

A-LEVEL H2 MATH 2020 – PAPER 1

Question 1

$$[\text{Ans: (i) } \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \text{ (ii) } 49.2^\circ]$$

$$(i) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ -6 \end{pmatrix} = -2 \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$$

$$\text{A vector normal to } \pi_1 = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$$

$$(ii) \pi_2 : \vec{r} \cdot \begin{pmatrix} 4 \\ 5 \\ -6 \end{pmatrix} = 0$$

Let θ be the acute angle between π_1 and π_2 .

$$\left| \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 5 \\ -6 \end{pmatrix} \right| = \left| \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \right| \left| \begin{pmatrix} 4 \\ 5 \\ -6 \end{pmatrix} \right| \cos \theta$$

$$|4 - 5 - 18| = \sqrt{1^2 + 1^2 + 3^2} \sqrt{4^2 + 5^2 + 6^2} \cos \theta$$

$$\cos \theta = \frac{|-19|}{\sqrt{11}\sqrt{77}}$$

$$\theta = \cos^{-1} \left(\frac{19}{\sqrt{11}\sqrt{77}} \right) = 49.2^\circ$$

Question 2

[Ans: $5x + 9y = 14$]

$$\frac{d}{dx} \left(\frac{x^2}{1+x^2} + \frac{y^2}{1+y^2} \right) = \frac{d}{dx} (x^3 y^5)$$

$$\frac{(1+x^2)(2x) - x^2(2x)}{(1+x^2)^2} + \left[\frac{(1+y^2)(2y) - y^2(2y)}{(1+y^2)^2} \right] \frac{dy}{dx} = x^3 \left(5y^4 \frac{dy}{dx} \right) + 3x^2 y^5$$

$$\frac{(1+x^2-x^2)(2x)}{(1+x^2)^2} + \left[\frac{(1+y^2-y^2)(2y)}{(1+y^2)^2} \right] \frac{dy}{dx} = 5x^3 y^4 \frac{dy}{dx} + 3x^2 y^5$$

$$\frac{2x}{(1+x^2)^2} + \frac{2y}{(1+y^2)^2} \frac{dy}{dx} = 5x^3 y^4 \frac{dy}{dx} + 3x^2 y^5$$

At (1,1),

$$\frac{2}{(1+1)^2} + \frac{2}{(1+1)^2} \frac{dy}{dx} = 5(1)(1) \frac{dy}{dx} + 3(1)(1)$$

$$\frac{1}{2} + \frac{1}{2} \frac{dy}{dx} = 5 \frac{dy}{dx} + 3$$

$$\frac{9}{2} \frac{dy}{dx} = -\frac{5}{2} \Rightarrow \frac{dy}{dx} = -\frac{5}{9}$$

Equation of tangent:

$$y - 1 = -\frac{5}{9}(x - 1)$$

$$-9y + 9 = 5x - 5 \Rightarrow 5x + 9y = 14$$

Question 3

[Ans: (i) show; $k = -9$ (ii) $f(x) = 3x - \frac{9}{2}x^2 + \frac{9}{2}x^3 + \dots$]

(i) $f(x) = \ln(1 + \sin 3x)$

$$f'(x) = \frac{1}{1 + \sin 3x} (3 \cos 3x) = \frac{3 \cos 3x}{1 + \sin 3x}$$

$$f''(x) = \frac{(1 + \sin 3x)[3(-3 \sin 3x)] - 3 \cos 3x(3 \cos 3x)}{(1 + \sin 3x)^2}$$

$$= \frac{-9 \sin 3x - 9 \sin^2 3x - 9 \cos^2 3x}{(1 + \sin 3x)^2}$$

$$= \frac{-9 - 9 \sin 3x}{(1 + \sin 3x)^2}$$

$$= \frac{-9(1 + \sin 3x)}{(1 + \sin 3x)^2} = -\frac{9}{1 + \sin 3x}, k = -9$$

(ii) $f(0) = \ln(1 + 0) = 0$

$$f'(0) = \frac{3(1)}{1+0} = 3$$

$$f''(0) = -\frac{9}{1+0} = -9$$

$$f'''(x) = -9(-1)(1 + \sin 3x)^{-2} (3 \cos 3x) = \frac{27 \cos 3x}{(1 + \sin 3x)^2}$$

