O-LEVEL A-MATHS 2020 – PAPER 1

Question 1

[Ans: $16x^2+7x+16=0$]

For $2x^2-5x+8=0$,

Sum of roots $=\alpha+\beta=-\dfrac{(-5)}{2}=\dfrac{5}{2}$

Product of roots $=\alpha\beta=\dfrac{8}{2}=4$

For the new quadratic equation,

Sum of new roots $=\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=\dfrac{\alpha^2+\beta^2}{\alpha\beta}=\dfrac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$$=\frac{\left(\frac{5}{2}\right)^2-2(4)}{4}=-\frac{7}{16}$$

Product of new roots $=\left(\dfrac{\alpha}{\beta}\right)\left(\dfrac{\beta}{\alpha}\right)=1$

New equation:

$x^2-\left(-\dfrac{7}{16}\right)x+1=0$

$16x^2+7x+16=0$

Question 2

[Ans: (a) $a=-1$, $b=\dfrac{7}{2}$, $c=-5$ (b) \$1500000]

(a) $\left[\left(\dfrac{50}{3}\right)^{-2}\times\sqrt{3^3}\right]\div\dfrac{5}{2}$

$$=\left[\left(2\times5^2\times3^{-1}\right)^{-2}\times3^{\frac{3}{2}}\right]\times\left(\frac{2}{5}\right)$$

$$=\left(2^{-2}3^{2}3^{\frac{3}{2}}5^{-4}\right)\times\left(2^{1}5^{-1}\right)=2^{-1}3^{\frac{7}{2}}5^{-5}$$

$\therefore a=-1,\ b=\dfrac{7}{2},\ c=-5$

(b) Value of painting at beginning of $2014=1000000$

Value of painting at beginning of $2015=1000000(1.07)$

Value of painting at beginning of $2016=1000000(1.07)(1.07)=1000000(1.07)^2$

Value of painting at beginning of $2017=1000000(1.07)^3$

⋮

Value of painting at beginning of $2020=1000000(1.07)^6=1500730.352\approx1500000$

Question 3

[Ans: $2+\frac{3}{2x-3}-\frac{4}{x+1}$]

$$\begin{array}{r} 2 \\ 2x^2-x-3\overline{\big)4x^2-7x+9} \\ -\left(4x^2-2x-6\right) \\ \hline -5x+15 \end{array}$$

$$\frac{4x^2-7x+9}{2x^2-x-3}=2+\frac{-5x+15}{2x^2-x-3}=2+\frac{-5x+15}{(2x-3)(x+1)}$$

Let $\frac{-5x+15}{(2x-3)(x+1)}=\frac{A}{2x-3}+\frac{B}{x+1}$

$-5x+15=A(x+1)+B(2x-3)$

When $x=-1$,

$-5(-1)+15=0+B\left[2(-1)-3\right]$

$-5B=20\Rightarrow B=-4$

When $x=0$,

$0+15=A(0+1)+(-4)(0-3)$

$12+A=15\Rightarrow A=3$

$$\therefore 2+\frac{-5x+15}{(2x-3)(x+1)}=2+\frac{3}{2x-3}-\frac{4}{x+1}$$

Question 4

[Ans: $-1<x<4$]

Given $y=\frac{2x-3}{x^2+4}$

$$\frac{dy}{dx}=\frac{(x^2+4)(2)-(2x-3)(2x)}{(x^2+4)^2}=\frac{-2x^2+6x+8}{(x^2+4)^2}$$

Let $\frac{dy}{dx}=0$

$$\frac{-2x^2+6x+8}{(x^2+4)^2}=0$$

$x^2-3x-4=0$

$(x+1)(x-4)=0$

$x=-1$ or $x=4$

From the graph of $y=\frac{2x-3}{x^2+4}$, y is increasing when $-1<x<4$.

