O-LEVEL ADDITIONAL MATH 2021 – PAPER 1

Question 1

[Ans: $4-\sqrt{3}$]

Width of rectangle

$$= \frac{\text{Area}}{\text{Length}} = \frac{6+5\sqrt{3}}{3+2\sqrt{3}}$$

$$= \frac{6+5\sqrt{3}}{3+2\sqrt{3}}\left(\frac{3-2\sqrt{3}}{3-2\sqrt{3}}\right)$$

$$= \frac{18-12\sqrt{3}+15\sqrt{3}-10(3)}{3^2-\left(2\sqrt{3}\right)^2}$$

$$= \frac{-12+3\sqrt{3}}{9-12} = \frac{-12+3\sqrt{3}}{-3}$$

$$= 4-\sqrt{3}$$

Question 2

[Ans: -5 and 4]

$xy+10=0$ (1)

$x+2y+1=0$

$x=-2y-1$ (2)

Sub. (2) into (1)

$$(-2y-1)y+10=0$$

$$-2y^2-y+10=0$$

$$(2y+5)(-y+2)=0$$

$$y=-\frac{5}{2} \text{ or } y=2$$

When $y=-\frac{5}{2}$, $x=-2\left(-\frac{5}{2}\right)-1=4$

When $y=2$, $x=-2(2)-1=-5$

The x-coordinate of P and Q are -5 and 4.

Question 3

[Ans: $-2(x+3)^2+21$; $(-3, 21)$]

$$3-12x-2x^2$$
$$=-2\left(x^2+6x-\frac{3}{2}\right)$$
$$=-2\left[(x+3)^2-3^2-\frac{3}{2}\right]$$
$$=-2\left[(x+3)^2-\frac{21}{2}\right]=-2(x+3)^2+21$$

Coordinates of turning point of $y=3-12x-2x^2$ is $(-3, 21)$.

Question 4

[Ans: $-\frac{3}{x}+\frac{4}{3}\ln(3x-5)+C$]

$$\int \frac{3}{x^2}+\frac{4}{3x-5}\,dx$$
$$=\int 3x^{-2}+\frac{4}{3x-5}\,dx$$
$$=3\left(\frac{x^{-1}}{-1}\right)+4\frac{\ln(3x-5)}{3}+C$$
$$=-\frac{3}{x}+\frac{4}{3}\ln(3x-5)+C$$

Queston 5

[Ans: $-\frac{3}{x}+\frac{2}{x^2}+\frac{6}{2x-3}$]

Let $\frac{13x-6}{x^2(2x-3)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{2x-3}$

$13x-6=Ax(2x-3)+B(2x-3)+Cx^2$

Let $x=0$,

$13(0)-6=0+B[2(0)-3]+0$

$-6=-3B\Rightarrow B=2$

$13x-6=Ax(2x-3)+2(2x-3)+Cx^2$

Let $x=\frac{3}{2}$,

$13\left(\frac{3}{2}\right)-6=0+0+C\left(\frac{3}{2}\right)^2$

$\frac{27}{2}=\frac{9}{4}C\Rightarrow C=6$

$13x-6=Ax(2x-3)+2(2x-3)+6x^2$

Let $x=1$,

$13(1)-6=A(1)[2(1)-3]+2[2(1)-3]+6(1)^2$

$7=-A+4\Rightarrow A=-3$

$\frac{13x-6}{x^2(2x-3)}=-\frac{3}{x}+\frac{2}{x^2}+\frac{6}{2x-3}$

Question 6

[Ans: (a) $k=20$ (b) $a=-5$]

(a) Let $P(x)=2x^3-x^2-13x+k$

$P(2)=6$

$2(2)^3-(2)^2-13(2)+k=6$

$-14+k=6\Rightarrow k=20$

(b) Given $P(x)=2x^3-x^2-13x-6$

Let $2x^3-x^2-13x-6=(2x^2+ax-3)(bx+c)$

By observation, $b=1$ and $c=2$

$\therefore 2x^3-x^2-13x-6=(2x^2+ax-3)(x+2)$

Coefficient of x^2: $-1=4+a\Rightarrow a=-5$

Question 7

[Ans: (a) explain (b) explain (c) $y = 5\sin 4x - 3$]

