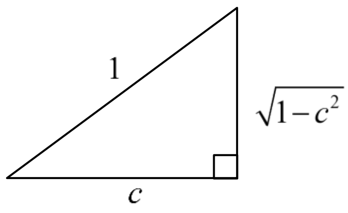
O-LEVEL A-MATHS 2019 – PAPER 1

Question 1

[Ans: (i) $\frac{\sqrt{1-c^2}}{c}$ (ii) $\frac{1}{\sqrt{1-c^2}}$]

(i) $\cos\theta = c = \frac{c}{1}$



From the triangle,

$$\tan\theta = \frac{\sqrt{1-c^2}}{c}$$

(ii) $\sin\theta = \frac{\sqrt{1-c^2}}{1} = \sqrt{1-c^2}$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\sqrt{1-c^2}}$$

Question 2

[Ans: $-1 < k < 1$]

$y = x^2 + (2k+1)x + 1$ (1)

$y = x$ (2)

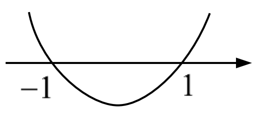
(1) = (2)

$$x^2 + (2k+1)x + 1 = x$$
$$x^2 + (2k+1)x - x + 1 = 0$$
$$x^2 + (2k+1-1)x + 1 = 0$$
$$x^2 + 2kx + 1 = 0$$

Since curve representing (1) does not intersect curve representing (2),

Discriminant < 0

$$(2k)^2 - 4(1)(1) < 0$$
$$k^2 - 1 < 0$$
$$(k-1)(k+1) < 0$$



$$-1 < k < 1$$

Question 3

[Ans: $A=\frac{1}{6}$, $B=-\frac{1}{3}$]

$$y = Ae^{2x} + Be^{-x}$$

$$\frac{dy}{dx} = A\left(e^{2x}\right)(2) + B\left(e^{-x}\right)(-1)$$

$$= 2Ae^{2x} - Be^{-x}$$

LHS

$$= \frac{dy}{dx} + 4y = e^{2x} - e^{-x}$$

$$= \left(2Ae^{2x} - Be^{-x}\right) + 4\left(Ae^{2x} + Be^{-x}\right)$$

$$= 6Ae^{2x} + 3Be^{-x}$$

$$\therefore 6A = 1 \Rightarrow A = \frac{1}{6}$$

$$3B = -1 \Rightarrow B = -\frac{1}{3}$$

Question 4

[Ans: $-\frac{2}{5}$]

$$A = 6(x)(x) = 6x^2$$

$$\frac{dA}{dx} = 12x$$

When $x = 10$,

$$\frac{dA}{dt} = -48 \text{ (given)}$$

$$\frac{dA}{dx} = 12(10) = 120$$

$$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$$

$$-48 = (120)\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{-48}{120} = -\frac{2}{5}$$

Queston 5

[Ans: (i) show (ii) 99 (iii) -0.236]

(i)

Minute	Number of germs present
0	N
1	$(1-0.21)N = 0.79N$
2	$(0.79)(0.79N) = (0.79)^2 N$
3	$(0.79)\left[(0.79)^2 N\right] = (0.79)^3 N$
$\vdots$	
n	$(0.79)^n N$ (shown)

(ii) $x = \dfrac{N-(0.79)^{20} N}{N} \times 100$

$= \dfrac{1-(0.79)^{20}}{1} \times 100 = 99$ (to 2 s.f.)

(iii) $(0.79)^n N = Ne^{kn}$

$(0.79)^n = \left(e^k\right)^n$

$e^k = 0.79$

$k = \ln 0.79 = -0.236$

Question 6

[Ans: (i) show (ii) explain]

(i) $\angle TQP = \angle STP$ (alt. segment th.)

$\angle QPT = \angle TPS$

$\angle PTQ = 90°$ ($\angle$ in a semicircle)

For triangle PQT,

$\angle TQP + \angle QPT + \angle PTQ = 180°$ ($\angle$ sum of triangle)

$\angle TQP + \angle QPT + 90° = 180°$

$\angle TQP + \angle QPT = 90°$

$\angle STP + \angle TPS = 90°$

For triangle PST,

$\angle STP + \angle TPS + \angle TSP = 180°$ ($\angle$ sum of triangle)

$90° + \angle TSP = 180°$

$\angle TSP = 90°$

$\angle TSR + \angle TSP = 180°$ (adj. $\angle$s on a st. line)

$\angle TSR + 90° = 180°$

$\angle TSR = 90°$ (shown)

(ii) As $\angle TSR = 90°$, TR is the diameter of a circle passing through S, R and T ($\angle$ in a semicircle).

