln\sqrt{10}\right)}\ln 4=1.20$ hours (3 s.f.)

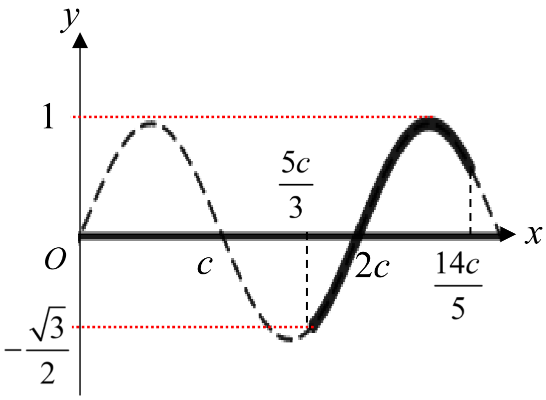
Q10	Suggested Solutions
(iii)	From part **(i)**, we have $x=\dfrac{x_0}{\left(\sqrt{10}\right)^t}$. Let the time from 2nd dose be *R*. Then $R=t-T$. Just after 2nd dose, amount of antibiotics in patient's body is $x_0+\dfrac{x_0}{\left(\sqrt{10}\right)^T}$. Replace initial dose $'x_o'$ with '$x_0+\dfrac{x_0}{\left(\sqrt{10}\right)^T}$'. Therefore, we have the amount of antibiotics in the patient's body after the second dose $\dfrac{\text{amount of antibiotics in body just after 2nd dose}}{\left(\sqrt{10}\right)^R}$ $=\dfrac{x_0+\dfrac{x_0}{\left(\sqrt{10}\right)^T}}{\left(\sqrt{10}\right)^{t-T}}$ and before the third dose is $=\dfrac{x_0}{\left(\sqrt{10}\right)^{t-T}}+\dfrac{x_0}{\left(\sqrt{10}\right)^{t-T+T}}$ $=x_0\left(10^{-\frac{1}{2}}\right)^{t-T}+x_0\left(10^{-\frac{1}{2}}\right)^t$ $=x_0\left(10^{\frac{1}{2}(T-t)}+10^{\left(-\frac{t}{2}\right)}\right)$

Q11	Suggested Solutions
(i)	We sketch the curves $y=\frac{1}{2}+\frac{1}{\lvert x-2\rvert-3}$ and $y=\frac{1}{x}-\frac{1}{3}$.  From graph, for $\frac{1}{2}+\frac{1}{\lvert x-2\rvert-3}\geq\frac{1}{x}-\frac{1}{3}$, $x<-1$ or $-0.6\leq x<0$ or $2\leq x\leq 3$ or $x>5$.
(ii)	Since $x<2$ (given domain is $-1<x<1$), $\frac{1}{2}+\frac{1}{\lvert x-2\rvert-3}=\frac{1}{2}+\frac{1}{-(x-2)-3}=\frac{1}{2}-\frac{1}{x+1}$ $y=\frac{1}{2}-\frac{1}{x+1}$ $\frac{1}{x+1}=\frac{1}{2}-y$ $\frac{1}{x+1}=\frac{1-2y}{2}$ $x+1=\frac{2}{1-2y}$ $x=\frac{2}{1-2y}-1$ $f^{-1}(x)=\frac{2}{1-2x}-1$ Consider graph of $f(x)=\frac{1}{2}-\frac{1}{x+1}$ for $-1<x<1$ in part **(i).** $D_{f^{-1}}=R_f=(-\infty,0)$

<table>
<tr><th>Q11</th><th>Suggested Solutions</th></tr>
<tr><td>(iii)</td><td>

$\mathrm{g}(x)=\sin\left(\dfrac{\pi x}{c}\right),\ x\in\mathbb{R},\ \dfrac{5c}{3}\le x<\dfrac{14c}{5}$

Period of g = $\dfrac{2\pi}{\pi/c}=2c$



When $x=\dfrac{5c}{3}$, $\sin\left(\dfrac{\pi x}{c}\right)=\sin\left(\dfrac{5\pi}{3}\right)=\sin\left(-\dfrac{\pi}{3}\right)=-\dfrac{\sqrt{3}}{2}$

