RAFFLES INSTITUTION
2019 YEAR 6 PRELIMINARY EXAMINATION

MATHEMATICS 9758/01 Suggested Solutions

<table>
<tr><th colspan="2">SOLUTION</th><th>COMMENTS</th></tr>
<tr><td>1(i)
[4]</td><td>$y=\frac{a}{x^3}+bx+c$
$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{3a}{x^4}+b$

At $x=-1.2$, $\frac{\mathrm{d}y}{\mathrm{d}x}=0$, we have
$-\frac{3a}{(-1.2)^4}+b=0 \qquad ...(1)$

At $(-1.2, 6.6)$, we have
$\frac{a}{(-1.2)^3}-1.2b+c=6.6 \qquad ...(2)$

At $(2.1, -4.5)$, we have
$\frac{a}{(2.1)^3}+2.1b+c=-4.5 \qquad ...(3)$

Using GC to solve (1), (2) and (3):
$a=-2.03260\approx-2.0$,
$b=-2.94068\approx-2.9$,
$c=1.89491\approx1.9$</td><td>Generally well done for most students, except for a small number.

These are the points to note:

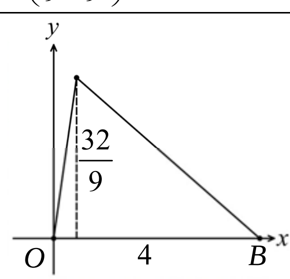
1. Some students could not get the 3 equations as they didn't realise that the point $(-1.2, 6.6)$ could result in 2 equations instead of only one.

2. There were a number of students who could get the 3 equations but they end up with the wrong solutions. Do be careful when keying the equations into the GC!

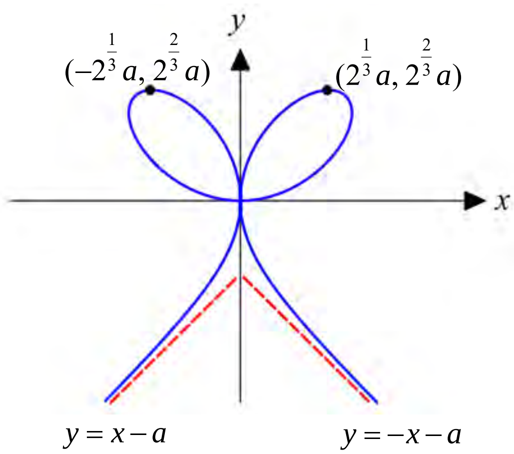
3. A number of students differentiated wrongly:
Eg. $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{3a}{x^2}+b$.</td></tr>
<tr><td>(ii)
[1]</td><td>$y=-2.9x+1.9$ (1 d.p.)</td><td>From equation of C, the other asymptote is $y=bx+c$. Most students were able to obtain a mark for this.</td></tr>
</table>

SOLUTION		COMMENTS
2 **[4]**	$\frac{1}{u}+\frac{1}{v}=\frac{1}{20}$ Differentiating w.r.t t, $-\frac{1}{u^2}\frac{\mathrm{d}u}{\mathrm{d}t}-\frac{1}{v^2}\frac{\mathrm{d}v}{\mathrm{d}t}=0$ --------------- (1) When $u=60$, $\frac{1}{60}+\frac{1}{v}=\frac{1}{20}$ $\frac{1}{v}=\frac{1}{20}-\frac{1}{60}$ $\frac{1}{v}=\frac{1}{30}$ $v=30$ Substituting $v=30$ and $\frac{\mathrm{d}u}{\mathrm{d}t}=-2$ into (1), $-\frac{1}{(60)^2}(-2)-\frac{1}{(30)^2}\frac{\mathrm{d}v}{\mathrm{d}t}=0$ $\frac{1}{1800}-\frac{1}{900}\frac{\mathrm{d}v}{\mathrm{d}t}=0$ $\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{900}{1800}$ $=\frac{1}{2}$ $\therefore$ Rate of increase of $v=\frac{1}{2}$ units/s	Probably 70% of students managed to do this. Some points to note: 1. Students are strongly advised to use implicit differentiation instead of making u or v the subject and applying quotient/product rule which often ends up with long and complicated expressions. 2. Some students mixed up differentiation and integration Eg. $\frac{\mathrm{d}}{\mathrm{d}u}\left(\frac{1}{u}\right)=\ln\|u\|$. 3. Students are again reminded of the need to write clearly and properly as they mixed up u and v and hence obtained the wrong answer eventually.

