O-LEVEL A-MATHS 2018 – PAPER 1

Question 1

[Ans: 0.381]

$$\sqrt{125^x} = \frac{5^{1-x}}{25}$$

$$\left[\left(5^3\right)^x\right]^{\frac{1}{2}} = \frac{5^{1-x}}{5^2}$$

$$5^{(3)(x)\left(\frac{1}{2}\right)} = 5^{1-x-2}$$

$$5^{\frac{3}{2}x} = 5^{-x-1}$$

$$\frac{3}{2}x = -x-1$$

$$\frac{5}{2}x = -1 \Rightarrow x = -\frac{2}{5}$$

$$\therefore \sqrt{125^x} = \sqrt{125^{-\frac{2}{5}}} = 0.381$$

Question 2

[Ans: (i) show (ii) $2+\sqrt{3}$]

(i) If A, B and C are angles of a triangle, then $A+B+C=180°$

RHS

$=-\tan\left(A+B\right)$

$=-\tan\left(180°-C\right)$

$=-\left(-\tan C\right)$

$=\tan C=$ LHS (shown)

(ii) $\tan C$

$=-\tan\left(A+B\right)$

$=-\tan\left(45°+60°\right)$

$$=-\frac{\tan 45°+\tan 60°}{1-\tan 45°\tan 60°}$$

$$=-\frac{1+\sqrt{3}}{1-\left(1\right)\left(\sqrt{3}\right)}$$

$$=-\frac{1+\sqrt{3}}{1-\sqrt{3}}\left(\frac{1+\sqrt{3}}{1-\sqrt{3}}\right)$$

$$=-\frac{1+2\left(1\right)\left(\sqrt{3}\right)+\left(\sqrt{3}\right)^2}{\left(1\right)^2-\left(\sqrt{3}\right)^2}$$

$$=-\frac{1+2\sqrt{3}+3}{1-3}$$

$$=-\frac{4+2\sqrt{3}}{-2}=\frac{4+2\sqrt{3}}{2}$$

$=2+\sqrt{3}$

Question 3

[Ans: $\frac{3}{2x-1}+\frac{2x-5}{x^2+4}$]

Let $\frac{7x^2-12x+17}{(2x-1)(x^2+4)}=\frac{A}{2x-1}+\frac{Bx+C}{x^2+4}$

$7x^2-12x+17=A(x^2+4)+(Bx+C)(2x-1)$

When $x=\frac{1}{2}$,

$$7\left(\frac{1}{2}\right)^2-12\left(\frac{1}{2}\right)+17=A\left[\left(\frac{1}{2}\right)^2+4\right]+0$$

$$\frac{51}{4}=\frac{17}{4}A\Rightarrow A=3$$

When $x=0$,

$$0-0+17=3(0+4)+(0+C)(0-1)$$

$$17=12-C\Rightarrow C=-5$$

When $x=1$,

$$7-12+17=3(1+4)+(B-5)(2-1)$$

$$12=15+B-5\Rightarrow B=2$$

$$\therefore\frac{7x^2-12x+17}{(2x-1)(x^2+4)}=\frac{3}{2x-1}+\frac{2x-5}{x^2+4}$$

Question 4

[Ans: (a) $a=-6$, $b=-11$ (b) $5-\sqrt{3}$]

(a) Since $3+2\sqrt{5}$ is a root of the equation $x^2+ax+b=0$,

$$\left(3+2\sqrt{5}\right)^2+a\left(3+2\sqrt{5}\right)+b=0$$

$$3^2+2(3)\left(2\sqrt{5}\right)+\left(2\sqrt{5}\right)^2+3a+2a\sqrt{5}+b=0$$

$$9+12\sqrt{5}+20+3a+2a\sqrt{5}+b=0$$

$$(29+3a+b)+(12+2a)\sqrt{5}=0$$

Since $a,b\in\mathbb{Z}$,

$$12+2a=0\Rightarrow a=-6$$

$$29+3a+b=0\Rightarrow 29+3(-6)+b=0\Rightarrow b=-11$$

(b) Area of rectangle $=(\text{Length})\times(\text{Breadth})$

$$\text{Breadth}=\frac{\text{Area of rectangle}}{\text{Length}}$$

$$=\frac{24+\sqrt{48}}{6+\sqrt{12}}=\frac{24+\sqrt{16\times3}}{6+\sqrt{4\times3}}=\frac{24+4\sqrt{3}}{6+2\sqrt{3}}=\frac{12+2\sqrt{3}}{3+\sqrt{3}}$$