$$f'''(0) = \frac{27(1)}{(1+0)^2} = 27$$

$$f(x) = 0 + (3)x + \frac{(-9)}{2!}x^2 + \frac{27}{3!}x^3 + \dots$$

$$= 3x - \frac{9}{2}x^2 + \frac{9}{2}x^3 + \dots$$

Question 4

$$[\text{Ans: (i) } \frac{1}{\sqrt{2}} \left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right) \text{ (ii) } z_4 = \sqrt{2} \left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right) \text{ or } z_4 = \sqrt{2} \left[\cos \left(-\frac{11\pi}{12} \right) + i \sin \left(-\frac{11\pi}{12} \right) \right]]$$

$$(i) |z_1| = \sqrt{1^2 + (\sqrt{3})^2} = 2; \arg(z_1) = \tan^{-1} \frac{\sqrt{3}}{1} = \frac{\pi}{3}$$

$$\therefore z_1 = 2e^{i\frac{\pi}{3}}$$

$$|z_2| = \sqrt{1^2 + 1^2} = \sqrt{2}; \arg(z_2) = -\tan^{-1} \frac{1}{1} = -\frac{\pi}{4}$$

$$\therefore z_2 = \sqrt{2}e^{-i\frac{\pi}{4}}$$

$$\therefore z_3 = 2e^{i\frac{\pi}{6}}$$

$$\begin{aligned} \frac{z_1}{z_2 z_3} &= \frac{2e^{i\frac{\pi}{3}}}{\left(\sqrt{2}e^{-i\frac{\pi}{4}}\right)\left(2e^{i\frac{\pi}{6}}\right)} = \frac{1}{\sqrt{2}} e^{i\left(\frac{\pi}{3} + \frac{\pi}{4} - \frac{\pi}{6}\right)} = \frac{1}{\sqrt{2}} e^{i\frac{5\pi}{12}} \\ &= \frac{1}{\sqrt{2}} \left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right) \end{aligned}$$

$$(ii) \text{ Let } z_4 = re^{i\theta}$$

$$\begin{aligned} \frac{z_1 z_4}{z_2 z_3} &= \left(\frac{z_1}{z_2 z_3} \right) z_4 = \left(\frac{1}{\sqrt{2}} e^{i\frac{5\pi}{12}} \right) (re^{i\theta}) \\ &= \frac{r}{\sqrt{2}} e^{i\left(\frac{5\pi}{12} + \theta\right)} = \frac{r}{\sqrt{2}} \left[\cos \left(\frac{5\pi}{12} + \theta \right) + i \sin \left(\frac{5\pi}{12} + \theta \right) \right] \end{aligned}$$

$$\left| \frac{z_1 z_4}{z_2 z_3} \right| = 1 \Rightarrow \frac{r}{\sqrt{2}} = 1 \Rightarrow r = \sqrt{2}$$

$$\operatorname{Re} \left(\frac{z_1 z_4}{z_2 z_3} \right) = 0$$

$$\cos \left(\frac{5\pi}{12} + \theta \right) = 0$$

$$\theta = \frac{\pi}{2} - \frac{5\pi}{12}, -\frac{\pi}{2} - \frac{5\pi}{12} = \frac{\pi}{12}, -\frac{11\pi}{12}$$

$$\therefore z_4 = \sqrt{2} \left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right) \text{ or } z_4 = \sqrt{2} \left[\cos \left(-\frac{11\pi}{12} \right) + i \sin \left(-\frac{11\pi}{12} \right) \right]$$

Question 5

[Ans: (a) \underline{a} and \underline{b} are parallel to each other(b)(i) The set of all possible positions of the point R consists of all the points lying on a line that passes through the point P and parallel to vector \underline{q} .(b)(ii) $3x - 5y + 2z = -5$; The set of all possible positions of the point R consists of all thepoints lying on a plane that passes through the point P and normal to vector $\begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix}$.]

