

A-LEVEL H2 MATHS 2018 – PAPER 2

Question 1

$$[\text{Ans: (i) } y = 3\left(\frac{2}{9}x + 4\right)^{\frac{3}{2}} + 45 \text{ (ii) } (54, 237)]$$

$$(i) \frac{dy}{dx} = \left(\frac{1}{3}y - 15\right)^{\frac{1}{3}}$$

$$\left(\frac{1}{3}y - 15\right)^{-\frac{1}{3}} \frac{dy}{dx} = 1$$

$$\int \left(\frac{1}{3}y - 15\right)^{-\frac{1}{3}} dy = \int dx$$

$$\frac{\left(\frac{1}{3}y - 15\right)^{\frac{2}{3}}}{\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)} = x + A$$

$$\left(\frac{1}{3}y - 15\right)^{\frac{2}{3}} = \frac{2}{9}x + B$$

When $x = 0$, $y = 69$,

$$\left[\frac{1}{3}(69) - 15\right]^{\frac{2}{3}} = \frac{2}{9}(0) + B \Rightarrow B = 4$$

$$\left(\frac{1}{3}y - 15\right)^{\frac{2}{3}} = \frac{2}{9}x + 4$$

$$\frac{1}{3}y - 15 = \left(\frac{2}{9}x + 4\right)^{\frac{3}{2}} \Rightarrow y = 3\left(\frac{2}{9}x + 4\right)^{\frac{3}{2}} + 45$$

(ii) When $\frac{dy}{dx} = 4$,

$$\left(\frac{1}{3}y - 15\right)^{\frac{1}{3}} = 4 \Rightarrow y = 237$$

$$\therefore \left[\frac{1}{3}(237) - 15\right]^{\frac{2}{3}} = \frac{2}{9}x + 4 \Rightarrow x = 54$$

\therefore coordinates of the point on the curve where gradient is 4 is (54, 237).

Question 2

[Ans: (a) $x = 2 - 3i, 2 + 3i$ or $\frac{1}{2}$; $s = 69, t = 13$ (b)(i) $-\frac{3}{2} - \frac{3\sqrt{3}}{2}i$ and $-\frac{3}{2} + \frac{3\sqrt{3}}{2}i$
 (ii) $w = 3e^{i(0)}, w = 3e^{-i\frac{2\pi}{3}}$ or $w = 3e^{i\frac{2\pi}{3}}$; sketch (iii) 0; 27]

