

- 1 Express $\frac{12}{x+1} - (7-x)$ as a single simplified fraction. [1]

Without using a calculator, solve $\frac{12}{x+1} \leq 7-x$. [3]

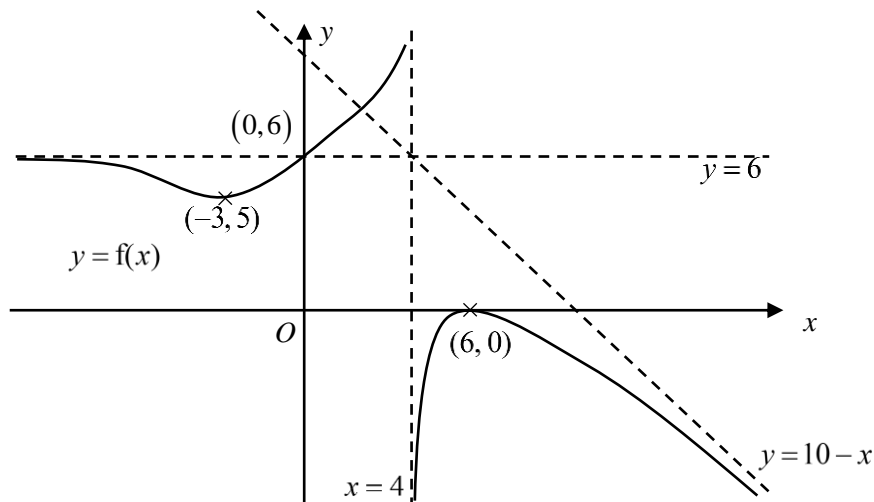
- 2 (i) Find $\frac{d}{dx} \tan^{-1}(x^2)$. [1]

(ii) Hence, or otherwise, evaluate $\int_0^1 x \tan^{-1}(x^2) dx$ exactly. [3]

- 3 (i) Find $\frac{d}{dx}(3x^2 2^x)$. [2]

(ii) Find the equation of the tangent to the curve $y = 3x^2 2^x$ at the point where $x = 1$, giving your answer in exact form. [3]

- 4 The graph for $y = f(x)$ is given below, where $y = 10 - x$, $y = 6$ and $x = 4$ are asymptotes. The turning points are $(-3, 5)$ and $(6, 0)$, and the graph intersects the y -axis at $(0, 6)$.



On separate diagrams, sketch the graphs of

- (i) $y = f(|x|)$, [3]

(ii) $y = \frac{1}{f(x)}$. [3]

5 Referred to the origin O , points P and Q have position vectors $3\mathbf{a}$ and $\mathbf{a} + \mathbf{b}$ respectively. Point M is a point on QP extended such that $PM:QM$ is 2:3.

(i) Find the position vector of point M in terms of \mathbf{a} and \mathbf{b} . [2]

(ii) Find $\overrightarrow{PQ} \times \overrightarrow{OM}$ in terms of \mathbf{a} and \mathbf{b} . [3]

(iii) State the geometrical meaning of $\frac{|\overrightarrow{PQ} \times \overrightarrow{OM}|}{|\overrightarrow{PQ}|}$. [1]

6 A curve C has equation $y = f(x)$, where the function f is defined by

$$f : x \mapsto \frac{12 - 3x}{x^2 + 4x - 5}, \quad x \in \mathbb{R}, x \neq -5, x \neq 1.$$

(i) Find algebraically the range of f . [3]

(ii) Sketch C , indicating all essential features. [4]

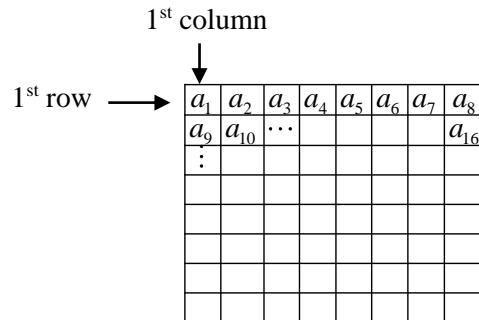
(iii) Describe a pair of transformations which transforms the graph of C on to the graph of $y = \frac{9 - x}{x^2 - 6x}$. [2]

7 Given that $\sin^{-1} y = \ln(1 + x)$, where $0 < x < 1$, show that $(1 + x) \frac{dy}{dx} = \sqrt{1 - y^2}$. [2]

(i) By further differentiation, find the Maclaurin expansion of y in ascending powers of x , up to and including the term in x^2 . [4]

(ii) Use your expansion from (i) and integration to find an approximate expression for $\int \frac{\sin(\ln(1 + x))}{x} dx$. Hence find an approximate value for $\int_0^{0.5} \frac{\sin(\ln(1 + x))}{x} dx$. [3]

- 8 (a) A sequence of numbers $a_1, a_2, a_3, \dots, a_{64}$ is such that $a_{n+1} = a_n + d$, where $1 \leq n \leq 63$ and d is a constant. The 64 numbers fill the 64 squares in the 8×8 grid in such a way that a_1 to a_8 fills the first row of boxes from left to right in that order. Similarly, a_9 to a_{16} fills the second row of boxes from left to right in that order.