(a) $\underline{a} \times \underline{b} = \underline{b} \times \underline{a}$

$$\underline{a} \times \underline{b} - \underline{b} \times \underline{a} = \underline{0}$$

$$\underline{a} \times \underline{b} + \underline{a} \times \underline{b} = \underline{0}$$

$$2\underline{a} \times \underline{b} = \underline{0} \Rightarrow \underline{a} \times \underline{b} = \underline{0}$$

 $\therefore \underline{a}$ and \underline{b} are parallel to each other.

(b) (i) $(\underline{r} - \underline{p}) \times \underline{q} = \underline{0} \Rightarrow \underline{r} - \underline{p} = \lambda \underline{q}, \lambda \in \mathbb{R}$

$$\therefore \underline{r} = \underline{p} + \lambda \underline{q}, \lambda \in \mathbb{R}$$

 \therefore The set of all possible positions of the point R consists of all the points lying on a line that passes through the point P and parallel to vector \underline{q} .

(ii) $(\underline{r} - \underline{p}) \cdot \underline{q} = 0$

$$\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix} \right) \cdot \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix} = 0$$

$$\begin{pmatrix} x+1 \\ y-2 \\ z-4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix} = 0$$

$$3x + 3 - 5y + 10 + 2z - 8 = 0$$

$$3x - 5y + 2z = -5$$

The set of all possible positions of the point R consists of all the points lying on aplane that passes through the point P and normal to vector $\begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix}$.

Question 6

[Ans: $t = -8$, $k = 2$; $-\frac{2}{5} + \frac{6}{5}i$]

$$\text{Let } f(z) = z^2(2+i) - 8iz + t.$$

$$f(k+ki) = 0$$

$$(k+ki)^2(2+i) - 8i(k+ki) + t = 0$$

$$k^2(1+i)^2(2+i) - 8ki + 8k + t = 0$$

$$k^2(-2+4i) - 8ki + 8k + t = 0$$

$$(-2k^2 + 8k + t) + (4k^2 - 8k)i = 0$$

$$\therefore 4k^2 - 8k = 0$$

$$4k(k-2) = 0$$

$$k = 0 \text{ (NA) or } k = 2$$

$$-2k^2 + 8k + t = 0$$

$$t = 2k^2 - 8k = 2(2)^2 - 8(2)$$

$$= -8$$

$$z^2(2+i) - 8iz - 8 = 0$$

$$z = \frac{-(-8i) \pm \sqrt{(-8i)^2 - 4(2+i)(-8)}}{2(2+i)}$$

$$= \frac{8i \pm (4+4i)}{2(2+i)}$$

$$z = \frac{8i - (4+4i)}{2(2+i)}$$

$$= -\frac{2}{5} + \frac{6}{5}i$$

or

$$z = \frac{8i + (4+4i)}{2(2+i)}$$

$$= 2 + 2i$$

$$\therefore \text{ the other root of the equation is } -\frac{2}{5} + \frac{6}{5}i.$$

Question 7

[Ans: (i) $2x + \frac{1}{4} \cos 4x + c$ (ii) $\frac{1}{8} \pi (2\pi + 1)$ (iii) $\frac{9}{4} \pi$]

$$(i) \int f(x) dx = \int 2 - \sin 4x dx = 2x + \frac{1}{4} \cos 4x + c$$

$$(ii) \int_0^{\frac{1}{2}\pi} xf(x) dx$$

$$u = x, \quad \frac{dv}{dx} = f(x)$$

$$\frac{du}{dx} = 1, \quad v = 2x + \frac{1}{4} \cos 4x$$

$$= \left[(x) \left(2x + \frac{1}{4} \cos 4x \right) \right]_0^{\frac{1}{2}\pi} - \int_0^{\frac{1}{2}\pi} 2x + \frac{1}{4} \cos 4x dx$$

$$= \left[\left(\frac{1}{2} \pi \right) \left(\pi + \frac{1}{4} \cos 2\pi \right) - (0) \right] - \left[x^2 + \frac{1}{16} \sin 4x \right]_0^{\frac{1}{2}\pi}$$

$$= \frac{1}{2} \pi^2 + \frac{1}{8} \pi - \left[\left(\frac{1}{4} \pi^2 + 0 \right) - (0) \right]$$

