RAFFLES INSTITUTION
2019 YEAR 6 PRELIMINARY EXAMINATION

CANDIDATE NAME

CLASS 19

MATHEMATICS **9758/01**

PAPER 1

3 hours

Candidates answer on the Question Paper
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

FOR EXAMINER'S USE					
Q1	**Q2**	**Q3**	**Q4**	**Q5**	**Q6**
Q7	**Q8**	**Q9**	**Q10**	**Q11**	**Total**

1 A curve C has equation

$$y = \frac{a}{x^3} + bx + c\,,$$

where a, b and c are constants. It is given that C has a stationary point $(-1.2, 6.6)$ and it also passes through the point $(2.1, -4.5)$.

(i) Find the values of a, b and c, giving your answers correct to 1 decimal place. [4]

(ii) One asymptote of C is the line with equation $x = 0$. Write down the equation of the other asymptote of C. [1]

2 Two variables u and v are connected by the equation $\frac{1}{u} + \frac{1}{v} = \frac{1}{20}$. Given that u and v both vary with time t, find an equation connecting $\frac{\mathrm{d}u}{\mathrm{d}t}$, $\frac{\mathrm{d}v}{\mathrm{d}t}$, u and v. Given also that u is decreasing at a rate of 2 units per second, calculate the rate of increase of v when $u = 60$ units.

[4]

3

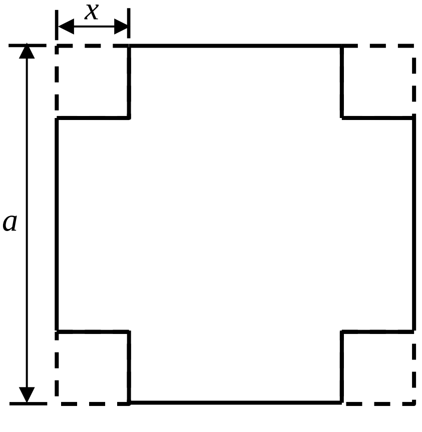


Fig. 1

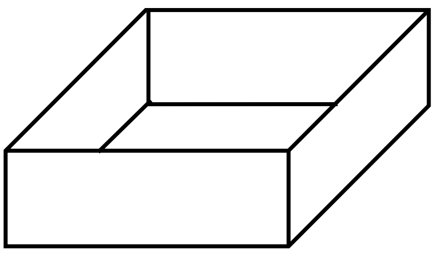


Fig. 2

Fig. 1 shows a square sheet of metal with side a cm. A square x cm by x cm is cut from each corner. The sides are then bent upwards to form an open box as shown in Fig. 2. Use differentiation to find, in terms of a, the maximum volume of the box, proving that it is a maximum. [6]

4 A curve C has parametric equations

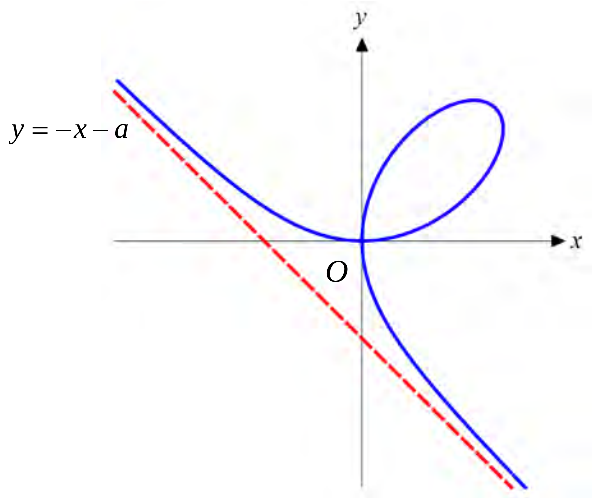
$$x = (1+t)^2, \qquad y = 2(1-t)^2.$$

(i) Find the coordinates of the point A where the tangent to C is parallel to the x-axis. [4]

(ii) The line $y = -x + d$ intersects C at the point A and another point B. Find the exact coordinates of B. [4]

(iii) Find the area of the triangle formed by A, B and the origin. [1]

5



The diagram shows the graph of Folium of Descartes with cartesian equation

$$x^3 + y^3 = 3axy,$$

where a is a positive constant. The curve passes through the origin, and has an oblique asymptote with equation $y = -x - a$.

