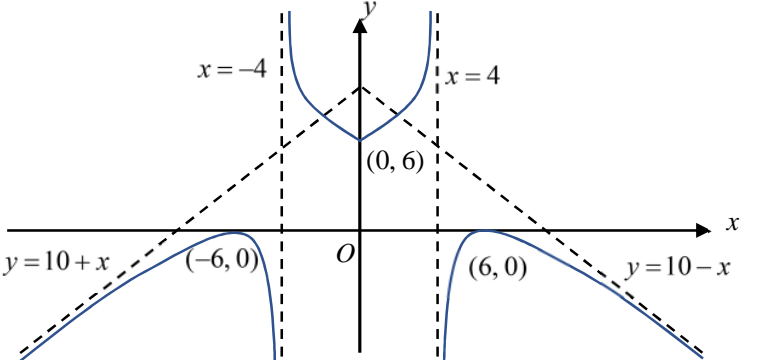
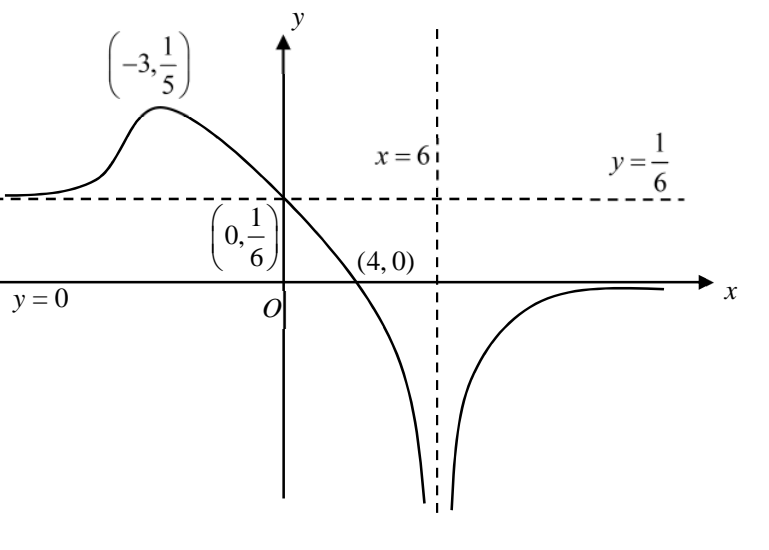
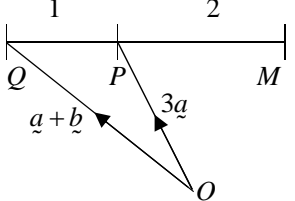
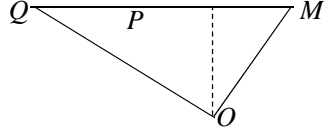


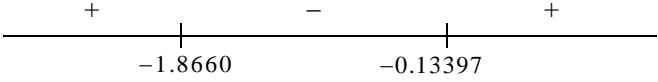
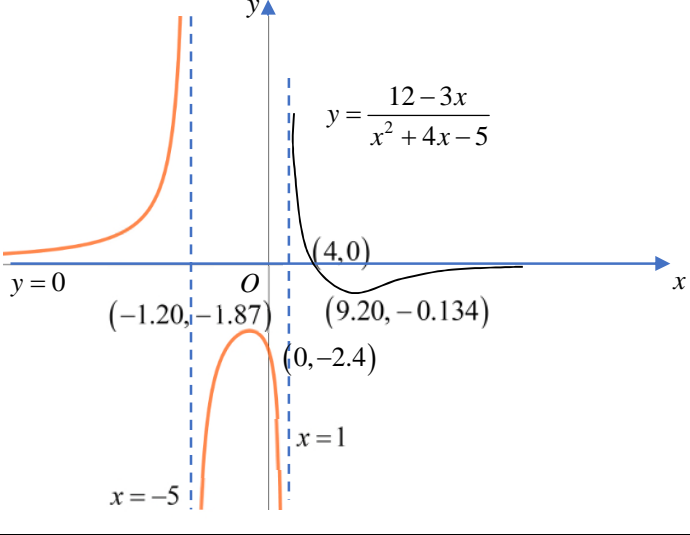
2019 H2 Maths Prelim Paper 1
Solution and Comments