$$= \frac{1}{4} \pi^2 + \frac{1}{8} \pi = \frac{1}{8} \pi (2\pi + 1)$$

$$(iii) \int_0^{\frac{1}{2}\pi} (f(x))^2 dx$$

$$= \int_0^{\frac{1}{2}\pi} (2 - \sin 4x)^2 dx$$

$$= \int_0^{\frac{1}{2}\pi} 4 - 4 \sin 4x + \sin^2 4x dx$$

$$= \int_0^{\frac{1}{2}\pi} 4 - 4 \sin 4x + \frac{1 - \cos 8x}{2} dx = \int_0^{\frac{1}{2}\pi} \frac{9}{2} - 4 \sin 4x - \frac{1}{2} \cos 8x dx$$

$$= \left[\frac{9}{2} x + \cos 4x - \frac{1}{16} \sin 8x \right]_0^{\frac{1}{2}\pi}$$

$$= \left[\frac{9}{4} \pi + \cos 2\pi - \frac{1}{16} \sin 4\pi \right] - \left[0 + \cos 0 - \frac{1}{16} \sin 0 \right]$$

$$= \left(\frac{9}{4} \pi + 1 \right) - (1) = \frac{9}{4} \pi$$

Question 8

[Ans: (a)(i) $\frac{95}{2}$ (ii) $\frac{3345}{2}$ (i) 20 (ii) 18](a) (i) Let d be the common difference of the arithmetic series.

5th term = 10

$$4 + (5-1)d = 10 \Rightarrow d = \frac{3}{2}$$

$$\therefore \text{30th term of this series} = 4 + (30-1)\left(\frac{3}{2}\right) = \frac{95}{2}$$

(ii) Sum of first 20 terms = $\frac{20}{2} \left[2(4) + (20-1)\left(\frac{3}{2}\right) \right] = 365$

Sum of first 50 terms = $\frac{50}{2} \left[2(4) + (50-1)\left(\frac{3}{2}\right) \right] = \frac{4075}{2}$

Required sum = $\frac{4075}{2} - 365 = \frac{3345}{2}$

(b) (i) Let r be the common ratio of the geometric series.

5th term = 1.6384

$$4r^{5-1} = 1.6384 \Rightarrow r^4 = 0.4096 \Rightarrow r = 0.8 (\because r > 0)$$

$$\therefore \text{Sum to infinity} = \frac{4}{1-0.8} = 20$$

(ii) Sum of first n terms > 19.6

$$\frac{4(1-0.8^n)}{1-0.8} > 19.6$$

$$1-0.8^n > 19.6 \left(\frac{0.2}{4}\right)$$

$$0.8^n < 1 - 19.6 \left(\frac{0.2}{4}\right) \Rightarrow 0.8^n < 0.02 \text{ (shown)}$$

$$n \ln 0.8 < \ln 0.02$$

$$n > \frac{\ln 0.02}{\ln 0.8}$$

$$n > 17.531$$

 \therefore smallest value of $n = 18$

Question 9

[Ans: (i) explain (ii) $\left\{ k \in \mathbb{R} : -\frac{1}{2} \leq k \leq \frac{9}{2} \right\}$ (iii) sketch (iv) $\frac{1}{2}\pi - \frac{4}{3}\ln 2$]

- (i) Let the gradient of a line be m_1 and the angle it makes with the positive x -axis be θ_1 .
Let the gradient of another line be m_2 and the angle it makes with the positive x -axis be θ_2 .

$$\text{Let } m_1 = 2 \Rightarrow \tan \theta_1 = 2 \Rightarrow \theta_1 = \tan^{-1}(2)$$

$$\text{Let } m_2 = -\frac{1}{2} \Rightarrow \tan \theta_2 = -\frac{1}{2} \Rightarrow \theta_2 = \pi - \tan^{-1}\left(\frac{1}{2}\right) = \pi + \tan^{-1}\left(-\frac{1}{2}\right)$$

Since $m_1 m_2 = (2)\left(-\frac{1}{2}\right) = -1$, the two lines are perpendicular to each other.

$$\therefore \theta_2 - \theta_1 = \frac{1}{2}\pi$$

$$\Rightarrow \left[\pi + \tan^{-1}\left(-\frac{1}{2}\right) \right] - \tan^{-1}(2) = \frac{1}{2}\pi$$

$$\therefore \tan^{-1}(2) - \tan^{-1}\left(-\frac{1}{2}\right) = \frac{1}{2}\pi$$