$\therefore$ this circle has its centre at the midpoint of TR.

Question 7

[Ans: (i) $64-24x+\frac{15}{4}x^2+\cdots$ (ii) $\frac{17}{40}$]

(i) $\left(2-\frac{x}{8}\right)^6$

$$=(2)^6+\binom{6}{1}(2)^5\left(-\frac{x}{8}\right)+\binom{6}{2}(2)^4\left(-\frac{x}{8}\right)^2+\cdots$$

$$=64-24x+\frac{15}{4}x^2+\cdots$$

(ii) $\left(4+kx+x^2\right)\left(2-\frac{x}{8}\right)^6$

$$=\left(4+kx+x^2\right)\left(64-24x+\frac{15}{4}x^2+\cdots\right)$$

Coefficient of $x=(4)(-24)+(k)(64)=-96+64k$

Coefficient of $x^2=(4)\left(\frac{15}{4}\right)+(k)(-24)+64=79-24k$

$$(-96+64k)+(79-24k)=0$$

$$40k=17\Rightarrow k=\frac{17}{40}$$

Question 8

[Ans: (i) $\frac{dy}{dx}=1-\frac{9}{(x-2)^2}$; $\frac{d^2y}{dx^2}=\frac{18}{(x-2)^3}$ (ii) $x=-1$ or 5 (iii) maximum (when $x=-1$); minimum (when $x=5$)]

(i) $\frac{dy}{dx}=1+\frac{(x-2)(2)-(2x+5)(1)}{(x-2)^2}$

$=1+\frac{2x-4-2x-5}{(x-2)^2}=1-\frac{9}{(x-2)^2}$

$\frac{d^2y}{dx^2}=-9(-2)(x-2)^{-3}(1)=\frac{18}{(x-2)^3}$

(ii) Let $\frac{dy}{dx}=0$

$1-\frac{9}{(x-2)^2}=0$

$\frac{9}{(x-2)^2}=1$

$(x-2)^2=9$

$x-2=-3$ or $x-2=3$

$x=-1$ $\quad$ $x=5$

(iii) When $x=-1$,

$\frac{d^2y}{dx^2}=\frac{18}{(-1-2)^3}=-\frac{2}{3}<0$

$\therefore$ this is a maximum point.

When $x=5$,

$\frac{d^2y}{dx^2}=\frac{18}{(5-2)^3}=\frac{2}{3}>0$

$\therefore$ this is a minimum point.

Question 9

[Ans: (i) explain (ii) $C(4,-2)$ (iii) $D(7,5)$]

(i) Since $OP \perp AB$ and $OA = OB = 2$, ABP is an isosceles triangle.
$\Rightarrow \angle PAB = \angle PBA = 45°$

Since triangle ABP and triangle CBP are congruent, $\angle CBP = \angle PBA = 45°$
$\Rightarrow \angle ABC = \angle PBA + \angle PBC = 45° + 45° = 90°$

$\therefore BC$ is parallel to the x-axis.

(ii) $BC = AB = 4$ and $OB = 2$
Since BC is parallel to the x-axis, $C(4,-2)$

(iii) Let $D(x, y)$.

Area of triangle $ABD = \frac{28}{2}$

$\frac{1}{2}(AB)(x) = 14$

$\frac{1}{2}(4)(x) = 14 \Rightarrow x = 7$

Area of triangle $BCD = \frac{28}{2}$

$\frac{1}{2}(BC)(2+y) = 14$

$\frac{1}{2}(4)(2+y) = 14 \Rightarrow 2 + y = 7 \Rightarrow y = 5$

$D(7,5)$

Question 10

[Ans: (a) $x=-\frac{1}{3},\ y=\frac{4}{3}$ (b) $11-4\sqrt{7}$]

(a) $3^{x+y}=\sqrt[3]{27}\Rightarrow 3^{x+y}=3^{1}\Rightarrow x+y=1$ (1)

$$\frac{4^{y}}{2^{x}}=\left(\frac{1}{2}\right)^{-3}\Rightarrow \frac{2^{2y}}{2^{x}}=\left(2^{-1}\right)^{-3}$$

$2^{2y-x}=2^{3}\Rightarrow 2y-x=3$ (2)