$\mathrm{R_g}=\left[-\dfrac{\sqrt{3}}{2},1\right]$

</td></tr>
<tr><td>(iv)</td><td>

Let $(\mathrm{fh})^{-1}\left(-\dfrac{1}{2}\right)=x$

$\mathrm{fh}\left((\mathrm{fh})^{-1}\left(-\dfrac{1}{2}\right)\right)=\mathrm{fh}(x)$

$-\dfrac{1}{2}=\mathrm{fh}(x)$

$\mathrm{f}^{-1}\left(-\dfrac{1}{2}\right)=\mathrm{f}^{-1}\mathrm{fh}(x)$

$\mathrm{f}^{-1}\left(-\dfrac{1}{2}\right)=\mathrm{h}(x)$

$\mathrm{h}(x)=\dfrac{2}{1-2(-0.5)}-1=0$

$\sin\left(\dfrac{\pi x}{c}\right)=0$

$\dfrac{\pi x}{c}=0\pm 2k\pi$ where $k\in\mathbb{Z}$

Since $\dfrac{5c}{3}\le x<\dfrac{7c}{3}$, therefore $\dfrac{5\pi}{3}\le\dfrac{\pi x}{c}<\dfrac{7\pi}{3}$.

$\dfrac{\pi x}{c}=2\pi$

$x=2c$

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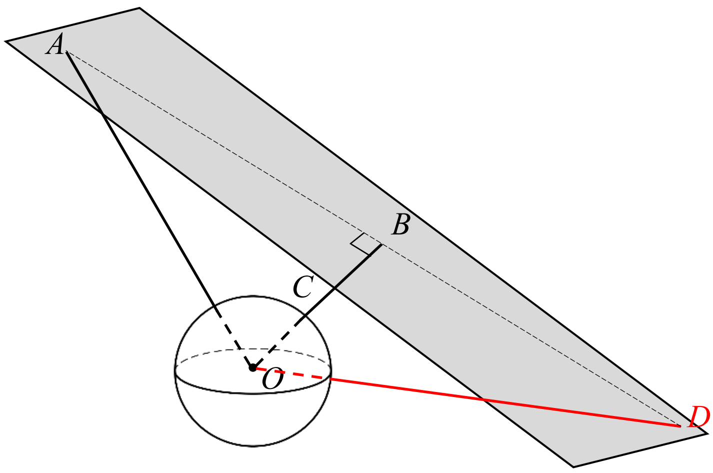
</table>

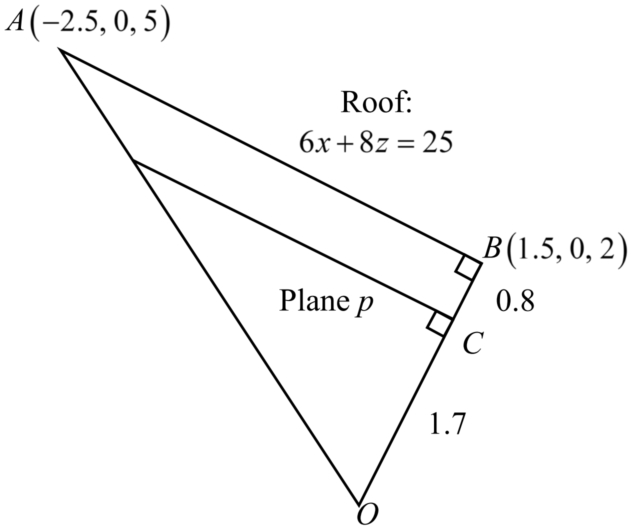
Q12	Suggested Solutions
(a)	$l_1:\ \mathbf{r}=\begin{pmatrix}10\\8\\8\end{pmatrix}+\lambda\begin{pmatrix}1\\14\\h\end{pmatrix}, \lambda\in\mathbb{R}$ $l_2:\ \mathbf{r}=\begin{pmatrix}s\\-10\\12\end{pmatrix}+\mu\begin{pmatrix}2\\2\\-5\end{pmatrix}, \mu\in\mathbb{R}$ l_1 and l_2 are perpendicular: $\begin{pmatrix}1\\14\\h\end{pmatrix}\cdot\begin{pmatrix}2\\2\\-5\end{pmatrix}=0$ $2+28-5h=0$ $h=6$ Suppose l_1 and l_2 intersect, $\begin{pmatrix}10+\lambda\\8+14\lambda\\8+6\lambda\end{pmatrix}=\begin{pmatrix}s+2\mu\\-10+2\mu\\12-5\mu\end{pmatrix}$ $\lambda \quad -2\mu \quad -s=-10$ $14\lambda \quad -2\mu \quad =-18$ $6\lambda \quad +5\mu \quad = 4$ By GC, $s=5$ Since the two lines do not intersect, $s\neq 5$. Hence $\underline{\underline{h=6}}$ and $\underline{\underline{s\in\mathbb{R},\ s\neq 5}}$.
(b)(i)	Given: *OB* is the 'shortest possible cord from the centre of the ball to the roof'. This implies that line $OB\perp$ plane.