(i) Given that (0, 0) is a stationary point on the curve, find, in terms of a, the coordinates of the other stationary point. [5]

(ii) Sketch the graph of

$$|x|^3 + y^3 = 3a|x|y,$$

including the equations of any asymptotes, coordinates of the stationary points and the point where the graph crosses the x-axis. [3]

6 **(a)** Given that $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$, find an expression for $\sum_{r=n+1}^{2n} \left(r(r-1)\right)$, simplifying your answer. [3]

(b) **(i)** Use the method of differences to find $\sum_{r=1}^{n} \left(\frac{(r+1)-\mathrm{e}r}{\mathrm{e}^r} \right)$. [3]

(ii) Hence find $\sum_{r=2}^{n} \left(\frac{\mathrm{e}+r-\mathrm{e}r}{\mathrm{e}^{r-1}} \right)$. [2]

7 **(a)** The complex numbers $3+2\mathrm{i}$, z, $4-6\mathrm{i}$ are the first three terms in a geometric progression. Without using a calculator, find the two possible values of z. [4]

(b) **(i)** The complex number w is such that $w = a + \mathrm{i}b$, where a and b are non-zero real numbers. The complex conjugate of w is denoted by w^*. Given that $\frac{w^2}{w^*}$ is a real number, find the possible values of w in terms of a only. [4]

(ii) Hence, find the exact possible arguments of w if a is positive. [2]

8 **(a)** Find the exact value of m such that

$$\int_0^3 \frac{1}{9+x^2}\,\mathrm{d}x = \int_0^{\frac{1}{2m}} \frac{1}{\sqrt{1-m^2x^2}}\,\mathrm{d}x.$$ [5]

(b) **(i)** Use the substitution $u = \sin 2x$ to show that

$$\int_0^{\frac{\pi}{4}} \sin^3 2x \cos^3 2x \,\mathrm{d}x = \frac{1}{2}\int_0^1 \left(u^3 - u^5\right)\,\mathrm{d}u.$$ [4]

(ii) Hence find the exact value of $\int_0^{\frac{\pi}{4}} \sin^3 2x \cos^3 2x \,\mathrm{d}x$. [2]

9 **(i)** Write down $\int \frac{1}{a^2 - v^2}\,\mathrm{d}v$, where a is a positive constant. [1]

(ii) In the motion of an object through a certain medium, the medium furnishes a resisting force proportional to the square of the velocity of the moving object. Suppose that a body falls vertically through the medium, the model used to describe the velocity, v ms^{-1} of the body at time t seconds after release from rest is given by the differential equation

$$\frac{a^2}{10}\frac{\mathrm{d}v}{\mathrm{d}t} = a^2 - v^2,$$

where a is a positive constant.

(a) Show that $v = a\left(\frac{\mathrm{e}^{\frac{20t}{a}} - 1}{\mathrm{e}^{\frac{20t}{a}} + 1}\right)$. [8]

(b) The rate of change of the displacement, x metres, of the body from the point of release is the velocity of the body. Given that $a = 2$, find the value of x when $t = 1$, giving your answer correct to 3 decimal places. [3]

10 The curve C_1 has equation $x^2 + y^2 = 25, \quad y \leq 0$.

The curve C_2 has equation $y = x + 3 + \frac{24}{x-3}$.

(i) Verify that $(-3, -4)$ lies on both C_1 and C_2. [1]

(ii) Sketch C_1 and C_2 on the same diagram, stating the coordinates of any stationary points, points of intersection with the axes and the equations of any asymptotes. [4]

The region bounded by C_1 and C_2 is R.