(a) Let $f(x) = 4x^4 - 20x^3 + sx^2 - 56x + t$

$$f(2 - 3i) = 0$$

$$4(2 - 3i)^4 - 20(2 - 3i)^3 + s(2 - 3i)^2 - 56(2 - 3i) + t = 0$$

$$332 + 828i + (-5 - 12i)s + t = 0$$

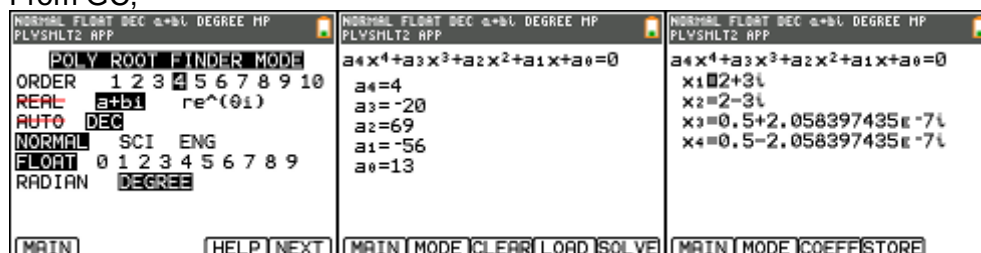
$$(332 - 5s + t) + i(828 - 12s) = 0$$

$$\therefore 828 - 12s = 0 \Rightarrow s = 69$$

$$\therefore 332 - 5(69) + t = 0 \Rightarrow t = 13$$

$$4x^4 - 20x^3 + 69x^2 - 56x + 13 = 0$$

From GC,



$$x = 2 - 3i, 2 + 3i \text{ or } \frac{1}{2}$$

(b) (i) Let $w = x + iy$

$$(x + iy)^3 = 27$$

$$x^3 + 3x^2(iy) + 3x(iy)^2 + (iy)^3 = 27$$

$$(x^3 - 3xy^2) + i(3x^2y - y^3) = 27$$

$$3x^2y - y^3 = 0$$

$$y(3x^2 - y^2) = 0$$

$$y(\sqrt{3}x + y)(\sqrt{3}x - y) = 0$$

$$y = 0 \text{ or } y = -\sqrt{3}x \text{ or } y = \sqrt{3}x$$

When $y = 0,$

$$x^3 - 3x(0)^2 = 27$$

$$x^3 = 27 \Rightarrow x = 3$$

[Continues on next page]

When $y = \pm\sqrt{3}x$,

$$x^3 - 3x(\pm\sqrt{3}x)^2 = 27$$

$$x^3 = -\frac{27}{8} \Rightarrow x = -\frac{3}{2}$$

$$\Rightarrow y = \pm \frac{3\sqrt{3}}{2}$$

\therefore the other possible values of w are $-\frac{3}{2} - \frac{3\sqrt{3}}{2}i$ and $-\frac{3}{2} + \frac{3\sqrt{3}}{2}i$.

(ii) For $w = 3$,

$$w = 3e^{i(0)}$$

For $w = -\frac{3}{2} - \frac{3\sqrt{3}}{2}i$,

$$|w| = \sqrt{\left(-\frac{3}{2}\right)^2 + \left(-\frac{3\sqrt{3}}{2}\right)^2} = 3; \arg w = -\pi + \tan^{-1} \frac{3\sqrt{3}/2}{3/2} = -\frac{2\pi}{3}$$

$$\therefore w = 3e^{-i\frac{2\pi}{3}}$$

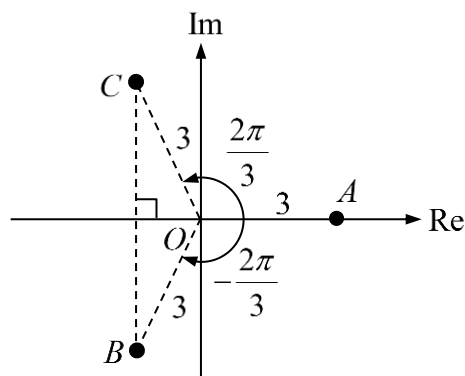
For $w = -\frac{3}{2} + \frac{3\sqrt{3}}{2}i$,

$$w = 3e^{i\frac{2\pi}{3}}$$

$$A: 3e^{i(0)}$$

$$B: 3e^{-i\frac{2\pi}{3}}$$

$$C: 3e^{i\frac{2\pi}{3}}$$



(iii) Sum

$$= 3 + \left(-\frac{3}{2} - \frac{3\sqrt{3}}{2}i\right) + \left(-\frac{3}{2} + \frac{3\sqrt{3}}{2}i\right) = 0$$

Product

$$= \left[3e^{i(0)}\right] \left[3e^{-i\frac{2\pi}{3}}\right] \left[3e^{i\frac{2\pi}{3}}\right] = 27e^{i\left(0 - \frac{2\pi}{3} + \frac{2\pi}{3}\right)} = 27e^{i(0)} = 27$$