Q12	**Suggested Solutions**
	$\overrightarrow{OB}$ = projection of $\overrightarrow{OA}$ onto the normal of the roof $=\left[\begin{pmatrix}-2.5\\0\\5\end{pmatrix}\cdot\frac{1}{\sqrt{6^2+8^2}}\begin{pmatrix}6\\0\\8\end{pmatrix}\right]\frac{1}{\sqrt{6^2+8^2}}\begin{pmatrix}6\\0\\8\end{pmatrix}$ $=\frac{1}{6^2+8^2}\left[\begin{pmatrix}-2.5\\0\\5\end{pmatrix}\cdot\begin{pmatrix}6\\0\\8\end{pmatrix}\right]\begin{pmatrix}6\\0\\8\end{pmatrix}$ $=\frac{-15+40}{100}\begin{pmatrix}6\\0\\8\end{pmatrix}$ $=\begin{pmatrix}1.5\\0\\2\end{pmatrix}$ $\therefore B(1.5,0,2)$ **Alternative Method** $l_{OB}:\ \mathbf{r}=\lambda\begin{pmatrix}6\\0\\8\end{pmatrix},\lambda\in\mathbb{R}$ Plane: $\mathbf{r}\cdot\begin{pmatrix}6\\0\\8\end{pmatrix}=25$ $\begin{pmatrix}6\lambda\\0\\8\lambda\end{pmatrix}\cdot\begin{pmatrix}6\\0\\8\end{pmatrix}=25$ $36\lambda+64\lambda=25$ $\lambda=0.25$ $\overrightarrow{OB}=0.25\begin{pmatrix}6\\0\\8\end{pmatrix}=\begin{pmatrix}1.5\\0\\2\end{pmatrix}$ $\therefore B(1.5,0,2)$

(b)(i)	Shortest distance from O to roof, OB $= \left\lvert \begin{pmatrix} 1.5 \\ 0 \\ 2 \end{pmatrix} \right\rvert$ $= \sqrt{(1.5)^2 + 2^2}$ $= 2.5$ Since the radius of the ball is 1 unit. Shortest distance between surface of ball to roof $= 2.5 - 1$ $= 1.5$ metres
(b) (ii)	 Point B is the foot of perpendicular of A onto line OB. $\overrightarrow{OB} = \dfrac{\overrightarrow{OA} + \overrightarrow{OD}}{2}$ $\overrightarrow{OD} = 2\overrightarrow{OB} - \overrightarrow{OA}$ $= 2\begin{pmatrix} 1.5 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} -2.5 \\ 0 \\ 5 \end{pmatrix}$ $= \begin{pmatrix} 5.5 \\ 0 \\ -1 \end{pmatrix}$ $l_{OD} : \mathbf{r} = \mu \begin{pmatrix} 5.5 \\ 0 \\ -1 \end{pmatrix}, \mu \in \mathbb{R}$