SOLUTION		COMMENTS
3 **[6]**	$V=(a-2x)^2 x = a^2x-4ax^2+4x^3$ $\dfrac{dV}{dx}=a^2-8ax+12x^2=0$ $(6x-a)(2x-a)=0$ $x=\dfrac{a}{6}$ or $x=\dfrac{a}{2}$ (rejected $\because V=0$) $\dfrac{d^2V}{dx^2}=-8a+24x=-4a<0$ when $x=\dfrac{a}{6}$ Maximum $V=\left(a-2\left(\dfrac{a}{6}\right)\right)^2\left(\dfrac{a}{6}\right)=\dfrac{2a^3}{27}$ cm^3.	This question is generally well done. Some points to note are • Some students did not justify why the value of $x=\dfrac{a}{2}$ was rejected. Many students checked that this gives a minimum value using the second derivative test, a good strategy. • Many students solve for *x* by completing the square (of which a number made careless mistakes) when a direct factorization is much faster. Good for some of them to relearn this method. • Students who used the first derivative test to check for the nature of the root need to be reminded that the expression for dV/dx need to be in factorized form. • A small number of students were not aware that the *a* in the question is a constant and proceed to differentiate wrt *a*. Such students also tend to bring in expressions involving surface area showing that they are not sure of what the question is asking.

SOLUTION		COMMENTS
4 **(i)** **[4]**	$x=(1+t)^2, \quad y=2(1-t)^2.$ $\frac{dx}{dt}=2(1+t), \quad \frac{dy}{dt}=-4(1-t)$ $\frac{dy}{dx}=\frac{-4(1-t)}{2(1+t)}=\frac{-2(1-t)}{1+t}$ Tangent parallel to x-axis, $\frac{dy}{dx}=\frac{-2(1-t)}{1+t}=0 \Rightarrow t=1$ Thus, the coordinates of point A is $(4,0)$.	This question is well done. The main mistakes for **(i)** are • giving $\frac{dy}{dt}=4(1-t)$ • the gradient of the tangent parallel to the x-axis is undefined.
(ii) **[4]**	Let the coordinates of B be $\left((1+b)^2, 2(1-b)^2\right)$, where $b \in \mathbb{R}$. $y=-x+d$ passes through $A(4,0) \Rightarrow d=4$ Sub $B: 2(1-b)^2=-(1+b)^2+4$ $2-4b+2b^2=-1-2b-b^2+4$ $3b^2-2b-1=0$ $(3b+1)(b-1)=0$ $b=-\frac{1}{3}$ or $b=1$ (point A) Thus, the coordinates of B is $\left(\frac{4}{9}, \frac{32}{9}\right)$.	This part of the question asked for the **exact** coordinates, which means that the use of the GC is **<u>not</u>** allowed.
(iii) **[1]**	Area of triangle OAB $=\frac{1}{2}(4)\left(\frac{32}{9}\right)$ $=\frac{64}{9}$  	This part is well done.

SOLUTION		COMMENTS
5 (i) [5]	$x^3+y^3=3axy$ Differentiate with respect to *x*, $3x^2+3y^2\frac{\mathrm{d}y}{\mathrm{d}x}=3a\left(x\frac{\mathrm{d}y}{\mathrm{d}x}+y\right)$ When $\frac{\mathrm{d}y}{\mathrm{d}x}=0$, $3x^2=3ay\Rightarrow y=\frac{x^2}{a}$ Substitute $y=\frac{x^2}{a}$ into the given curve $x^3+y^3=3axy$, $x^3+\frac{x^6}{a^3}=3x^3$ $x^6-2a^3x^3=0$ $x^3\left(x^3-2a^3\right)=0$ $x=0$ or $x=2^{\frac{1}{3}}a$ and, $y=0$ or $y=2^{\frac{2}{3}}a$ The required coordinates are $(0,0)$ (shown) and $\left(2^{\frac{1}{3}}a,2^{\frac{2}{3}}a\right)$. For the following working, note that we need $y^2\neq ax$ to get $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of *x* and *y*. $3x^2+3y^2\frac{\mathrm{d}y}{\mathrm{d}x}=3a\left(x\frac{\mathrm{d}y}{\mathrm{d}x}+y\right)$ $\left(y^2-ax\right)\frac{\mathrm{d}y}{\mathrm{d}x}=ay-x^2$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{ay-x^2}{y^2-ax}$ if $y^2\neq ax$ ...(*) Letting (*) = 0 and solving will also give (0,0) as an answer, but would need to be rejected as $y^2\neq ax$.	Differentiation was well done. A few students made the mistakes: - did not apply chain rule - did not apply product rule Many did not substitute $y=\frac{x^2}{a}$ back into the given curve. Note a stationary point **lies on the curve** and has 0 gradient. It does **NOT** lie on the asymptote $y=-x-a$ or any other lines such as $y=x$. Many made mistakes when manipulating the indices. Some students did not simply their answers. In addition, please leave the answers in exact form when the question explicitly states that the coordinates are to be expressed "in terms of *a*".