(ii) $y = \frac{1}{x^2 + 1}$ (1) $y = \frac{k}{3x + 4}$ (2)

$$(1) = (2)$$

$$\frac{1}{x^2 + 1} = \frac{k}{3x + 4}$$

$$3x + 4 = kx^2 + k$$

$$kx^2 - 3x + (k - 4) = 0$$

Let Discriminant ≥ 0

$$(-3)^2 - 4k(k - 4) \geq 0$$

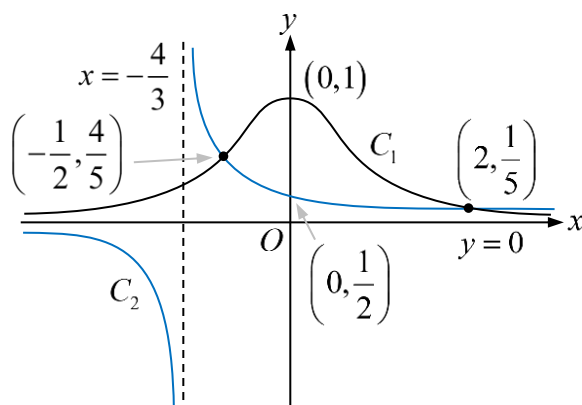
$$4k^2 - 16k - 9 \leq 0$$

$$(2k + 1)(2k - 9) \leq 0$$

$$-\frac{1}{2} \leq k \leq \frac{9}{2}$$

$$\therefore \left\{ k \in \mathbb{R} : -\frac{1}{2} \leq k \leq \frac{9}{2} \right\}$$

(iii) $C_1: y = \frac{1}{x^2+1}$, $C_2: y = \frac{2}{3x+4}$



(iv) Area

$$\begin{aligned}
 &= \int_{-\frac{1}{2}}^2 \frac{1}{x^2+1} - \frac{2}{3x+4} dx \\
 &= \left[\tan^{-1} x - \frac{2}{3} \ln|3x+4| \right]_{-\frac{1}{2}}^2 \\
 &= \left[\tan^{-1}(2) - \frac{2}{3} \ln 10 \right] - \left[\tan^{-1}\left(-\frac{1}{2}\right) - \frac{2}{3} \ln \frac{5}{2} \right] \\
 &= \left[\tan^{-1}(2) - \tan^{-1}\left(-\frac{1}{2}\right) \right] + \left(\frac{2}{3} \ln \frac{5}{2} - \frac{2}{3} \ln 10 \right) \\
 &= \frac{1}{2} \pi + \frac{2}{3} \ln \left(\frac{5/2}{10} \right) = \frac{1}{2} \pi + \frac{2}{3} \ln \left(\frac{1}{4} \right) = \frac{1}{2} \pi - \frac{4}{3} \ln 2
 \end{aligned}$$

Question 10

[Ans: (i) $\frac{dP}{dt} = -0.03P$ (ii) $P = Ce^{-0.03t}$; the number of sheep will go to zero

(iii) $\frac{dP}{dt} = n - 0.03P$ (iv) $P = \frac{100}{3}n + De^{-0.03t}$ (v) 15]

$$(i) \frac{dP}{dt} = -0.03P$$

$$(ii) \frac{1}{P} \frac{dP}{dt} = -0.03$$

$$\int \frac{1}{P} dP = -0.03 \int dt$$

$$\ln|P| = -0.03t + A$$

$$|P| = e^{-0.03t+A} = e^A e^{-0.03t}$$

$$P = e^A e^{-0.03t}$$

$$= Be^{-0.03t}, B = e^A$$

When $t \rightarrow \infty$, $P \rightarrow 0$

\therefore if this situation continues over many years, the number of sheep will go to zero.