Question 3

[Ans: (i) $D(-5, -4, 3)$ (ii) $4x + 45y + 20z = 200$ (iii) 58.6° (iv) 6.88 units]

$$\begin{aligned} \text{(i)} \quad \overrightarrow{OD} &= \overrightarrow{OA} + \overrightarrow{AD} \\ &= \overrightarrow{OA} + \overrightarrow{BC} = \overrightarrow{OA} + (\overrightarrow{OC} - \overrightarrow{OB}) = \begin{pmatrix} 5 \\ -4 \\ 1 \end{pmatrix} + \left[\begin{pmatrix} -5 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 5 \\ 4 \\ 0 \end{pmatrix} \right] = \begin{pmatrix} -5 \\ -4 \\ 3 \end{pmatrix} \end{aligned}$$

$$\therefore D(-5, -4, 3)$$

$$\text{(ii)} \quad \overrightarrow{BC} = \begin{pmatrix} -5 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 5 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -10 \\ 0 \\ 2 \end{pmatrix}; \quad \overrightarrow{BE} = \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix} - \begin{pmatrix} 5 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -5 \\ -4 \\ 10 \end{pmatrix}$$

$$\begin{pmatrix} -10 \\ 0 \\ 2 \end{pmatrix} \times \begin{pmatrix} -5 \\ -4 \\ 10 \end{pmatrix} = \begin{pmatrix} 8 \\ 90 \\ 40 \end{pmatrix} = 2 \begin{pmatrix} 4 \\ 45 \\ 20 \end{pmatrix}$$

$$\text{Face } BCE : r \cdot \begin{pmatrix} 4 \\ 45 \\ 20 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 45 \\ 20 \end{pmatrix} = 200$$

\therefore cartesian equation of face BCE is $4x + 45y + 20z = 200$.

$$\text{(iii)} \quad \overrightarrow{AB} = \begin{pmatrix} 5 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 5 \\ -4 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \\ -1 \end{pmatrix}; \quad \overrightarrow{AD} = \begin{pmatrix} -5 \\ -4 \\ 3 \end{pmatrix} - \begin{pmatrix} 5 \\ -4 \\ 1 \end{pmatrix} = \begin{pmatrix} -10 \\ 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 8 \\ -1 \end{pmatrix} \times \begin{pmatrix} -10 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 16 \\ 10 \\ 80 \end{pmatrix} = 2 \begin{pmatrix} 8 \\ 5 \\ 40 \end{pmatrix}$$

Let the angle between face BCE and the base of the pyramid be θ .

$$\begin{pmatrix} 8 \\ 5 \\ 40 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 45 \\ 20 \end{pmatrix} = \left| \begin{pmatrix} 8 \\ 5 \\ 40 \end{pmatrix} \right| \left| \begin{pmatrix} 4 \\ 45 \\ 20 \end{pmatrix} \right| \cos \theta$$

$$1057 = \sqrt{1689} \sqrt{2441} \cos \theta$$

$$\cos \theta = \frac{1057}{\sqrt{1689} \sqrt{2441}} \Rightarrow \theta = 58.6^\circ$$

[Continues on next page]

(iv) Let M be the midpoint of edge AD .

$$\overrightarrow{OM} = \frac{(1)OA + (1)OD}{1+1} = \frac{1}{2} \left[\begin{pmatrix} 5 \\ -4 \\ 1 \end{pmatrix} + \begin{pmatrix} -5 \\ -4 \\ 3 \end{pmatrix} \right] = \begin{pmatrix} 0 \\ -4 \\ 2 \end{pmatrix}$$

Shortest distance

$$= \frac{\begin{vmatrix} \begin{pmatrix} 0 \\ -4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 45 \\ 20 \end{pmatrix} \\ \left\| \begin{pmatrix} 4 \\ 45 \\ 20 \end{pmatrix} \right\| \end{vmatrix} - \frac{200}{\left\| \begin{pmatrix} 4 \\ 45 \\ 20 \end{pmatrix} \right\|}}{\left\| \begin{pmatrix} 4 \\ 45 \\ 20 \end{pmatrix} \right\|} = \frac{-140}{\sqrt{2441}} - \frac{200}{\sqrt{2441}} = 6.88$$

Question 4

[Ans: (i) $-2x^2 - \frac{4}{3}x^4 - \frac{64}{45}x^6$; $x = \frac{1}{4}\pi$ (ii) $-2x - \frac{4}{3}x^3 - \frac{64}{225}x^5 + c$; -1.0644 (iii) -1.0670]