<table>
<tr>
<td>(ii)
[3]</td>
<td>$x^3 + y^3 = 3axy \xrightarrow{\text{Replace } x \text{ by } |x|} |x|^3 + y^3 = 3a|x|y$

</td>
<td>Some common mistakes:
- incomplete graph
- graph not symmetrical about y-axis
- (0, 0) was not drawn as stationary point with zero gradient
- stationary points are the points with “maximum” y values, NOT the points furthest away from the origin.
- label the asymptotes wrongly
- curve not approaching the asymptotes as $x \to \infty$
- not labelling/labelling the required coordinates wrongly</td>
</tr>
</table>

SOLUTION		COMMENTS
6 **(a)** **[3]**	$\sum_{r=n+1}^{2n}\left(r\left(r-1\right)\right)$ $=\sum_{r=n+1}^{2n}\left(r^2-r\right)$ $=\sum_{r=n+1}^{2n}r^2-\sum_{r=n+1}^{2n}r$ $=\sum_{r=1}^{2n}r^2-\sum_{r=1}^{n}r^2-\frac{n}{2}\left(n+1+2n\right)$ $=\frac{1}{6}\left(2n\right)\left(2n+1\right)\left(4n+1\right)-\frac{1}{6}\left(n\right)\left(n+1\right)\left(2n+1\right)-\frac{1}{2}n\left(3n+1\right)$ $=\frac{1}{6}n\left[2\left(2n+1\right)\left(4n+1\right)-\left(n+1\right)\left(2n+1\right)-3\left(3n+1\right)\right]$ $=\frac{1}{6}n\left(14n^2-2\right)$ $=\frac{1}{3}n\left(7n^2-1\right)$ Alternatively, $\sum_{r=n+1}^{2n}\left(r^2-r\right)$ $=\sum_{r=n+1}^{2n}r^2-\sum_{r=n+1}^{2n}r$ $=\sum_{r=1}^{2n}r^2-\sum_{r=1}^{n}r^2-\left(\sum_{r=1}^{2n}r-\sum_{r=1}^{n}r\right)$ $=\frac{1}{6}\left(2n\right)\left(2n+1\right)\left(4n+1\right)-\frac{1}{6}\left(n\right)\left(n+1\right)\left(2n+1\right)-\frac{1}{2}\left(2n\right)\left(2n+1\right)$ $+\frac{1}{2}n\left(n+1\right)$ $=\frac{1}{6}n\left[2\left(2n+1\right)\left(4n+1\right)-\left(n+1\right)\left(2n+1\right)-6\left(2n+1\right)+3\left(n+1\right)\right]$ $=\frac{1}{6}n\left(14n^2-2\right)$ $=\frac{1}{3}n\left(7n^2-1\right)$	This part was well done. However, quite a number of students lost 1 mark for not simplifying their answer. Another common mistake seen was miscalculation of the number of terms in the sum of AP , $\sum_{r=n+1}^{2n}r$.
(b) **(i)** **[3]**	$\sum_{r=1}^{n}\left(\frac{(r+1)-\mathrm{e}\,r}{\mathrm{e}^r}\right)=\sum_{r=1}^{n}\left(\frac{(r+1)}{\mathrm{e}^r}-\frac{\mathrm{e}\,r}{\mathrm{e}^r}\right)$ $=\sum_{r=1}^{n}\left(\frac{(r+1)}{\mathrm{e}^r}-\frac{r}{\mathrm{e}^{r-1}}\right)$	This part was well done. Some common mistakes seen were: • Missing out the $-$ sign in -1