Qn	Solution	Comments
1	$\frac{12}{x+1} - (7-x) = \frac{12 + (x-7)(x+1)}{x+1}$ $= \frac{x^2 - 6x + 5}{x+1}$	
	$\frac{12}{x+1} \leq 7-x$ $\frac{x^2 - 6x + 5}{x+1} \leq 0$ $\frac{(x-1)(x-5)}{x+1} \leq 0$ $x < -1 \text{ or } 1 \leq x \leq 5$	
2i	$\frac{d}{dx} \tan^{-1}(x^2) = \frac{2x}{1+(x^2)^2}$ $= \frac{2x}{1+x^4}$	
ey2ii	$\int_0^1 x \tan^{-1}(x^2) dx = \left[\frac{x^2}{2} \tan^{-1}(x^2) \right]_0^1 - \int_0^1 \frac{x^2}{2} \left(\frac{2x}{1+x^4} \right) dx$ $= \frac{1}{2} \left(\frac{\pi}{4} \right) - \left[\frac{1}{4} \ln(1+x^4) \right]_0^1$ $= \frac{\pi}{8} - \frac{1}{4} \ln 2$	
3i	$\frac{d}{dx} (3x^2 2^x) = 3(x^2 2^x \ln 2 + 2^{x+1} x)$ $= 3x2^x (x \ln 2 + 2)$	
3ii	<p>At $x=1$, gradient = $6(\ln 2 + 2)$ $y = 6$</p> <p>Equation of tangent: $y - 6 = 6(\ln 2 + 2)(x - 1)$ $y = 6x(\ln 2 + 2) - 6 \ln 2 - 6$</p>	

2019 H2 Maths Prelim Paper 1
Solution and Comments

Qn	Solution	Comments
4i		
4ii		
5i	$\overline{OP} = \frac{2\overline{OQ} + \overline{OM}}{3}$ $\overline{OM} = 3\overline{OP} - 2\overline{OQ}$ $= 3(3\mathbf{a}) - 2(\mathbf{a} + \mathbf{b})$ $= 7\mathbf{a} - 2\mathbf{b}$ 	
5ii	$\overline{PQ} \times \overline{OM} = (\mathbf{a} + \mathbf{b} - 3\mathbf{a}) \times (7\mathbf{a} - 2\mathbf{b})$ $= (\mathbf{b} - 2\mathbf{a}) \times (7\mathbf{a} - 2\mathbf{b})$ $= 7\mathbf{b} \times \mathbf{a} - 2\mathbf{b} \times \mathbf{b} - 14\mathbf{a} \times \mathbf{a} + 4\mathbf{a} \times \mathbf{b}$ <p>Since $\mathbf{a} \times \mathbf{a} = \mathbf{b} \times \mathbf{b} = \mathbf{0}$ and $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$,</p> $\overline{PQ} \times \overline{OM} = 3\mathbf{b} \times \mathbf{a}$	
5iii	$\frac{ \overline{PQ} \times \overline{OM} }{ \overline{PQ} } = \left \overline{OM} \times \frac{\overline{PQ}}{ \overline{PQ} } \right $ <p>$\frac{ \overline{PQ} \times \overline{OM} }{ \overline{PQ} }$ is the shortest distance from O to line PQ.</p> 	