Queston 5

[Ans: explain]

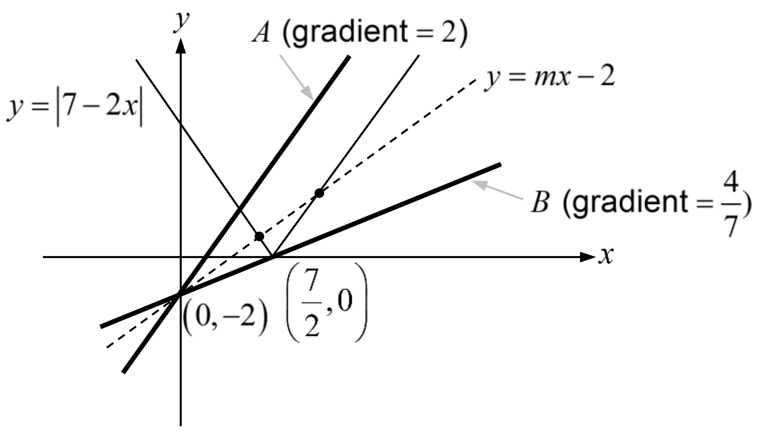
For $y = |7-2x|$, when $y = 0$,

$$|7-2x| = 0 \Rightarrow x = \frac{7}{2}$$

Gradient of line passing through $(0, -2)$ and $\left(\frac{7}{2}, 0\right)$

$$= \frac{0-(-2)}{\frac{7}{2}-0} = \frac{4}{7}$$

For line $y = mx - 2$ to cut $y = |7-2x|$ at two distinct points, line $y = mx - 2$ will need to be pivoted at $(0, -2)$ and being between the two lines A and B.



$\therefore$ based on gradient of $y = mx - 2$, $\frac{4}{7} < m < 2$.

Question 6

[Ans: $\left(-\sqrt[4]{4}, 4\right)$, minimum; $\left(\sqrt[4]{4}, 4\right)$, minimum]

$$y = x^2 + \frac{4}{x^2} = x^2 + 4x^{-2}$$

$$\frac{dy}{dx} = 2x + 4\left(-2x^{-3}\right) = 2x - 8x^{-3} = 2x - \frac{8}{x^3}$$

$$\frac{d^2y}{dx^2} = 2 - 8\left(-3x^{-4}\right) = 2 + 24x^{-4} = 2 + \frac{24}{x^4}$$

Let $\frac{dy}{dx} = 0$

$$2x - \frac{8}{x^3} = 0$$

$$2x = \frac{8}{x^3}$$

$$x^4 = 4 \Rightarrow x = \pm\sqrt[4]{4}$$

When $x = -\sqrt[4]{4}$,

$$y = \left(-\sqrt[4]{4}\right)^2 + \frac{4}{\left(-\sqrt[4]{4}\right)^2} = 2 + \frac{4}{2} = 4$$

$$\frac{d^2y}{dx^2} = 2 + \frac{24}{\left(-\sqrt[4]{4}\right)^4} = 2 + \frac{24}{4} = 8 > 0$$

$\therefore$ There is a stationary point at $\left(-\sqrt[4]{4}, 4\right)$, and it is a minimum point.

When $x = \sqrt[4]{4}$,

$$y = \left(\sqrt[4]{4}\right)^2 + \frac{4}{\left(\sqrt[4]{4}\right)^2} = 2 + \frac{4}{2} = 4$$

$$\frac{d^2y}{dx^2} = 2 + \frac{24}{\left(\sqrt[4]{4}\right)^4} = 2 + \frac{24}{4} = 8 > 0$$

$\therefore$ There is a stationary point at $\left(\sqrt[4]{4}, 4\right)$, and it is a minimum point.

Question 7

[Ans: $199.5°$ or $340.5°$]

$3\cos A = \sec A - 5\tan A$

$3\cos A = \frac{1}{\cos A} - \frac{5\sin A}{\cos A}$

$3\cos^2 A = 1 - 5\sin A$

$3\left(1-\sin^2 A\right) = 1 - 5\sin A$

$3\sin^2 A - 5\sin A - 2 = 0$

$\left(3\sin A + 1\right)\left(\sin A - 2\right) = 0$

$3\sin A + 1 = 0$ or $\sin A - 2 = 0$

$\sin A = -\frac{1}{3}$ $\sin A = 2$ (NA)