	$\begin{aligned} = \; & \cancel{\frac{2}{e^1}} - \frac{1}{e^0} \\ + \; & \cancel{\frac{3}{e^2}} - \cancel{\frac{2}{e^1}} \\ + \; & \cancel{\frac{4}{e^3}} - \cancel{\frac{3}{e^2}} \\ & \quad \vdots \\ + \; & \cancel{\frac{n}{e^{n-1}}} - \cancel{\frac{n-1}{e^{n-2}}} \\ + \; & \frac{n+1}{e^n} - \cancel{\frac{n}{e^{n-1}}} \\ = \; & \frac{n+1}{e^n} - \frac{1}{e^0} \\ = \; & \frac{n+1}{e^n} - 1 \end{aligned}$	• Writing the last term wrongly as $\frac{n+1}{e^{n+1}}$
(b) **(ii)** **[2]**	$\sum_{r=2}^{n}\left(\frac{e+r-er}{e^{r-1}}\right) = \sum_{r+1=2}^{n}\left(\frac{e+(r+1)-e(r+1)}{e^{(r+1)-1}}\right)$ $=\sum_{r=1}^{n-1}\left(\frac{(r+1)-er}{e^{r}}\right)$ $=\frac{n}{e^{n-1}}-1$	Take note of the "Hence" method required in this part. Many students failed to show proper working of using answer to **(b)(i)**. There were also a lot of mistakes in the change of lower and upper index of the summation.

SOLUTION		**COMMENTS**
7 **(a)** **[4]**	$\frac{z}{3+2i}=\frac{4-6i}{z}$ $\Rightarrow z^2=(4-6i)(3+2i)=24-10i$ Let $z=a+bi, \quad a,b\in\mathbb{R}$ $(a+bi)^2=24-10i$ $\Rightarrow a^2-b^2+2abi=24-10i$ Comparing Real and Imaginary parts, $a^2-b^2=24$ $2ab=-10\Rightarrow ab=-5$ $\Rightarrow \frac{25}{b^2}-b^2=24$	Students need to understand that when a question states "without using a calculator", they need to show all working no matter how trival it seemed to them. Many lose marks for failing to show the proper factorization of

	$\Rightarrow b^4+24b^2-25=0$ ------- (*) $\Rightarrow (b^2+25)(b^2-1)=0$ $\Rightarrow b=\pm 1$ $\Rightarrow a=\mp 5$ The two possible complex numbers are $-5+\mathrm{i}$ or $5-\mathrm{i}$.	the quartic equation (*).
(b) (i) [4]	$\frac{w^2}{w^*} = \frac{(a+\mathrm{i}b)^2}{a-\mathrm{i}b}$ $= \frac{(a+\mathrm{i}b)^3}{a^2+b^2}$ $= \frac{a^3+3\mathrm{i}a^2b-3ab^2-\mathrm{i}b^3}{a^2+b^2}$ $= \frac{(a^3-3ab^2)+\mathrm{i}(3a^2b-b^3)}{a^2+b^2}$ $\frac{w^2}{w^*}$ is purely real $\mathrm{Im}\left(\frac{w^2}{w^*}\right)=0 \Rightarrow \frac{3a^2b-b^3}{a^2+b^2}=0$ $\Rightarrow b(3a^2-b^2)=0$ $\Rightarrow b=0$ (rejected, since $b\neq 0$) or $b=\pm a\sqrt{3}$ Thus, $w=a+\mathrm{i}a\sqrt{3}$ or $w=a-\mathrm{i}a\sqrt{3}$. **<u>Alternative method 1:</u>** $\arg\left(\frac{w^2}{w^*}\right)=k\pi$, where $k\in\mathbb{Z}$ $3\arg w=k\pi$ $\arg w=\frac{k\pi}{3}$ Since a and b are non-zero, $\tan^{-1}\left(\left\lvert\frac{b}{a}\right\rvert\right)=\frac{\pi}{3}$ $\lvert b\rvert=\lvert a\rvert\sqrt{3}$ $b=a\sqrt{3}$ or $-a\sqrt{3}$ Thus, $w=a+\mathrm{i}a\sqrt{3}$ or $w=a-\mathrm{i}a\sqrt{3}$.	Generally well done for students who use the first method, except for a few who forgot to reject $b = 0$. Students who use the argument method (Alternative Method 1) mostly wrote $\arg w=\tan^{-1}\left(\frac{b}{a}\right)$ Students who wrote this were not penalized in this exam, but please do note