(1) + (2)

$$3y=4\Rightarrow y=\frac{4}{3}$$

Sub. $y=\frac{4}{3}$ into (1)

$$x+\frac{4}{3}=1\Rightarrow x=-\frac{1}{3}$$

(b) Volume of cylinder $=\left(3\sqrt{7}-6\right)\pi$

$$\pi r^{2}\left(2+\sqrt{7}\right)=\left(3\sqrt{7}-6\right)\pi$$

$$r^{2}=\frac{3\sqrt{7}-6}{2+\sqrt{7}}$$

$$=\frac{3\sqrt{7}-6}{2+\sqrt{7}}\left(\frac{2-\sqrt{7}}{2-\sqrt{7}}\right)$$

$$=\frac{\left(3\sqrt{7}\right)(2)-3(7)-12+6\sqrt{7}}{2^{2}-\left(\sqrt{7}\right)^{2}}$$

$$=\frac{-33+12\sqrt{7}}{-3}=11-4\sqrt{7}$$

Question 11

[Ans: (i) $t = 1$ s or 3 s (ii) -6 cm/s^2 (iii) 12 cm]

(i) $v = \frac{ds}{dt} = 3t^2 - 12t + 9$

When the dot is instantaneously at rest,

$v = 0$

$3t^2 - 12t + 9 = 0$

$t^2 - 4t + 3 = 0$

$(t-1)(t-3) = 0$

$t = 1$ or 3

(ii) $a = \frac{dv}{dt} = 6t - 12$

When $t = 1$,

$a = 6(1) - 12 = -6$

(iii) Since the dot was instantaneously at rest when $t = 1$ and when $t = 3$, it would have changed direction twice between $t = 0$ and $t = 4$. Therefore the total distance travelled will not be the displacement, s, when $t = 4$.

(iv) $s = t^3 - 6t^2 + 9t$

When $t = 0$, $s = 0$

When $t = 1$, $s = (1)^3 - 6(1)^2 + 9(1) = 4$

When $t = 3$, $s = (3)^3 - 6(3)^2 + 9(3) = 0$

When $t = 4$, $s = (4)^3 - 6(4)^2 + 9(4) = 4$

$\therefore$ total distance

$= 4 \times 3 = 12$

Question 12

[Ans: (i) ± 2 (ii) -4; 2 (iii) $180°$ (iv) $720°$ (v) sketch (vi) 3]

(i) Least of $f(x) = -2$

Greatest of $f(x) = 2$

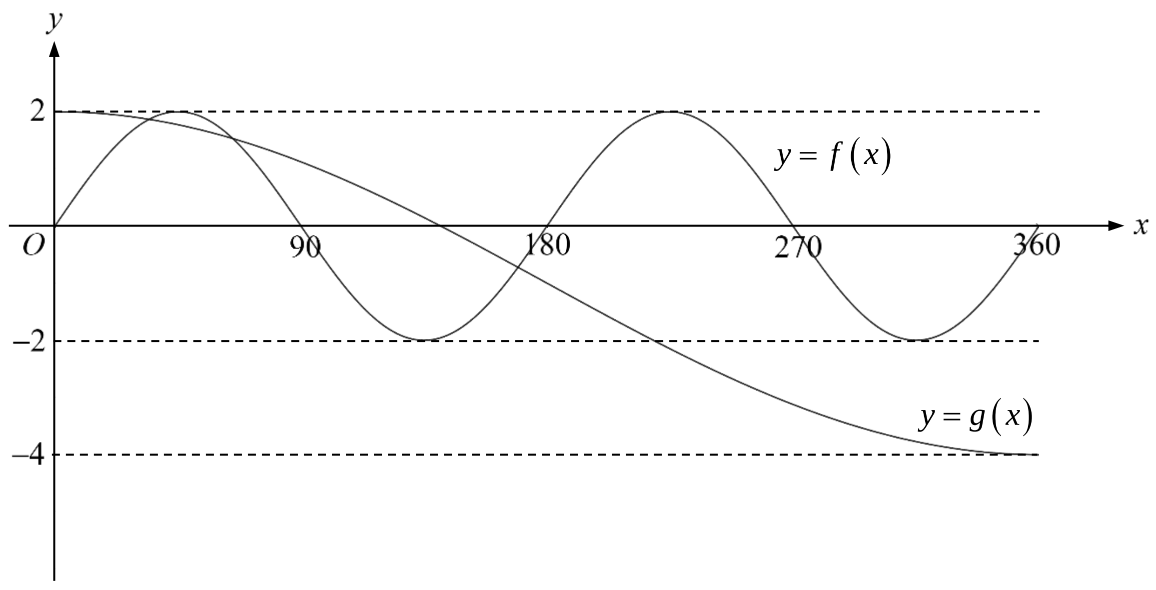
(ii) Least of $g(x) = -3 - 1 = -4$

Greatest of $g(x) = 3 - 1 = 2$

(iii) Period of $f(x) = \dfrac{360°}{2} = 180°$

(iv) Period of $g(x) = \dfrac{360°}{\frac{1}{2}} = 720°$

(v)



(vi) Number of solutions $= 3$