(i) $\ln(\cos 2x)$

$$\begin{aligned} &= \ln \left[1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \dots \right] \\ &= \ln \left(1 - 2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \dots \right) \\ &= \ln \left[1 + \left(-2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 + \dots \right) \right] \\ &\approx \left(-2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 \right) - \frac{\left(-2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 \right)^2}{2} + \frac{\left(-2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 \right)^3}{3} \\ &= \left(-2x^2 + \frac{2}{3}x^4 - \frac{4}{45}x^6 \right) - \frac{1}{2} \left(4x^4 - \frac{4}{3}x^6 - \frac{4}{3}x^6 \right) + \frac{1}{3}(-8x^6) \\ &= -2x^2 - \frac{4}{3}x^4 - \frac{64}{45}x^6 \end{aligned}$$

For $0 \leq x \leq \frac{1}{4}\pi$, $0 \leq \cos 2x \leq 1$. \therefore the expansion will be undefined when $\cos 2x = 0$ as $\ln 0$ is undefined.

$$\therefore \cos 2x = 0 \Rightarrow 2x = \frac{1}{2}\pi \Rightarrow x = \frac{1}{4}\pi$$

(ii) $\int \frac{\ln(\cos 2x)}{x^2} dx$

$$\begin{aligned} &\approx \int \frac{-2x^2 - \frac{4}{3}x^4 - \frac{64}{45}x^6}{x^2} dx \\ &= \int -2 - \frac{4}{3}x^2 - \frac{64}{45}x^4 dx \\ &= -2x - \frac{4}{9}x^3 - \frac{64}{225}x^5 + c \end{aligned}$$

$$\begin{aligned} \int_0^{0.5} \frac{\ln(\cos 2x)}{x^2} dx &= \left[-2x - \frac{4}{9}x^3 - \frac{64}{225}x^5 \right]_0^{0.5} \\ &= -1.0644 \end{aligned}$$

(iii) From GC, $\int_0^{0.5} \frac{\ln(\cos 2x)}{x^2} dx = -1.0670$

Question 5

[Ans: (i) explain; state (ii) $H_0 : \mu = 65000$; $H_1 : \mu < 65000$ (iii) $\sigma^2 > 9420000$]

- (i) As we do not know if the time to failure of the fan follows a normal distribution, in order to apply the Central Limit Theorem to assume the sample mean distribution of the time to failure of the fan follows a normal distribution, the sample size should be reasonably large. Therefore the manager should take a sample of at least 30 fans.

The fans should be randomly chosen.

- (ii) Let X be the time to failure of the fan, and μ be the population mean of X .

$$H_0 : \mu = 65000$$

$$H_1 : \mu < 65000$$

- (iii) $n = 43$

$$\bar{x} = 64230$$

Let σ^2 be the population variance.

Test Statistics,

$$n = 43 \text{ (large)}$$

$$\bar{X} \sim N\left(65000, \frac{\sigma^2}{43}\right) \text{ by CLT}$$

$$\Rightarrow Z = \frac{\bar{X} - 65000}{\sqrt{\frac{\sigma^2}{43}}} \sim N(0,1)$$

To not reject H_0 ,

$$p\text{-value} > 0.05$$

$$P(\bar{X} < 64230) > 0.05$$

$$P\left(Z < \frac{64230 - 65000}{\sqrt{\frac{\sigma^2}{43}}}\right) > 0.05$$

$$\frac{64230 - 65000}{\sqrt{\frac{\sigma^2}{43}}} > -1.6449$$

$$\frac{64230 - 65000}{-1.6449} < \sqrt{\frac{\sigma^2}{43}}$$

$$\sqrt{\sigma^2} > \frac{64230 - 65000}{-1.6449} \sqrt{43}$$

$$\sigma^2 > 9420000$$

Question 6

[Ans: (i) show (ii) $\frac{5}{9} < p < \frac{2}{3}$ (iii) 0.430](i) Let X be the number of left fork that the bug takes out of 8 forks.