	<u>Alternative method 2:</u> Let $\dfrac{w^2}{w^*} = k$, where $k \in \mathbb{R}$ $\dfrac{(a+\mathrm{i}b)^2}{a-\mathrm{i}b} = k$ $a^2 - b^2 + \mathrm{i}(2ab) = ka - \mathrm{i}(kb)$ Comparing real and imaginary parts, $a^2 - b^2 = ka$ ----------- (1) $2ab = -kb$ ------------ (2) From (2), $k = -2a$ Substitute into (1): $a^2 - b^2 = -2a^2$ $b^2 = 3a^2$ $b = a\sqrt{3}$ or $-a\sqrt{3}$	that this is not correct.
(b) **(ii)** **[2]**	Basic angle = $\tan^{-1}\left(\dfrac{\sqrt{3}}{1}\right) = \dfrac{\pi}{3}$ Thus, $\arg(w) = \dfrac{\pi}{3}$ or $\arg(w) = -\dfrac{\pi}{3}$.	

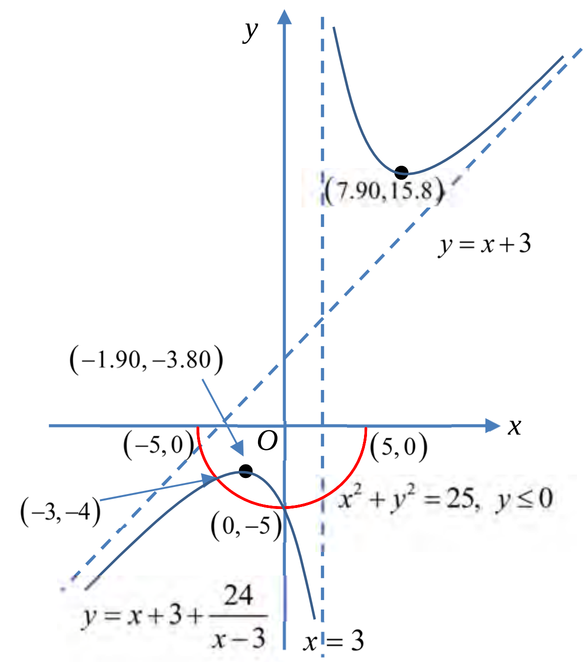
SOLUTION		COMMENTS
8(a) **[5]**	$\int_0^3 \frac{1}{9+x^2}\,\mathrm{d}x = \left[\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)\right]_0^3$ $= \frac{1}{3}\left(\frac{\pi}{4}\right)$ $= \frac{\pi}{12}.$ $\int_0^{\frac{1}{2m}} \frac{1}{\sqrt{1-m^2x^2}}\,\mathrm{d}x = \frac{1}{m}\int_0^{\frac{1}{2m}} \frac{m}{\sqrt{1-(mx)^2}}\,\mathrm{d}x$ $= \frac{1}{m}\left[\sin^{-1}(mx)\right]_0^{\frac{1}{2m}}$ $= \frac{1}{m}\left[\sin^{-1}\left(\frac{1}{2}\right) - 0\right]$ $= \frac{\pi}{6m}.$ So $\int_0^{\frac{1}{2m}} \frac{1}{\sqrt{1-m^2x^2}}\,\mathrm{d}x = \int_0^3 \frac{1}{9+x^2}\,\mathrm{d}x$ $\frac{\pi}{6m} = \frac{\pi}{12}$ $m = 2$	Common mistakes: a) Exclude the $\frac{1}{m}$ in the $\frac{1}{m}\sin^{-1}(mx)$; b) Wrong value for $\tan^{-1}\left(\frac{3}{3}\right)$ or $\sin^{-1}\left(\frac{1}{2}\right)$. Some students resorted to using calculator to compute $\frac{\sin^{-1}\left(\frac{1}{2}\right)}{\tan^{-1}\left(\frac{3}{3}\right)}$, which illustrated that they do not have knowledge of the special angles. Note that since question asks for exact value of m, calculator should not be used in the computation.
(b) **[6]**	$u = \sin 2x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2\cos 2x$ When $x = 0,\ u = 0.$ When $x = \frac{\pi}{4},\ u = 1.$ $\int_0^{\frac{\pi}{4}} \sin^3 2x\cos^3 2x\,\mathrm{d}x = \int_0^{\frac{\pi}{4}} \sin^3 2x\cos^2 2x\cos 2x\mathrm{d}x$ $= \frac{1}{2}\int_0^{\frac{\pi}{4}} (\sin^3 2x)\left(1-\sin^2 2x\right)(2\cos 2x)\mathrm{d}x$ $= \frac{1}{2}\int_0^1 u^3\left(1-u^2\right)\,\mathrm{d}u$ $= \frac{1}{2}\int_0^1 \left(u^3 - u^5\right)\,\mathrm{d}u$	Some students did this: $\frac{\mathrm{d}u}{\mathrm{d}x} = -2\cos 2x$ which is less than desirable. Very common presentation error: $\frac{1}{2}\int_0^1 (u^3)\left(\cos^3 2x\right)\frac{\mathrm{d}u}{\cos 2x}$ and the variations of mixing x and u in the integrand, or even integrating wrt to u but the limits are still in x. Some students failed to use the "hence" for the last numerical