(iii) Find the exact volume of solid obtained when R is rotated through 2π radians about the x-axis. [6]

The region bounded by C_1, the x-axis and the vertical asymptote of C_2, where $x > 3$, is S.

(iv) Write down the equation of the curve obtained when C_1 is translated by 3 units in the negative x-direction.

Hence, or otherwise, find the volume of solid obtained when S is rotated through 2π radians about the vertical asymptote of C_2. [4]

11 Path integration is a predominant mode of navigation strategy used by many animals to return home by the *shortest* possible route during a food foraging journey. In path integration, animals continuously compute a homebound global vector relative to their starting position by integrating the angles steered and distances travelled during the entire foraging run. Once a food item has been found, the animal commences its homing run by using the homebound global vector, which was acquired during the outbound run.

(a) A Honeybee's hive is located at the origin *O*. The Honeybee travels 6 units in the direction $-\mathbf{i}+2\mathbf{j}-2\mathbf{k}$ before moving 15 units in the direction $3\mathbf{i}-4\mathbf{k}$. The Honeybee is now at point *A*.

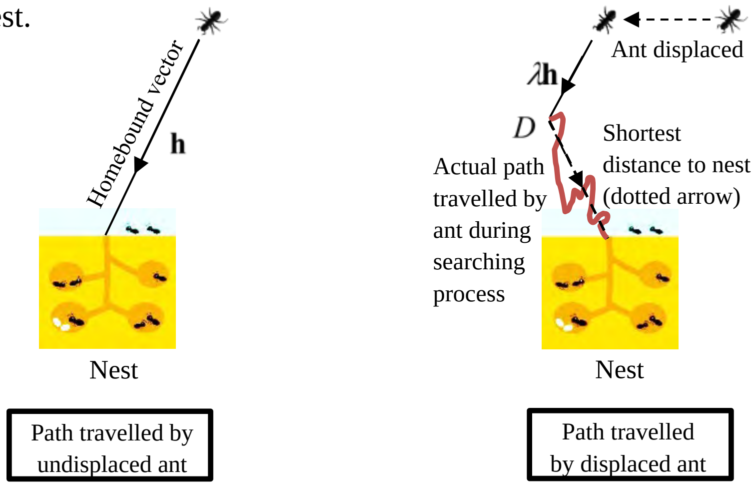
(i) Show that the homebound global vector $\overrightarrow{AO}$ is $-7\mathbf{i}-4\mathbf{j}+16\mathbf{k}$. Hence find the exact distance the Honeybee is from its hive. [3]

(ii) Explain why path integration may fail. [1]

A row of flowers is planted along the line $\dfrac{x-3}{5}=y+2,\ z=2$.

(iii) The Honeybee will take the shortest distance from point *A* to the row of flowers. Find the position vector of the point along the row of flowers which the Honeybee will fly to. [4]

(b) To further improve their chances of returning home, apart from relying on the path integration technique, animals depend on visual landmarks to provide directional information. When an ant is displaced to distant locations where familiar visual landmarks are absent, its initial path is guided solely by the homebound global vector, $\mathbf{h}$, until it reaches a point *D* and begins a search for their nest (see diagram). During the searching process, the distance travelled by the ant is 2.4 times the shortest distance back to the nest.



Let an ant's nest be located at the origin *O*. The ant has completed its foraging journey and is at a point with position vector $4\mathbf{i}+3\mathbf{j}$. A boy picks up the ant and displaces it 4 units in the direction $-\mathbf{i}$. Given $\lambda(-4\mathbf{i}-3\mathbf{j})$ as the initial path taken by the ant before it begins a search for its nest, find the value of λ which gives the minimum total distance travelled by the ant back to the nest. [4]

[It is not necessary to verify the nature of the minimum point in this part.]