$$X \sim B(8, p)$$

Probability for the bug to finish at D

$$= P(X = 5)$$

$$= \binom{8}{5} p^5 (1-p)^{8-5} = 56p^5 q^3 \text{ (shown)}$$

(ii) Given $P(X = 5)$ is the highest probability.

$$P(X = 5) > P(X = 4) \quad \text{and}$$

$$P(X = 5) > P(X = 6)$$

$$56p^5 q^3 > \binom{8}{4} p^4 q^4$$

$$56p^5 q^3 > \binom{8}{6} p^6 q^2$$

$$56p > 70q$$

$$56q > 28p$$

$$56p > 70(1-p)$$

$$56(1-p) > 28p$$

$$126p > 70$$

$$84p < 56$$

$$p > \frac{5}{9}$$

$$p < \frac{2}{3}$$

$$\therefore \frac{5}{9} < p < \frac{2}{3}$$

(iii) Required probability

$$= (1-0.1)^8$$

$$= 0.430$$

Question 7

[Ans: (i) $(1-a)(1-b)$; prove (ii) $1-a-c$; draw (iii)]

$$\begin{aligned}
 \text{(i)} \quad & P(A' \cap B') \\
 &= 1 - P(A \cup B) \\
 &= 1 - [P(A) + P(B) - P(A \cap B)] \\
 &= 1 - [P(A) + P(B) - P(A)P(B)] \\
 &= 1 - (a + b - ab) \\
 &= 1 - a - b + ab \\
 &= (1-a) - b(1-a) = (1-a)(1-b)
 \end{aligned}$$

$P(A' \cap B') = (1-a)(1-b) = P(A')P(B')$
 $\therefore A'$ and B' are independent events. (proven)

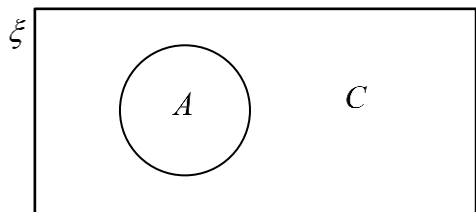
$$\begin{aligned}
 \text{(ii)} \quad & P(A' \cap C') \\
 &= 1 - P(A \cup C) \\
 &= 1 - [P(A) + P(C) - P(A \cap C)] \\
 &= 1 - (a + c - 0) = 1 - a - c
 \end{aligned}$$

If A' and C' are mutually exclusive events,

$$P(A' \cap C') = 0$$

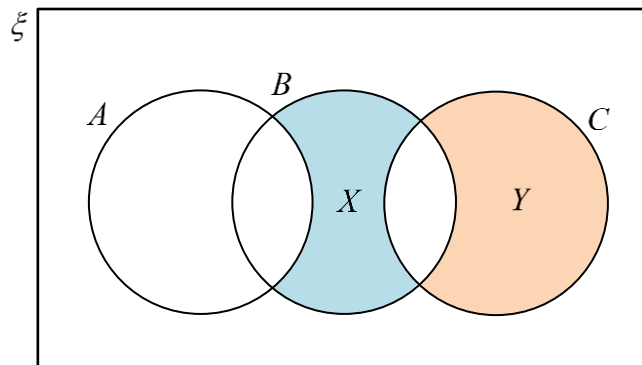
$$1 - a - c = 0$$

$$a + c = 1$$



[Continues on next page]

(iii)



Probability represented by the shaded regions

$$= 1 - P(A' \cap B' \cap C') - P(A) - P(B \cap C)$$

$$= 1 - \frac{1}{10} - \frac{2}{5} - \frac{1}{5} = \frac{3}{10}$$

$$P(A \cap B) = P(A)P(B) = \frac{2}{5}b$$

$\therefore \min P(A \cap B)$ and $\max P(A \cap B)$ depends directly on b being a minimum or maximum respectively.