	$\int_0^{\frac{\pi}{4}} \sin^3 2x\cos^3 2x \, dx = \frac{1}{2}\int_0^1 \left(u^3 - u^5\right) du$ $= \frac{1}{2}\left[\frac{u^4}{4} - \frac{u^6}{6}\right]_0^1$ $= \frac{1}{2}\left[\frac{1}{4} - \frac{1}{6}\right]$ $= \frac{1}{24}$	computation and reworked from the start.

SOLUTION		COMMENTS
9(i) **[1]**	$\int \frac{1}{a^2 - v^2} \, dv = \frac{1}{2a}\ln\left\|\frac{a+v}{a-v}\right\| + c$	This part was very poorly done. Most students either missed out on the modulus sign or the +c.
(ii) **(a)** **[8]**	$\frac{a^2}{10}\frac{dv}{dt} = a^2 - v^2$ $\frac{1}{a^2 - v^2}\frac{dv}{dt} = \frac{10}{a^2}$ $\int \frac{1}{a^2 - v^2} \, dv = \int \frac{10}{a^2} dt$ $\frac{1}{2a}\ln\left\|\frac{a+v}{a-v}\right\| = \frac{10}{a^2}t + C$, where C is an arbitrary constant $\ln\left\|\frac{a+v}{a-v}\right\| = \frac{20t}{a} + D$, where $D = 2aC$ $\frac{a+v}{a-v} = Ae^{\frac{20t}{a}}$, where $A = \pm e^D$ Given that $v = 0$ when $t = 0$, $A = 1$.	Most students were able to do this part relatively well. A few points to take note 1. There should be a modulus in the natural log. 2. The modulus will be taken care of when we take $A = \pm e^D$ and then substitute initial conditions to calculate for *A*. 3. Some students substituted the initial conditions too early to calculate *D*, obtaining *D*=0. This is insufficient because upon simplifying the equation by removing the modulus sign, you will still get $\pm$. In this case you will need to substitute the initial conditions again to decide that the positive form is to be retained. Otherwise, students generally have no problems expanding

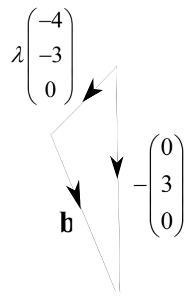
	$\dfrac{a+v}{a-v}=\mathrm{e}^{\frac{20t}{a}}$ $a+v=(a-v)\mathrm{e}^{\frac{20t}{a}}$ $\left(1+\mathrm{e}^{\frac{20t}{a}}\right)v=a\left(\mathrm{e}^{\frac{20t}{a}}-1\right)$ $v=a\left(\dfrac{\mathrm{e}^{\frac{20t}{a}}-1}{\mathrm{e}^{\frac{20t}{a}}+1}\right)$ (shown)	and rearranging the terms to obtain the final answer.
(ii) **[3]**	Since $a=2$, $v=2\left(\dfrac{\mathrm{e}^{10t}-1}{\mathrm{e}^{10t}+1}\right)$. $x=2\displaystyle\int_0^1 \frac{\mathrm{e}^{10t}-1}{\mathrm{e}^{10t}+1}\,\mathrm{d}t$ $=1.723$ metres (3 d.p.)	In this part of the question, many students wrongly substituted $t=1$ to find the value of v at that point. Do note that we are looking for x, not v. Many students did the integration manually without using GC, which led to many mistakes. Some examples: 1. Wrong method of integration. 2. Applying indefinite integration and then forgetting or wrongly calculating the value of the arbitrary constant c. Some students left their answer in 3 significant figures instead of 3 decimal places.