2019 H2 Maths Prelim Paper 1
Solution and Comments

Qn	Solution	Comments
6i	<p>Let $y = \frac{12-3x}{x^2+4x-5} \Rightarrow yx^2 + (4y+3)x + (-5y-12) = 0$</p> <p>When x is real \Rightarrow discriminant ≥ 0</p> $\Rightarrow (4y+3)^2 - 4y(-5y-12) \geq 0$ $\Rightarrow 16y^2 + 24y + 9 + 20y^2 + 48y \geq 0$ $\Rightarrow 36y^2 + 72y + 9 \geq 0$ $\Rightarrow 36(y+1.8660)(y+0.13397) \geq 0$  <p>$y \leq -1.8660$ or $y \geq -0.13397$</p> $R_f = (-\infty, -1.87] \cup [-0.134, \infty)$	
6ii		
6iii	$y = \frac{12-3x}{(x-1)(x+5)}$ <p>Stretch parallel to the y-axis with x-axis invariant with factor $\frac{1}{3}$:</p> $y = \frac{4-x}{(x-1)(x+5)}$ <p>Translate in the positive x-direction by 5 units:</p> $y = \frac{4-(x-5)}{(x-5-1)(x-5+5)} \quad \text{ie.} \quad y = \frac{9-x}{(x-6)(x)}$	

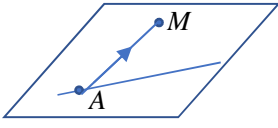
2019 H2 Maths Prelim Paper 1
Solution and Comments

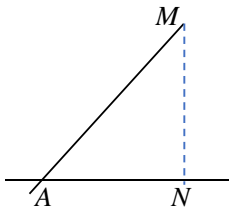
Qn	Solution	Comments
7	$\sin^{-1} y = \ln(1+x)$ $\frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = \frac{1}{1+x}$ $(1+x) \frac{dy}{dx} = \sqrt{1-y^2}$	
7i	$(1+x) \frac{dy}{dx} = \sqrt{1-y^2}$ $\frac{dy}{dx} + (1+x) \frac{d^2y}{dx^2} = -\frac{y}{\sqrt{1-y^2}} \left(\frac{dy}{dx} \right)$ <p>When $x = 0, y = 0, \frac{dy}{dx} = 1, \frac{d^2y}{dx^2} = -\frac{dy}{dx} = -1$</p> $y = 0 + x - \frac{1}{2!}x^2 + \dots \approx x - \frac{1}{2}x^2$	
7ii	$\sin^{-1} y = \ln(1+x)$ $y = \sin(\ln(1+x)) \approx x - \frac{1}{2}x^2$ $\int \frac{\sin(\ln(1+x))}{x} dx \approx \int \left(1 - \frac{1}{2}x\right) dx$ $= x - \frac{x^2}{4} + C$ $\int_0^{0.5} \frac{\sin(\ln(1+x))}{x} dx \approx \left[x - \frac{x^2}{4} \right]_0^{0.5}$ $= 0.4375$	
8i	<p>The sum of the numbers in the first row:</p> $\frac{8}{2}[2a_1 + 7d] = 58 \Rightarrow 4a_1 + 14d = 29 \text{ ----- (1)}$ <p>The sum of the numbers in the third column: $a_3, a_{11}, a_{19}, \dots, a_{59}$ are in arithmetic progression with common difference $8d$.</p> $a_3 + a_{11} + a_{19} + \dots + a_{59} = \frac{8}{2}[2(a_1 + 2d) + 7(8d)] = 376$ $\Rightarrow a_1 + 30d = 47 \text{ ----- (2)}$ <p>By GC, $a_1 = 2, d = 1.5$</p>	
8ii	<p>Sum to infinity $= 2 \Rightarrow \frac{a}{1-r} = 2 \text{ ----- (1)}$</p> <p>Sum of the terms from the 4th term to 9th term:</p> $\frac{ar^3(1-r^6)}{1-r} = -\frac{63}{256} \text{ ----- (2)}$ <p>Sub (1) into (2):</p>	

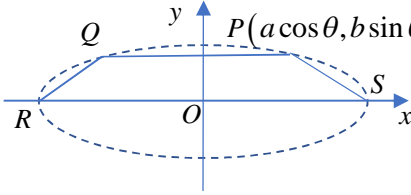
2019 H2 Maths Prelim Paper 1
Solution and Comments