(b) (iii)

Side view



Let C be the point on the ball that is nearest to the roof.

$$\overrightarrow{OC}=1.7\frac{\overrightarrow{OB}}{\left|\overrightarrow{OB}\right|}=1.7\frac{\begin{pmatrix}1.5\\0\\2\end{pmatrix}}{\sqrt{1.5^2+2^2}}=0.68\begin{pmatrix}1.5\\0\\2\end{pmatrix}=\begin{pmatrix}1.02\\0\\1.36\end{pmatrix}$$

$$\begin{pmatrix}1.02\\0\\1.36\end{pmatrix}\cdot\begin{pmatrix}6\\0\\8\end{pmatrix}=17$$

Equation of plane p is $6x+8z=17$.

Alternative Method

For any plane with equation $\mathbf{r}\cdot\mathbf{n}=D$:

Shortest distance from O to plane $=\dfrac{|D|}{|\mathbf{n}|}$

Shortest distance from O to plane p $=2.5-0.8=1.7$

Since plane p is parallel to the roof, $\mathbf{n} \mathbin{/\!/} \begin{pmatrix}6\\0\\8\end{pmatrix}$.

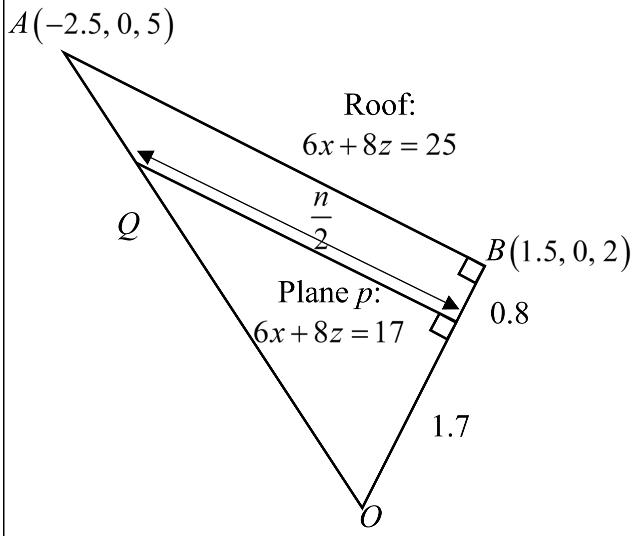
Therefore,

$$1.7=\frac{|D|}{\sqrt{6^2+8^2}}\Rightarrow|D|=17$$

Since plane p are the roof are on the same side of O, $D=17$ (same sign as ‘25’ from equation of the roof).

Equation of plane p is $6x+8z=17$.

(b) (iv)	Side view



$AB = \sqrt{(1.5+2.5)^2 + (2-5)^2} = 5$

By similar triangles,

$$\frac{n/2}{5} = \frac{1.7}{1.7+0.8}$$

$$n = 6.8$$

Therefore, largest **integer** n is 6.

Alternative Method

Let Q be the point of intersection between plane p and OA.

$$\alpha\begin{pmatrix}-2.5\\0\\5\end{pmatrix}\cdot\begin{pmatrix}6\\0\\8\end{pmatrix} = 17$$

$$\alpha(-15+40) = 17$$

$$\alpha = \frac{17}{25}$$

$$\overrightarrow{OQ} = \frac{17}{25}\begin{pmatrix}-2.5\\0\\5\end{pmatrix} = \begin{pmatrix}-1.7\\0\\3.4\end{pmatrix}$$

$$OQ = \sqrt{(-1.7)^2 + (3.4)^2} = \sqrt{14.45}$$

By Pythagoras Theorem,

$$OQ^2 = \left(\frac{n}{2}\right)^2 + (1.7)^2$$

$$\frac{n^2}{4} = 14.45 - (1.7)^2$$

$$n^2 = 46.24$$

$$n = 6.8 \text{ since } n > 0$$

Therefore, largest **integer** n is 6.