(a) $a = \dfrac{2-(-8)}{2} = 5$

$c = 2 - 5 = -3$

(b) $\dfrac{2\pi}{b} = \dfrac{\pi}{2}$

$b = 4$

(c) $y = 5\sin 4x - 3$

Question 8

[Ans: (a) show (b) $A = 1600$; minimum]

(a) $CQ = 80 - 2x$

$DP = 50 - x$

$$A = (50)(80) - \frac{1}{2}(80)(x) - \frac{1}{2}(50)(80-2x) - \frac{1}{2}(2x)(50-x)$$

$$= 4000 - 40x - 2000 + 50x - 50x + x^2$$

$= x^2 - 40x + 2000$ (shown)

(b) $\dfrac{dA}{dx} = 2x - 40$

Let $\dfrac{dA}{dx} = 0$

$2x - 40 = 0 \Rightarrow x = 20$

Stationay value of $A = 20^2 - 40(20) + 2000 = 1600$

$\dfrac{d^2A}{dx^2} = 2 > 0$

This value of A is at a minimum.

Question 9

[Ans: (a) show (b) $(-6,0)$ (c) 60 units²]

(a) $BC = CD$

$\sqrt{(0-6)^2 + (k-6)^2} = \sqrt{(6-h)^2 + (6-0)^2}$

$36 + (k-6)^2 = (6-h)^2 + 36$

$(k-6)^2 = (6-h)^2$

$(k-6)^2 - (6-h)^2 = 0$

$[(k-6)+(6-h)][(k-6)-(6-h)] = 0$

$(k-h)(k+h-12) = 0$

$k = h$ (NA) or $k+h-12=0 \Rightarrow h+k=12$ (shown)

(b) For $h = 4$,

$4 + k = 12 \Rightarrow k = 8$

Let $A(a,0)$.

$AB = AD$

$\sqrt{(a-0)^2 + (0-8)^2} = \sqrt{(a-4)^2 + (0-0)^2}$

$a^2 + 64 = a^2 - 8a + 16$

$8a = -48 \Rightarrow a = -6$

$\therefore A(-6,0)$

(c) Area of kite

$= \frac{1}{2}\begin{vmatrix} -6 & 4 & 6 & 0 & -6 \\ 0 & 0 & 6 & 8 & 0 \end{vmatrix}$

$= \frac{1}{2}[(72)-(-48)] = 60$

Question 10

[Ans: (a) show (b) $\theta = 76.7°, 166.7°$]

(a) LHS

$$= \frac{\sin\theta}{1-\cos\theta} - \frac{1}{\sin\theta}$$

$$= \frac{\sin^2\theta - (1-\cos\theta)}{\sin\theta(1-\cos\theta)}$$

$$= \frac{1-\cos^2\theta - 1 + \cos\theta}{\sin\theta(1-\cos\theta)}$$

$$= \frac{\cos\theta - \cos^2\theta}{\sin\theta(1-\cos\theta)}$$

$$= \frac{\cos\theta(1-\cos\theta)}{\sin\theta(1-\cos\theta)} = \cot\theta = \text{RHS}$$

(b) $\frac{\sin 2\theta}{1-\cos 2\theta} - \frac{1}{\sin 2\theta} = -2$

$\cot 2\theta = -2$

$\tan 2\theta = -\frac{1}{2}$

Reference $\angle = \tan^{-1}\frac{1}{2} = 26.565°$

$0° \le \theta \le 180° \Rightarrow 0° \le 2\theta \le 360°$

$2\theta = 180° - 26.565°, 360° - 26.565°$

$\theta = 76.7°, 166.7°$

Question 11

[Ans: (a) prove (b) prove]

(a) $\angle TCA = \angle ABC$ (alternate segment theorem)

$\angle TCA = \angle CAB$ (alternate $\angle$ s, $BA // CT$)

$\therefore \angle ABC = \angle CAB$

$\Rightarrow \Delta ABC$ is isosceles (base $\angle$ s of isosceles Δ) (proven)

(b) Let $\angle ABC = \angle CAB = \angle TCA = \theta$

$\angle TAC = \angle ABC = \theta$ (alternate segment theorem)

$\angle ABC + \angle CAB + \angle BCA = 180°$

$\Rightarrow \theta + \theta + \angle BCA = 180° \Rightarrow 2\theta + \angle BCA = 180°$

$\angle TAC + \angle TCA + \angle CTA = 180°$

$\Rightarrow \theta + \theta + \angle CTA = 180° \Rightarrow 2\theta + \angle CTA = 180°$