Qn	Solution	Comments
	$2r^3(1-r^6) = -\frac{63}{256}$ $\Rightarrow 512r^3 - 512r^9 = -63$ $\Rightarrow 512r^9 - 512r^3 - 63 = 0$ <p>From the GC, $r = 1.02(\text{NA})$, -0.977 or -0.5 as $-1 < r < 1$ for the sum to infinity to exist.</p>	
9i	<p>Coefficients of equation are real $\Rightarrow b + \sqrt{2}i$ is also a root.</p> $z^3 - az - 66 = (z - b + \sqrt{2}i)(z - b - \sqrt{2}i)(z + k)$ $= \left[(z - b)^2 - (\sqrt{2}i)^2 \right] (z + k)$ $= (z^2 - 2bz + b^2 + 2)(z + k)$ $= z^3 + (-2b + k)z^2 + (b^2 + 2 - 2bk)z + k(b^2 + 2)$ <p>Comparing coefficients of z^2, z and constant:</p> $-2b + k = 0$ $(b^2 + 2) - 2kb = -a$ $k(b^2 + 2) = -66$ <p>Solving the equations,</p> $k = 2b \Rightarrow 2b(b^2 + 2) = -66 \Rightarrow b = -3$ <p>Alternatively</p> <p>Substitute $z = b - \sqrt{2}i$</p> $(b - \sqrt{2}i)^3 - a(b - \sqrt{2}i) - 66 = 0$ $b^3 + 3b^2(-\sqrt{2}i) + 3b(-\sqrt{2}i)^2 + (-\sqrt{2}i)^3 - ab + \sqrt{2}ai - 66 = 0$ <p>Comparing real and imaginary parts</p> $b^3 - 6b - ab - 66 = 0 \quad \text{--- (1)}$ $-3\sqrt{2}b^2 + 2\sqrt{2} + \sqrt{2}a = 0 \quad \text{--- (2)}$ <p>From (2): $a = 3b^2 - 2$ --- (3)</p> <p>Substitute (3) into (1):</p> $b^3 - 6b - b(3b^2 - 2) - 66 = 0$ $2b^3 + 4b + 66 = 0$ $b = -3$ <p>$b = -3 \Rightarrow a = 25$</p> $\frac{w}{w^*} = \frac{-3 - \sqrt{2}i}{-3 + \sqrt{2}i} \times \frac{-3 - \sqrt{2}i}{-3 - \sqrt{2}i} = \frac{7 + 6\sqrt{2}i}{11}$	

2019 H2 Maths Prelim Paper 1
Solution and Comments

Qn	Solution	Comments
9ii	w is a root $\Rightarrow w^3 - aw - 66 = 0 \Rightarrow aw + 66 = w^3 \Rightarrow aw^2 + 66w = w^4$ $w^4 = (re^{i\theta})^4 = r^4 e^{i(4\theta)}$ $\therefore aw^2 + 66w = w^4 = r^4$ $\arg(aw^2 + 66w) = \arg(r^4 e^{i(4\theta)})$ $-\pi < \theta < -\frac{3\pi}{4} \Rightarrow -4\pi < 4\theta < -3\pi \Rightarrow 0 < 4\theta + 4\pi < \pi$ $\therefore \arg(aw^2 + 66w) = 4\theta + 4\pi$	
10i	<p>Equation of l_1 is $\vec{r} = \begin{pmatrix} 7 \\ 0 \\ -2 \end{pmatrix} + s \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}, s \in \mathbb{R}$</p> $\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix} = 4 - 6 + 2 = 0$ <p>\therefore line l_1 is perpendicular to normal vector of π. \therefore line l_1 is parallel to π.</p> $\begin{pmatrix} 7 \\ 0 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix} = 28 + 0 + 2 = 30$ <p>$\therefore (7, 0, -2)$ is in π. Thus, l_1 is in π.</p>	
10ii	<p>Let $A(7, 0, -2)$.</p> $\overrightarrow{AM} = \begin{pmatrix} 6 \\ -5 \\ 11 \end{pmatrix} - \begin{pmatrix} 7 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} -1 \\ -5 \\ 13 \end{pmatrix}$  <p>normal vector of plane $= \begin{pmatrix} -1 \\ -5 \\ 13 \end{pmatrix} \times \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -29 \\ 11 \\ 2 \end{pmatrix}$</p> $\begin{pmatrix} 7 \\ 0 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} -29 \\ 11 \\ 2 \end{pmatrix} = -207$ <p>Cartesian equation of the plane is $-29x + 11y + 2z = -207$</p>	
10iii	<p>Since N is a point on l_1,</p> $\overrightarrow{ON} = \begin{pmatrix} 7 \\ 0 \\ -2 \end{pmatrix} + s \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}, \text{ for some } s \in \mathbb{R}$	