$$(iii) \frac{dP}{dt} = n - 0.03P$$

$$(iv) \frac{1}{n - 0.03P} \frac{dP}{dt} = 1$$

$$\int \frac{1}{n - 0.03P} dP = \int dt$$

$$\frac{1}{-0.03} \ln|n - 0.03P| = t + C$$

$$\ln|n - 0.03P| = -0.03t - 0.03C$$

$$|n - 0.03P| = e^{-0.03t - 0.03C} = e^{-0.03C} e^{-0.03t}$$

$$n - 0.03P = \pm e^{-0.03C} e^{-0.03t}$$

$$n - 0.03P = \pm e^{-0.03C} e^{-0.03t}$$

$$P = \frac{100}{3}n \pm \frac{100}{3} e^{-0.03C} e^{-0.03t}$$

$$= \frac{100}{3}n + De^{-0.03t}, D = \pm \frac{100}{3} e^{-0.03C}$$

(v) When $t \rightarrow \infty$, $P \rightarrow 500$

$$\therefore \frac{100}{3}n = 500 \Rightarrow n = 15$$

Question 11

[Ans: (i) show (ii) $x = \sqrt{4a + a^2}$; $\tan \theta = \frac{2}{\sqrt{4a + a^2}}$ (iii) explain (iv) show; $\frac{\pi}{4}$

(v) $0.0801 \leq \theta < \frac{\pi}{2}$]

$$\begin{aligned}
 \text{(i) } \tan \theta &= \tan(AKD - AKC) \\
 &= \frac{\tan AKD - \tan AKC}{1 + (\tan AKD)(\tan AKC)} \\
 &= \frac{\frac{4+a}{x} - \frac{a}{x}}{1 + \left(\frac{4+a}{x}\right)\left(\frac{a}{x}\right)} \\
 &= \frac{4+a-a}{\frac{x^2+4a+a^2}{x^2}} = \frac{4x}{x^2+4a+a^2} \text{ (shown)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) Let } y = \tan \theta &= \frac{4x}{x^2+4a+a^2} \\
 \frac{dy}{dx} &= \frac{(x^2+4a+a^2)(4) - (4x)(2x)}{(x^2+4a+a^2)^2} = \frac{-4x^2+16a+4a^2}{(x^2+4a+a^2)^2}
 \end{aligned}$$

$$\text{Let } \frac{dy}{dx} = 0$$

$$\frac{-4x^2+16a+4a^2}{(x^2+4a+a^2)^2} = 0$$

$$-4x^2+16a+4a^2 = 0$$

$$x^2 = 4a+a^2 \Rightarrow x = \sqrt{4a+a^2}$$

$$\text{When } x = \sqrt{4a+a^2},$$

$$\begin{aligned}
 \tan \theta &= \frac{4\sqrt{4a+a^2}}{(4a+a^2)+4a+a^2} = \frac{4\sqrt{4a+a^2}}{2(4a+a^2)} \\
 &= \frac{2}{\sqrt{4a+a^2}}
 \end{aligned}$$

(iii) At the optimal point, although $\tan \theta$ is maximized, a player may decide not to take the kick if the physical distance to get the ball between C and D is perceived to be beyond what he/she can likely achieve.

(iv) When θ is the optimal angle, $x = \sqrt{4a + a^2}$,

$$\begin{aligned}\tan KDA &= \frac{x}{4+a} = \frac{\sqrt{4a+a^2}}{4+a} = \frac{\sqrt{a}\sqrt{4+a}}{4+a} \\ &= \frac{\sqrt{a}}{\sqrt{4+a}} = \sqrt{\frac{a}{4+a}} \text{ (shown)}\end{aligned}$$

When a is much greater than 4,

$$\tan KDA \approx \sqrt{\frac{a}{a}} = 1$$

$$KDA \approx \frac{\pi}{4}$$

(v) At optimal angle,

$$\tan \theta = \frac{2}{\sqrt{4a+a^2}}$$

θ is the greatest when $a=0$ (A coincides with C),

$\tan \theta$ is undefined

$$\theta \rightarrow \frac{\pi}{2}$$

θ is the least when $a = \frac{50-4}{2} = 23$ (A coincides with X),

$$\tan \theta = \frac{2}{\sqrt{4(23)+(23)^2}}$$

$$\theta = \tan^{-1} \frac{2}{\sqrt{621}} = 0.080086$$

$$\therefore 0.0801 \leq \theta < \frac{\pi}{2}$$