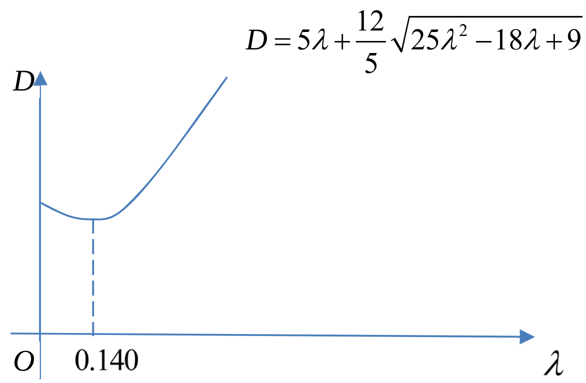
SOLUTION		COMMENTS
10 (i) [1]	For $x^2 + y^2 = 25, \quad y \le 0$ $(-3)^2 + (-4)^2 = 25.$ For $y = x + 3 + \dfrac{24}{x-3}$, $x = -3,\ y = -3 + 3 + \dfrac{24}{-3-3} = -4.$	This part is done well.

(ii) **[4]**	 Asymptotes : $y = x+3$ and $x = 3$	Graphs were generally sketched well. The only errors seen were related to inappropriate points i.e. (5, 0) being further to the right of (7.90, 15.8) or the curve C_2 not being sketched completely. A significant number of candidates used differentiation to compute the turning points when the graphing calculator could have been used.
(iii) **[6]**	Volume of solid obtained when R is rotated through 2π radians about the x-axis $= \pi \int_{-3}^{0} (25 - x^2) - \left(x + 3 + \frac{24}{x-3}\right)^2 \, dx$ $= \pi \int_{-3}^{0} \left[25 - x^2 - (x+3)^2 - 48\left(\frac{x+3}{x-3}\right) - \frac{24^2}{(x-3)^2} \right] dx$ $= \pi \int_{-3}^{0} \left[25 - x^2 - (x+3)^2 - 48\left(1 + \frac{6}{x-3}\right) - \frac{24^2}{(x-3)^2} \right] dx$ $= \pi \left[25x - \frac{x^3}{3} - \frac{(x+3)^3}{3} - 48\left(x + 6\ln\lvert x-3 \rvert\right) + \frac{24^2}{(x-3)} \right]_{-3}^{0}$ $= \pi \left(-\frac{27}{3} - 48(6\ln 3) + \frac{24^2}{-3} - 25(-3) + \frac{(-3)^3}{3} + 48(-3 + 6\ln 6) - \frac{24^2}{-6} \right)$ $= \pi(-9 - 192 + 75 - 9 - 144 + 96 - 288\ln 3 + 288\ln 6)$ $= \pi(288\ln 2 - 183)$	The instruction "Find the exact volume" was ignored by some candidates who had set up the integrals correctly but went on to use the calculator to compute the volume. It was necessary to realize that the points of intersection (–3, –4) and (0, –5) were already verified and found in parts **(i)** and **(ii)** respectively. Note $x + 3 + \frac{24}{x-3} \ne \frac{x^2 - 15}{x-3}$.

(iv) **[4]**	Translating the graph of $x^2+y^2=25$ 3 units in the negative x-direction, we have $(x+3)^2+y^2=25$. $(x+3)^2+y^2=25$ $(x+3)^2=25-y^2$ $x=-3\pm\sqrt{25-y^2}$ $x=-3+\sqrt{25-y^2}$ for $x\geq 3$ Volume of solid obtained when S is rotated through 2π radians about the line $x=3$ $=\pi\int_{-4}^{0}\left(-3+\sqrt{25-y^2}\right)^2\ \mathrm{d}y$ $=28.6$ (3 s.f.)	Most candidates were able to write down the equation for the translated graph but many went on to use the original equation and make y the subject to find the volume generated using integration of the region rotated about the x-axis when the question stated "rotated … about the vertical asymptote" which meant that the y-axis would be a more likely option.