Basic $\measuredangle = \sin^{-1}\frac{1}{3}$

$A = 180° + \sin^{-1}\frac{1}{3}, 360° - \sin^{-1}\frac{1}{3}$

$= 199.5°, 340.5°$

Question 8

[Ans: (a) $a = -3$ (b) $x = 3$, $\frac{-1-\sqrt{5}}{2}$ or $\frac{-1+\sqrt{5}}{2}$]

(a) Let $f(x) = x^3 + ax$

$f(2) = f(-1)$

$(2)^3 + a(2) = (-1)^3 + a(-1)$

$8 + 2a = -1 - a$

$3a = -9 \Rightarrow a = -3$

(b) Let $g(x) = x^3 - 2x^2 - 4x + 3$

$g(3) = (3)^3 - 2(3)^2 - 4(3) + 3 = 0$

$\therefore x - 3$ is a factor of $g(x)$.

$$\begin{array}{r|l} & x^2 + x - 1 \\ x-3 & x^3 - 2x^2 - 4x + 3 \\ & -(x^3 - 3x^2) \\ \hline & x^2 - 4x \\ & -(x^2 - 3x) \\ \hline & -x + 3 \\ & -(-x + 3) \\ \hline & 0 \end{array}$$

$\therefore g(x) = 0$

$(x-3)(x^2 + x - 1) = 0$

$x = 3$ or $x^2 + x - 1 = 0$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

$$= \frac{-1-\sqrt{5}}{2} \text{ or } \frac{-1+\sqrt{5}}{2}$$

Question 9

[Ans: (i) 5, $(-2,3)$ (ii) $y=-\frac{3}{4}x+\frac{31}{4}$]

(i) $x^2+y^2+4x-6y-12=0$

$\left(x^2+4x\right)+\left(y^2-6y\right)-12=0$

$\left(x+2\right)^2-2^2+\left(y-3\right)^2-3^2-12=0$

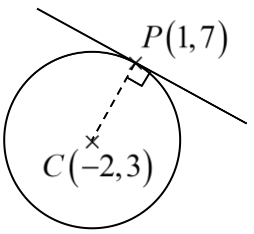
$\left(x+2\right)^2+\left(y-3\right)^2=25$

$\left[x-(-2)\right]^2+\left(y-3\right)^2=5^2$

Radius $=5$, center is at $(-2,3)$

(ii) $(1)^2+(7)^2+4(1)-6(7)-12=0$

$\therefore (1,7)$ lies on the circumference of the circle.



Gradient of $CP=\frac{7-3}{1-(-2)}=\frac{4}{3}$

Equation of tangent:

$y-7=-\frac{3}{4}(x-1)$

$y=-\frac{3}{4}x+\frac{31}{4}$

Question 10

[Ans: (i) $\frac{5\pi}{12}, \frac{7\pi}{12}$ (ii) $\left(\frac{1}{2} - \frac{\sqrt{3}\pi}{12}\right)$ units^2]

(i) Let $\cos 2x = -\frac{\sqrt{3}}{2}$

Basic $\measuredangle = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

$2x = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}$

$x = \frac{5\pi}{12}, \frac{7\pi}{12}$

x-coordinate of A is $\frac{5\pi}{12}$, and of B is $\frac{7\pi}{12}$.

(ii) Area

$= \int_{\frac{5\pi}{12}}^{\frac{7\pi}{12}} \left(-\frac{\sqrt{3}}{2} - \cos 2x\right) dx$

$= \left[-\frac{\sqrt{3}}{2}x - \frac{1}{2}\sin 2x\right]_{\frac{5\pi}{12}}^{\frac{7\pi}{12}}$

$= \left[-\frac{\sqrt{3}}{2}\left(\frac{7\pi}{12}\right) - \frac{1}{2}\sin 2\left(\frac{7\pi}{12}\right)\right] - \left[-\frac{\sqrt{3}}{2}\left(\frac{5\pi}{12}\right) - \frac{1}{2}\sin 2\left(\frac{5\pi}{12}\right)\right]$

$= \frac{1}{2}\left[\sin\left(\frac{5\pi}{6}\right) - \sin\left(\frac{7\pi}{6}\right)\right] + \frac{\sqrt{3}}{2}\left(\frac{5\pi}{12} - \frac{7\pi}{12}\right)$