Qn	Solution	Comments
	$\overrightarrow{MN} \cdot \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} = 0$ $\left[\left(\begin{pmatrix} 7 \\ 0 \\ -2 \end{pmatrix} + s \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \right) - \begin{pmatrix} 6 \\ -5 \\ 11 \end{pmatrix} \right] \cdot \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} = 0$ $\left[\begin{pmatrix} 1 \\ 5 \\ -13 \end{pmatrix} + s \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \right] \cdot \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} = 0$ $42 + 14s = 0$ $s = -3$ $\therefore \overrightarrow{ON} = \begin{pmatrix} 7-3 \\ 3(-3) \\ -2-2(-3) \end{pmatrix} = \begin{pmatrix} 4 \\ -9 \\ 4 \end{pmatrix}$ <p>Alternatively,</p> $\overrightarrow{AN} = \left(\overrightarrow{AM} \cdot \frac{1}{\sqrt{14}} \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \right) \frac{1}{\sqrt{14}} \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$ $= \frac{1}{14} \left(\left(\begin{pmatrix} 6 \\ -5 \\ 11 \end{pmatrix} - \begin{pmatrix} 7 \\ 0 \\ -2 \end{pmatrix} \right) \cdot \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \right) \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$ $= \frac{1}{14} \left(\begin{pmatrix} -1 \\ -5 \\ 13 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \right) \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$ $= \frac{-1-15-26}{14} \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$ $= \begin{pmatrix} -3 \\ -9 \\ 6 \end{pmatrix}$ $\therefore \overrightarrow{ON} = \begin{pmatrix} 7 \\ 0 \\ -2 \end{pmatrix} + \begin{pmatrix} -3 \\ -9 \\ 6 \end{pmatrix} = \begin{pmatrix} 4 \\ -9 \\ 4 \end{pmatrix}$ <p>Area of triangle = $\frac{1}{2} \overrightarrow{OM} \times \overrightarrow{ON}$</p> $= \frac{1}{2} \left \begin{pmatrix} 6 \\ -5 \\ 11 \end{pmatrix} \times \begin{pmatrix} 4 \\ -9 \\ 4 \end{pmatrix} \right $ $= \frac{1}{2} \left \begin{pmatrix} 79 \\ 20 \\ -34 \end{pmatrix} \right $ $= 44.2 \text{ unit}^2$ 	
\10iv	<p>Let θ be the angle between the normal of π and l_2.</p> $\overrightarrow{MN} = \begin{pmatrix} 4 \\ -9 \\ 4 \end{pmatrix} - \begin{pmatrix} 6 \\ -5 \\ 11 \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \\ -7 \end{pmatrix}$	