For $\max b$, let $P(\text{Region } Y) = 0$:

$$\text{Probability represented by the shaded regions} = \frac{3}{10}$$

$$P(\text{Region } X) + P(\text{Region } Y) = \frac{3}{10}$$

$$P(A' \cap B \cap C') + 0 = \frac{3}{10}$$

$$P(B) - P(A \cap B) - P(B \cap C) = \frac{3}{10}$$

$$b - \frac{2}{5}b - \frac{1}{5} = \frac{3}{10} \Rightarrow b = \frac{5}{6}$$

For $\min b$, let $P(\text{Region } X) = 0$:

$$P(A' \cap B \cap C') = 0$$

$$P(B) - P(A \cap B) - P(B \cap C) = 0$$

$$b - \frac{2}{5}b - \frac{1}{5} = 0 \Rightarrow b = \frac{1}{3}$$

$$\therefore \frac{1}{3} \leq b \leq \frac{5}{6}$$

$$\frac{2}{15} \leq \frac{2}{5}b \leq \frac{1}{3} \Rightarrow \frac{2}{15} \leq P(A \cap B) \leq \frac{1}{3}$$

The maximum value of $P(A \cap B)$ is $\frac{1}{3}$, and the minimum value of $P(A \cap B)$ is $\frac{2}{15}$.

Question 8

[Ans: (i) probability distribution (ii) 0 ; explain (iii) show; $g(n) = 22n^2 + 78n + 36$]

(i)

	3	4	5
3	6	7	8
4	7	8	9
5	8	9	10

$$P(S=6) = \left(\frac{2}{n+5}\right)\left(\frac{1}{n+4}\right) = \frac{2}{(n+5)(n+4)}$$

$$P(S=7) = \left(\frac{2}{n+5}\right)\left(\frac{3}{n+4}\right) + \left(\frac{3}{n+5}\right)\left(\frac{2}{n+4}\right) = \frac{12}{(n+5)(n+4)}$$

$$P(S=8) = \left(\frac{2}{n+5}\right)\left(\frac{n}{n+4}\right) + \left(\frac{3}{n+5}\right)\left(\frac{2}{n+4}\right) + \left(\frac{n}{n+5}\right)\left(\frac{2}{n+4}\right) = \frac{4n+6}{(n+5)(n+4)}$$

$$P(S=9) = \left(\frac{3}{n+5}\right)\left(\frac{n}{n+4}\right) + \left(\frac{n}{n+5}\right)\left(\frac{3}{n+4}\right) = \frac{6n}{(n+5)(n+4)}$$

$$P(S=10) = \left(\frac{n}{n+5}\right)\left(\frac{n-1}{n+4}\right) = \frac{n(n-1)}{(n+5)(n+4)}$$

s	6	7	8	9	10
$P(S=s)$	$\frac{2}{(n+5)(n+4)}$	$\frac{12}{(n+5)(n+4)}$	$\frac{4n+6}{(n+5)(n+4)}$	$\frac{6n}{(n+5)(n+4)}$	$\frac{n(n-1)}{(n+5)(n+4)}$

(ii) When $n=1$, $P(S=10) = \frac{(1)(1-1)}{(1+5)(1+4)} = 0$

As there needs to be two balls numbered 5 in order to get sum of the numbers on the two balls taken to be 10, it is impossible for S to be 10 since now there is just 1 ball numbered 5. Therefore $P(S=10) = 0$ is expected.

(iii) $E(S)$

$$= \sum sP(S=s)$$

$$= 6 \left[\frac{2}{(n+5)(n+4)} \right] + 7 \left[\frac{12}{(n+5)(n+4)} \right] + 8 \left[\frac{4n+6}{(n+5)(n+4)} \right] + 9 \left[\frac{6n}{(n+5)(n+4)} \right]$$