SOLUTION		COMMENTS
11(i) **[3]**	$\overrightarrow{OA} = 6\left[\frac{1}{3}\begin{pmatrix}-1\\2\\-2\end{pmatrix}\right] + 15\left[\frac{1}{5}\begin{pmatrix}3\\0\\-4\end{pmatrix}\right] = \begin{pmatrix}-2\\4\\-4\end{pmatrix} + \begin{pmatrix}9\\0\\-12\end{pmatrix} = \begin{pmatrix}7\\4\\-16\end{pmatrix}$ Homebound global vector = $\overrightarrow{AO} = \begin{pmatrix}-7\\-4\\16\end{pmatrix}$ (shown) Distance from the hive $= \left\|\begin{pmatrix}-7\\-4\\16\end{pmatrix}\right\| = \sqrt{7^2+4^2+16^2} = \sqrt{321}$	A significant minority of students forgot to scale the directions to unit vectors. Some students missed this part.
(ii) **[1]**	The homebound vector may be blocked by obstacles along the path.	Any reasonable response is acceptable. However, anything blaming the poor bee is not, and neither are general statements that something can go wrong. Only a few students realized that (b) actually provided a plausible answer here.
(iii) **[4]**	Equation of line: $l: \mathbf{r} = \begin{pmatrix}3\\-2\\2\end{pmatrix} + \mu\begin{pmatrix}5\\1\\0\end{pmatrix}, \mu \in \mathbb{R}$ Let N be the foot of perpendicular from A to the line. $\overrightarrow{ON} = \begin{pmatrix}3\\-2\\2\end{pmatrix} + \mu\begin{pmatrix}5\\1\\0\end{pmatrix}$, for some $\mu \in \mathbb{R}$ $\overrightarrow{AN} = \begin{pmatrix}3\\-2\\2\end{pmatrix} + \mu\begin{pmatrix}5\\1\\0\end{pmatrix} - \begin{pmatrix}7\\4\\-16\end{pmatrix} = \begin{pmatrix}-4+5\mu\\-6+\mu\\18\end{pmatrix}$ Since $\overrightarrow{AN}$ is perpendicular to the line,	This part is generally handled proficiently by most students.

	$\overrightarrow{AN}\cdot\begin{pmatrix}5\\1\\0\end{pmatrix}=0$ $\begin{pmatrix}-4+5\mu\\-6+\mu\\18\end{pmatrix}\cdot\begin{pmatrix}5\\1\\0\end{pmatrix}=0$ $\Rightarrow -20+25\mu-6+\mu=0$ $\Rightarrow \mu=1$ $\Rightarrow \overrightarrow{ON}=\begin{pmatrix}8\\-1\\2\end{pmatrix}$	
(b) **[4]**	Let **b** be the direction vector back to the nest from any point along the homebound vector.  $\lambda\begin{pmatrix}-4\\-3\\0\end{pmatrix}+\mathbf{b}=-\begin{pmatrix}0\\3\\0\end{pmatrix}$ $\mathbf{b}=\lambda\begin{pmatrix}4\\3\\0\end{pmatrix}-\begin{pmatrix}0\\3\\0\end{pmatrix}=\begin{pmatrix}4\lambda\\3\lambda-3\\0\end{pmatrix}$ Distance to travel from the position vector $3\mathbf{j}$ back to the origin, D = Distance travelled via homebound vector + Distance travelled via searching process $=\left\|\begin{pmatrix}-4\lambda\\-3\lambda\\0\end{pmatrix}\right\|+2.4\left\|\begin{pmatrix}4\lambda\\3\lambda-3\\0\end{pmatrix}\right\|$ $=\sqrt{16\lambda^2+9\lambda^2}+\frac{12}{5}\sqrt{16\lambda^2+(3\lambda-3)^2}$ $=5\lambda+\frac{12}{5}\sqrt{25\lambda^2-18\lambda+9}$ Differentiating with respect to λ, $\frac{\mathrm{d}D}{\mathrm{d}\lambda}=5+\frac{6}{5}\left(16\lambda^2+(3\lambda-3)^2\right)^{-1/2}\left(32\lambda+6(3\lambda-3)\right)$ $5+\frac{6}{5}\left(16\lambda^2+(3\lambda-3)^2\right)^{-1/2}\left(32\lambda+6(3\lambda-3)\right)=0$ From G.C., $\lambda\approx 0.140$ OR	Students' success in this part mostly relies on interpreting the question correctly and quality of responses various greatly. Many students who got here squared to remove the radical. This is not only unnecessary, but also introduced a spurious solution that needs to be rejected.



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