Qn	Solution	Comments
	$\cos \theta = \frac{\begin{pmatrix} -2 \\ -4 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix}}{\sqrt{69}\sqrt{21}} = \frac{7}{\sqrt{69}\sqrt{21}}$ $\theta = 79.403^\circ$ <p>Acute angle between π and $l_2 = 90^\circ - 79.403^\circ = 10.6^\circ$</p>	
11i	<p>Given the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,</p> $\text{LHS} = \frac{(a \cos \theta)^2}{a^2} + \frac{(b \sin \theta)^2}{b^2}$ $= \cos^2 \theta + \sin^2 \theta$ $= 1 = \text{RHS}$ <p>$P(a \cos \theta, b \sin \theta)$ lies on the curve.</p> <p>Q is $(-a \cos \theta, b \sin \theta)$. [by symmetry in the y-axis]</p> 	
11ii	<p>$V = \text{cylinder} + 2 \text{ identical cones}$</p> $= \pi (b \sin \theta)^2 (2a \cos \theta) + 2 * \frac{1}{3} \pi (b \sin \theta)^2 (a - a \cos \theta)$ $= \frac{2}{3} \pi b^2 \sin^2 \theta (3a \cos \theta + a - a \cos \theta)$ $= \frac{2}{3} \pi a b^2 \sin^2 \theta (2 \cos \theta + 1)$ $\therefore k = \frac{2}{3} a b^2$	
11iii	<p>At $\theta = \theta_1$,</p> $\frac{dV}{d\theta} = 0$ $\frac{dV}{d\theta} = \frac{2}{3} \pi a b^2 [\sin^2 \theta (-2 \sin \theta) + (2 \cos \theta + 1)(2 \sin \theta \cos \theta)] = 0$ $\sin \theta (-2 \sin^2 \theta + 2 \cos \theta + 4 \cos^2 \theta) = 0$ $\sin \theta = 0 \quad \text{or} \quad -2(1 - \cos^2 \theta) + 2 \cos \theta + 4 \cos^2 \theta = 0$ <p>$\left(\text{NA since } 0 < \theta < \frac{\pi}{2} \right)$</p> $6 \cos^2 \theta + 2 \cos \theta - 2 = 0$ $3 \cos^2 \theta + \cos \theta - 1 = 0 \text{ (shown)}$ $\cos \theta = 0.43425, -0.76759 \left(\text{NA } \because 0 < \theta < \frac{\pi}{2} \right)$ $\Rightarrow \theta_1 = 1.1216 \approx 1.12$ $\frac{dV}{d\theta} = \frac{2}{3} \pi a b^2 [-2 \sin^3 \theta + (2 \cos \theta + 1) \sin 2\theta]$ $\frac{d^2V}{d\theta^2} = \frac{2}{3} \pi a b^2 [-6 \sin^2 \theta \cos \theta + (-2 \sin \theta) \sin 2\theta$ $+ 2 \cos 2\theta (2 \cos \theta + 1)]$	

Qn	Solution	Comments
	<p>When $\theta_1 = 1.1216$, $\frac{d^2V}{d\theta^2} = -3.90\pi ab^2$</p> <p>Since a and b are positive, $\frac{d^2V}{d\theta^2} < 0$</p> <p>Hence, θ_1 gives a maximum value of V.</p>	
11iv	$\begin{aligned} \text{Volume} &= \pi \int_{a \cos\left(\frac{\pi}{6}\right)}^a y^2 dx \\ &= \pi \int_{\frac{\sqrt{3}a}{2}}^a b^2 \left(1 - \frac{x^2}{a^2}\right) dx \\ &= \pi b^2 \int_{\frac{\sqrt{3}a}{2}}^a \left(1 - \frac{x^2}{a^2}\right) dx \\ &= \pi b^2 \left[x - \frac{x^3}{3a^2} \right]_{\frac{\sqrt{3}a}{2}}^a \\ &= \pi b^2 \left\{ \left(a - \frac{a^3}{3a^2} \right) - \left(\frac{\sqrt{3}}{2}a - \frac{3\sqrt{3}a^3}{24a^2} \right) \right\} \\ &= \pi ab^2 \left(\frac{2}{3} - \frac{3\sqrt{3}}{8} \right) \text{ (or } 0.0171\pi ab^2 \text{)} \end{aligned}$	
12i	$\begin{aligned} \frac{dv}{dt} &= 10 - 0.001v^2 = \frac{10000 - v^2}{1000} \\ \Rightarrow \int \frac{1}{10000 - v^2} dv &= \int \frac{1}{1000} dt \\ \Rightarrow \frac{1}{200} \ln \left \frac{100+v}{100-v} \right &= \frac{t}{1000} + d \\ \Rightarrow \ln \left \frac{100+v}{100-v} \right &= \frac{t}{5} + d' \\ \Rightarrow \left \frac{100+v}{100-v} \right &= Ce^{\frac{t}{5}} \\ \Rightarrow \frac{100+v}{100-v} &= De^{\frac{t}{5}} \\ t=0, v=0 &\Rightarrow D=1 \\ \frac{100+v}{100-v} = e^{\frac{t}{5}} &\Rightarrow 100+v = e^{\frac{t}{5}}(100-v) \Rightarrow v \left(e^{\frac{t}{5}} + 1 \right) = 100e^{\frac{t}{5}} - 100 \\ v &= 100 \left(\frac{e^{\frac{t}{5}} - 1}{e^{\frac{t}{5}} + 1} \right) \text{ (shown)} \end{aligned}$	
12ii	<p>Method 1:</p> $v = 100 \left(\frac{e^{\frac{t}{5}} - 1}{e^{\frac{t}{5}} + 1} \right) = 100 \left(1 - \frac{2}{e^{\frac{t}{5}} + 1} \right)$	