$= \frac{1}{2}\left[\frac{1}{2} - \left(-\frac{1}{2}\right)\right] - \frac{\sqrt{3}\pi}{12}$

$= \frac{1}{2} - \frac{\sqrt{3}\pi}{12}$

Question 11

[Ans: (i) $30\,\text{m/s}$ (ii) $222\,\text{m}$ (iii) $1.41\,\text{m/s}^2$]

(i) At A, $v=15\left(\frac{0}{20}+e^{0}\right)=15$

At B, $v=2(15)=30$

(ii) Distance, s

$$=\int 15\left(\frac{t}{20}+e^{kt}\right)dt=15\left(\frac{t^2}{40}+\frac{e^{kt}}{k}\right)+C$$

When $t=0$,

$s=0$

$$15\left(\frac{0^2}{40}+\frac{e^{0}}{k}\right)+C=0\Rightarrow C=-\frac{15}{k}$$

When $t=10$

$v=30$

$$15\left(\frac{10}{20}+e^{10k}\right)=30$$

$$\frac{1}{2}+e^{10k}=2$$

$$e^{10k}=\frac{3}{2}$$

$$10k=\ln\frac{3}{2}\Rightarrow k=\frac{1}{10}\ln\frac{3}{2}$$

At B, $t=10$,

$$s=15\left[\frac{10^2}{40}+\frac{e^{\left(\frac{1}{10}\ln\frac{3}{2}\right)(10)}}{\left(\frac{1}{10}\ln\frac{3}{2}\right)}\right]-\frac{15}{\frac{1}{10}\ln\frac{3}{2}}=222$$

(iii) Acceration, $a=\frac{dv}{dt}=15\left(\frac{1}{20}+ke^{kt}\right)$

When $t=2$,

$$a=15\left[\frac{1}{20}+\left(\frac{1}{10}\ln\frac{3}{2}\right)e^{\left(\frac{1}{10}\ln\frac{3}{2}\right)(2)}\right]=1.41$$

Question 12

[Ans: (i) show (ii) $B > A$ (iii) $\frac{4}{5}$ (iv) $\frac{24}{25}$ (v) show (vi) $\frac{13}{9}$]

(i) LHS

$$= \frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)}$$

$$= \frac{(\sin\alpha\cos\beta+\cos\alpha\sin\beta)+(\sin\alpha\cos\beta-\cos\alpha\sin\beta)}{(\cos\alpha\cos\beta-\sin\alpha\sin\beta)+(\cos\alpha\cos\beta+\sin\alpha\sin\beta)}$$

$$= \frac{2\sin\alpha\cos\beta}{2\cos\alpha\cos\beta}$$

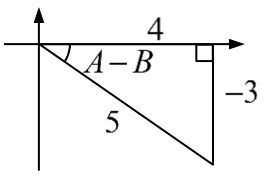
$= \frac{\sin\alpha}{\cos\alpha} = \tan\alpha =$ RHS (shown)

(ii) Since $\sin(A-B) < 0$ and angles A and B are acute,

$A - B < 0 \Rightarrow A < B$

$\therefore \measuredangle B$ must be bigger than $\measuredangle A$

(iii) Given $\sin(A-B) = -\frac{3}{5}$



$$\cos(A-B) = \frac{4}{5}$$

(iv) $\tan(A+B) = \frac{24}{7}$

$$\frac{\sin(A+B)}{\cos(A+B)} = \frac{24}{7}$$

$$\sin(A+B) = \frac{24}{7}\cos(A+B)$$

$$= \frac{24}{7}\left(\frac{7}{25}\right) = \frac{24}{25}$$

(v) $\tan A$

$$= \frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)}$$

$= \dfrac{\frac{24}{25}+\left(-\frac{3}{5}\right)}{\frac{7}{25}+\frac{4}{5}} = \frac{1}{3}$ (shown)

(vi) $\tan(A+B)=\frac{24}{7}$

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{24}{7}$$

$$\frac{\frac{1}{3}+\tan B}{1-\frac{1}{3}\tan B}=\frac{24}{7}$$

$$\frac{7}{3}+7\tan B=24-8\tan B$$

$$15\tan B=\frac{65}{3}$$

$$\tan B=\frac{13}{9}$$