Given that the sums of the numbers in the **first row** and in the **third column** are 58 and 376 respectively, find the values of a_1 and d . [4]

- (b) A geometric series has first term a and common ratio r , where a and r are non-zero. The sum to infinity of the series is 2. The sum of the six terms of this series from the 4th term to the 9th term is $-\frac{63}{256}$. Show that $512r^9 - 512r^3 - 63 = 0$.

Find the two possible values of r , justifying the choice of your answers. [5]

- 9 One of the roots of the equation $z^3 - az - 66 = 0$, where a is real, is w .

(i) Given that $w = b - \sqrt{2}i$, where b is real, find the exact values of a and $\frac{w}{w^*}$. [6]

(ii) Given instead that $w = re^{i\theta}$, where $r > 0$, $-\pi < \theta < -\frac{3\pi}{4}$, find $|aw^2 + 66w|$ and $\arg(aw^2 + 66w)$ in terms of r and θ . [4]

- 10 The point M has position vector relative to the origin O , given by $6\mathbf{i} - 5\mathbf{j} + 11\mathbf{k}$. The line l_1 has equation $x - 7 = \frac{y}{3} = \frac{z + 2}{-2}$, and the plane π has equation $4x - 2y - z = 30$.

(i) Show that l_1 lies in π . [2]

(ii) Find a cartesian equation of the plane containing l_1 and M . [3]

The point N is the foot of perpendicular from M to l_1 . The line l_2 is the line passing through M and N .

(iii) Find the position vector of N and the area of triangle OMN . [5]

(iv) Find the acute angle between l_2 and π , giving your answer correct to the nearest 0.1° . [3]

- 11** [It is given that the volume of a cylinder with base radius r and height h is $\pi r^2 h$ and the volume of a cone with the same base radius and height is a third of a cylinder.]

A manufacturer makes double-ended coloured pencils that allow users to have two different colours in one pencil. The manufacturer determines that the shape of each coloured pencil is formed by rotating a trapezium $PQRS$ completely about the x -axis, such that it is a solid made up of a cylinder and two cones. The volume, $V \text{ cm}^3$, of the coloured pencil should be as large as possible.

It is given that the points P , Q , R and S lie on the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are positive constants. The points R and S are $(-a, 0)$ and $(a, 0)$ respectively, and the line PQ is parallel to the x -axis.

- (i) Verify that $P(a \cos \theta, b \sin \theta)$, where $0 < \theta < \frac{\pi}{2}$, lies on the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Write down the coordinates of the point Q . [2]
- (ii) Show that V can be expressed as $V = k\pi \sin^2 \theta (2 \cos \theta + 1)$, where k is a constant in terms of a and b . [3]
- (iii) Given that $\theta = \theta_1$ is the value of θ which gives the maximum value of V , show that θ_1 satisfies the equation $3 \cos^2 \theta + \cos \theta - 1 = 0$. Hence, find the value of θ_1 . [4]

At $\theta = \frac{\pi}{6}$, the manufacturer wants to change one end of the coloured pencil to a rounded-end eraser. The eraser is formed by rotating the arc PS completely about the x -axis.

- (iv) Find the volume of the eraser in terms of a and b . [3]

- 12** A ball-bearing is dropped from a point O and falls vertically through the atmosphere. Its speed at O is zero, and t seconds later, its velocity is $v \text{ ms}^{-1}$ and its displacement from O is $x \text{ m}$. The rate of change of v with respect to t is given by $10 - 0.001v^2$.

- (i) Show that $v = 100 \left(\frac{e^{\frac{t}{5}} - 1}{e^{\frac{t}{5}} + 1} \right)$. [4]

- (ii) Find the value of v_0 , where v_0 is the value approached by v for large values of t . [1]

- (iii) By using chain rule, form an equation relating $\frac{dx}{dt}$, $\frac{dv}{dt}$ and $\frac{dv}{dx}$. Given that $v = \frac{dx}{dt}$, form a differential equation relating v and x . Show that

$$v = 100 \sqrt{1 - e^{-\frac{x}{500}}}. \quad [5]$$

- (iv) Find the distance of the ball-bearing from O after 5 seconds, giving your answer correct to 2 decimal places. [3]

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