2019 H2 Maths Prelim Paper 1
Solution and Comments

Qn	Solution	Comments
	<p>When $t \rightarrow \infty$, $\frac{1}{e^{\frac{t}{5}} + 1} \rightarrow 0$, $v \rightarrow 100 \therefore v_0 = 100$</p> <p>Method 2:</p> $v = 100 \left(\frac{e^{\frac{t}{5}} - 1}{e^{\frac{t}{5}} + 1} \right) = 100 \left(\frac{e^{\frac{t}{5}} \left(1 - e^{-\frac{t}{5}} \right)}{e^{\frac{t}{5}} \left(1 + e^{-\frac{t}{5}} \right)} \right)$ <p>When $t \rightarrow \infty$, $e^{-\frac{t}{5}} \rightarrow 0$, $v \rightarrow 100 \therefore v_0 = 100$</p> <p>Method 3:</p> <p>When $t \rightarrow \infty$, $\frac{1}{e^{\frac{t}{5}} + 1} \rightarrow 0$, $v \rightarrow 100 \therefore v_0 = 100$</p> $v = 100 \left(\frac{e^{\frac{t}{5}} - 1}{e^{\frac{t}{5}} + 1} \right) \rightarrow 100 \left(\frac{e^{\frac{t}{5}}}{e^{\frac{t}{5}}} \right)$ <p>$v \rightarrow 100 \therefore v_0 = 100$</p>	
12iii	$\frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt}$ $\Rightarrow 10 - 0.001v^2 = \frac{dv}{dx}(v)$ $\Rightarrow \int 1 dx = \int \frac{v}{10 - 0.001v^2} dv = \int \frac{1000v}{10000 - v^2} dv$ $\Rightarrow -500 \int \frac{-2v}{10000 - v^2} dv = x + c$ $\Rightarrow \ln 10000 - v^2 = -\frac{x}{500} + c'$ $\Rightarrow 10000 - v^2 = Ae^{-\frac{x}{500}}$ $\Rightarrow 10000 - v^2 = Be^{-\frac{x}{500}}$ <p>$x = 0, v = 0 \Rightarrow B = 10000$</p> $10000 - v^2 = 10000e^{-\frac{x}{500}}$ $v^2 = 10000 - 10000e^{-\frac{x}{500}}$ $v \geq 0 \Rightarrow v = 100\sqrt{1 - e^{-\frac{x}{500}}} \quad (\text{shown})$	
12iv	<p>When $t = 5, v = 100 \left(\frac{e - 1}{e + 1} \right) = 46.2117$</p> <p>When $v = 46.2117, 46.2117 = 100\sqrt{1 - e^{-\frac{x}{500}}}$</p> <p>$\therefore x = 120.11$</p> <p>The required distance is 120.11 m.</p>	