$\therefore \angle BCA = \angle CTA$ (proven)

Question 12

[Ans: (a) show (b) $x^3+2x^2-81=0$]

(a) $6^x=5\times 3^{x+1}$

$6^x=5\times 3^x\times 3$

$\dfrac{6^x}{3^x}=15\Rightarrow\left(\dfrac{6}{3}\right)^x=15$

$2^x=15$

$\lg 2^x=\lg 15$

$x\lg 2=\lg 15\Rightarrow x=\dfrac{\lg 15}{\lg 2}$ (shown)

(b) $\log_3 x+\log_9(x+2)=2$

$\log_3 x+\dfrac{\log_3(x+2)}{\log_3 9}=2$

$\log_3 x+\dfrac{\log_3(x+2)}{2\log_3 3}=2\Rightarrow\log_3 x+\dfrac{\log_3(x+2)}{2}=2$

$2\log_3 x+\log_3(x+2)=4\log_3 3$

$\log_3 x^2+\log_3(x+2)=\log_3 3^4$

$\log_3\left[x^2(x+2)\right]=\log_3 81$

$x^2(x+2)=81\Rightarrow x^3+2x^2-81=0$

Question 13

[Ans: (a)(i) 19.3 cm (ii) 0.107 cm/s (b) $-\dfrac{5}{3}$]

(a) (i) Volume, $V=500\times 60=30000$

$\dfrac{4}{3}\pi r^3=30000\Rightarrow r^3=\dfrac{22500}{\pi}\Rightarrow r=\sqrt[3]{\dfrac{22500}{\pi}}=19.3$

(ii) $V=\dfrac{4}{3}\pi r^3$

$\dfrac{dV}{dr}=4\pi r^2$

After 1 minute,

$\dfrac{dV}{dt}=500$ (given), $r=\sqrt[3]{\dfrac{22500}{\pi}}$ (from (i))

$\dfrac{dV}{dr}=4\pi r^2=4\pi\left(\sqrt[3]{\dfrac{22500}{\pi}}\right)^2$

$\dfrac{dV}{dt}=\dfrac{dV}{dr}\times\dfrac{dr}{dt}\Rightarrow 500=\left[4\pi\left(\sqrt[3]{\dfrac{22500}{\pi}}\right)^2\right]\times\dfrac{dr}{dt}\Rightarrow\dfrac{dr}{dt}=0.107$

(b) $PV=k$

$$V=\frac{k}{P}=kP^{-1}$$

$$\frac{dV}{dP}=k\left(-P^{-2}\right)=-\frac{k}{P^2}$$

$$=-\frac{PV}{P^2}=-\frac{V}{P}$$

When $V=2$,
$P=1.2$ (given)

$$\frac{dV}{dP}=-\frac{2}{1.2}=-\frac{5}{3}$$

Question 14

[Ans: (a) $Q\left(\frac{11}{2},0\right)$ (b) $\frac{139}{9}$ units²]

(a) $y=\left(4+3x\right)^{\frac{1}{2}}$

$$\frac{dy}{dx}=\frac{1}{2}\left(4+3x\right)^{-\frac{1}{2}}\left(3\right)=\frac{3}{2\sqrt{4+3x}}$$

At $P\left(4,4\right)$,

$$\frac{dy}{dx}=\frac{3}{2\sqrt{4+3\left(4\right)}}=\frac{3}{8}$$

Equation of normal:

$$y-4=-\frac{8}{3}\left(x-4\right)\Rightarrow y=-\frac{8}{3}x+\frac{44}{3}$$

At Q,

$$y=0\Rightarrow-\frac{8}{3}x+\frac{44}{3}=0\Rightarrow x=\frac{11}{2}$$

$$\therefore Q\left(\frac{11}{2},0\right)$$

(b) Area

$$=\int_0^4\left(4+3x\right)^{\frac{1}{2}}dx+\frac{1}{2}\left(\frac{11}{2}-4\right)\left(4\right)$$

$$=\left[\frac{\left(4+3x\right)^{\frac{3}{2}}}{\left(3\right)\left(\frac{3}{2}\right)}\right]_0^4+3=\frac{2}{9}\left[\left(4+12\right)^{\frac{3}{2}}-\left(4\right)^{\frac{3}{2}}\right]+3=\frac{139}{9}$$