$$+ 10 \left[\frac{n(n-1)}{(n+5)(n+4)} \right]$$

$$= \frac{12 + 84 + 32n + 48 + 54n + 10n^2 - 10n}{(n+5)(n+4)}$$

$$= \frac{10n^2 + 76n + 144}{(n+5)(n+4)}$$

$$= \frac{(10n+36)(n+4)}{(n+5)(n+4)} = \frac{10n+36}{n+5} \text{ (shown)}$$

[Continues on next page]

$$\begin{aligned}
E(S^2) &= \sum s^2 P(S = s) \\
&= 6^2 \left[\frac{2}{(n+5)(n+4)} \right] + 7^2 \left[\frac{12}{(n+5)(n+4)} \right] + 8^2 \left[\frac{4n+6}{(n+5)(n+4)} \right] + 9^2 \left[\frac{6n}{(n+5)(n+4)} \right] \\
&\quad + 10^2 \left[\frac{n(n-1)}{(n+5)(n+4)} \right] \\
&= \frac{72 + 588 + 256n + 384 + 486n + 100n^2 - 100n}{(n+5)(n+4)} \\
&= \frac{100n^2 + 642n + 1044}{(n+5)(n+4)}
\end{aligned}$$

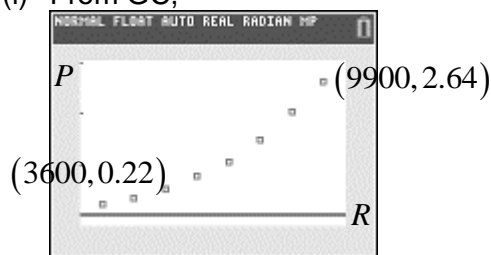
$$\begin{aligned}
\text{Var}(S) &= E(S^2) - [E(S)]^2 \\
&= \frac{100n^2 + 642n + 1044}{(n+5)(n+4)} - \left(\frac{10n+36}{n+5} \right)^2 \\
&= \frac{(100n^2 + 642n + 1044)(n+5) - (10n+36)^2(n+4)}{(n+5)^2(n+4)} \\
&= \frac{(100n^2 + 642n + 1044)(n+5) - (100n^2 + 720n + 1296)(n+4)}{(n+5)^2(n+4)} \\
&= \frac{22n^2 + 78n + 36}{(n+5)^2(n+4)}
\end{aligned}$$

$$\therefore g(n) = 22n^2 + 78n + 36$$

Question 9

- [Ans: (i) draw; explain (ii) $r = 0.969$; $r = 0.993$; explain; $P = 2.85 \times 10^{-8} R^2 - 0.283$
 (iii) $R = 6450$; explain (iv) $P = 0.0273$; explain (v) $P = 1.03 \times 10^{-4} R^2 - 0.283$]

(i) From GC,



The scatter diagram suggests that the relationship between P and R is non-linear, where P increases at an increasing rate as R increases. Therefore it will not be well modelled by the linear equation $P = aR + b$.

(ii) For model $P = aR + b$,

L1	L2	L3	L4	L5	2
3600	0.22				
4500	0.34				
5400	0.52				
6300	0.78				
7200	1.06				
8100	1.48				
9000	2.04				
9900	2.64				
*****	*****				

Lrc1)=0.22

```

LinReg(a+bx)
Xlist:L1
Ylist:L2
FreqList:
Store RegEQ:
Calculate
    
```

```

LinReg
y=a+bx
a=-1.418571429
b=3.783068783E-4
r^2=0.9395255922
r=0.9692912835
    
```

$r = 0.969$

For model $P = aR^2 + b$,

L1	L2	L3	L4	L5	3
3600	0.22	1.31E7			
4500	0.34	2.03E7			
5400	0.52	2.92E7			
6300	0.78	3.97E7			
7200	1.06	5.18E7			
8100	1.48	6.56E7			
9000	2.04	8.1E7			
9900	2.64	9.8E7			
*****	*****	*****			

Lrc1)=12960000

```

LinReg(a+bx)
Xlist:L3
Ylist:L2
FreqList:
Store RegEQ:
Calculate
    
```

```

LinReg
y=a+bx
a=-0.2826076107
b=2.845744476E-8
r^2=0.9861260773
r=0.9930388096
    
```

$r = 0.993$

Since $|r|$ for model $P = aR^2 + b$ is closer to 1, it will better model the relationship between P and R .

From GC, equation: $P = 2.85 \times 10^{-8} R^2 - 0.283$

(iii) When $P = 0.9$,

$$0.9 = 2.8457 \times 10^{-8} R^2 - 0.28261$$

$$R = 6450$$

As $P = 0.9$ is within the data range, $0.22 \leq P \leq 2.64$, and r is very close to 1, the estimate is reliable.

(iv) When $R = 3300$,

$$P = 2.8457 \times 10^{-8} (3300)^2 - 0.28261$$

$$= 0.0273$$

As $R = 3300$ is outside the data range, $3600 \leq R \leq 9900$, the estimate is unreliable.

[Continues on next page]

- (v) Given R is the speed of the fan in revolution per second.
Let R_m be speed of the fan in revolution per minute.

$$P = 2.8457 \times 10^{-8} R_m^2 - 0.28261 \text{ and } R_m = 60R$$

$$\begin{aligned} P &= 2.8457 \times 10^{-8} (60R)^2 - 0.28261 \\ &= 1.03 \times 10^{-4} R^2 - 0.283 \end{aligned}$$

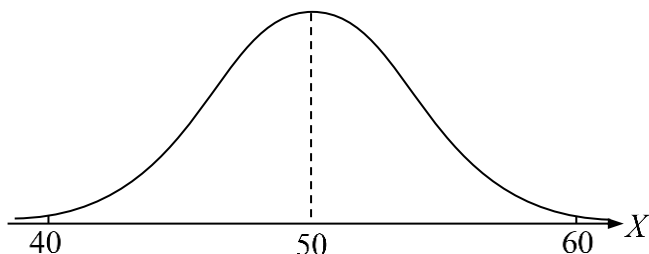
Question 10

[Ans: (i) sketch (ii) 0.605 (iii) 0.773 (iv) 0.126 (v) 136 (vi) 0.185]

Let X be the mass of one type of specialist light bulb.

$$X \sim N(50, 1.5^2)$$

(i)



(ii) $P(X < 50.4) = 0.605$

(iii) Let Y be the mass of an empty box.

$$Y \sim N(75, 2^2)$$

$$E(Y_1 + \dots + Y_4) = 4E(Y) = 4(75) = 300$$

$$\text{Var}(Y_1 + \dots + Y_4) = 4\text{Var}(Y) = 4(2^2) = 16$$

$$\therefore Y_1 + \dots + Y_4 \sim N(300, 16)$$

$$P(Y_1 + \dots + Y_4 > 297) = 0.773$$

(iv) $E(X + Y) = E(X) + E(Y) = 50 + 75 = 125$

$$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) = 1.5^2 + 2^2 = 6.25$$

$$\therefore X + Y \sim N(125, 6.25)$$

$$P(124.9 < X + Y < 125.7) = 0.126$$

(v) $E(1.3X + Y) = 1.3E(X) + E(Y) = 1.3(50) + 75 = 140$

$$\text{Var}(1.3X + Y) = 1.3^2\text{Var}(X) + \text{Var}(Y) = 1.3^2(1.5^2) + 2^2 = 7.8025$$

$$\therefore 1.3X + Y \sim N(140, 7.8025)$$

$$P(1.3X + Y > k) = 0.9$$

$$k = 136$$

[Continues on next page]

(vi) Let W be the mass of a box containing a bulb with its padding.

$$W = 1.3X + Y \sim N(140, 7.8025)$$

$$E(W_1 + \dots + W_4) = 4E(W) = 4(140) = 560$$

$$\text{Var}(W_1 + \dots + W_4) = 4\text{Var}(W) = 4(7.8025) = 31.21$$

$$W_1 + \dots + W_4 \sim N(560, 31.21)$$

$$P(W_1 + \dots + W_4 > 565) = 0.185$$