$$=\frac{12+2\sqrt{3}}{3+\sqrt{3}}\left(\frac{3-\sqrt{3}}{3-\sqrt{3}}\right)$$

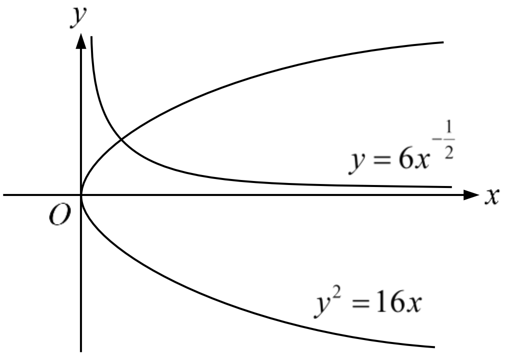
$$=\frac{36-12\sqrt{3}+6\sqrt{3}-6}{(3)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{30-6\sqrt{3}}{6}=5-\sqrt{3}$$

Queston 5

[Ans: (i) sketch (ii) $\left(\frac{3}{2}, 2\sqrt{6}\right)$]

(i)



(ii) $y^2 = 16x$ (1) $\quad y = 6x^{-\frac{1}{2}} \Rightarrow y^2 = \left(6x^{-\frac{1}{2}}\right)^2 = 36x^{-1} \Rightarrow y^2 = \frac{36}{x}$ (2)

$(1)-(2)$

$$16x - \frac{36}{x} = 0$$
$$16x^2 - 36 = 0$$
$$x^2 = \frac{9}{4}$$
$$x = \frac{3}{2} \ (\because x > 0)$$

Sub. $x = \frac{3}{2}$ into (1)

$$y^2 = 16\left(\frac{3}{2}\right) = 24$$
$$y = \sqrt{24} \ \left(\because y > 0\right)$$
$$= 2\sqrt{6}$$

$\therefore$ coordinates of intersection is $\left(\frac{3}{2}, 2\sqrt{6}\right)$.

Question 6

[Ans: (i) show (ii) $x = 3^{\frac{8}{3}}$]

(i) LHS

$$= \log_3 x + \log_9 x$$

$$= \log_3 x + \frac{\log_3 x}{\log_3 9} = \log_3 x + \frac{\log_3 x}{\log_3 3^2} = \log_3 x + \frac{\log_3 x}{2\log_3 3}$$

$$= \log_3 x + \frac{1}{2}\log_3 x$$

$$= \left(1 + \frac{1}{2}\right)\log_3 x = \frac{3}{2}\log_3 x$$

$$= \frac{3\lg x}{2\lg 3}$$

(ii) $\log_3 x + \log_9 x = 4$

$$\frac{3\lg x}{2\lg 3} = 4$$

$$\lg x = \frac{8}{3}\lg 3$$

$$\lg x = \lg 3^{\frac{8}{3}}$$

$$x = 3^{\frac{8}{3}}$$

Question 7

[Ans: (i) $\frac{81}{2}$ s (ii) $\frac{2}{15}$ cm/s]

(i) When $x = 9$,

$$V = \frac{1}{3}\pi(9)^2(36-9) = 729\pi$$

Time taken

$$= \frac{729\pi}{18\pi} = \frac{81}{2}$$

(ii) $V = \frac{1}{3}\pi x^2(36-x) = \frac{1}{3}\pi(36x^2 - x^3)$

$$\frac{dV}{dx} = \frac{1}{3}\pi(72x - 3x^2) = \pi x(24-x)$$

When $x = 9$,

$$\frac{dV}{dt} = 18\pi \text{ (given)}$$

$$\frac{dV}{dx} = \pi(9)(24-9) = 135\pi$$

$$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$$

$$18\pi = 135\pi\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{18}{135} = \frac{2}{15}$$

Question 8

[Ans: (i) show: $5-3\cos 2x$ (ii) π ; 3 (iii) sketch]

(i) $8\sin^2 x + 2\cos^2 x$

$$= 8\left(\frac{1-\cos 2x}{2}\right) + 2\left(\frac{1+\cos 2x}{2}\right)$$

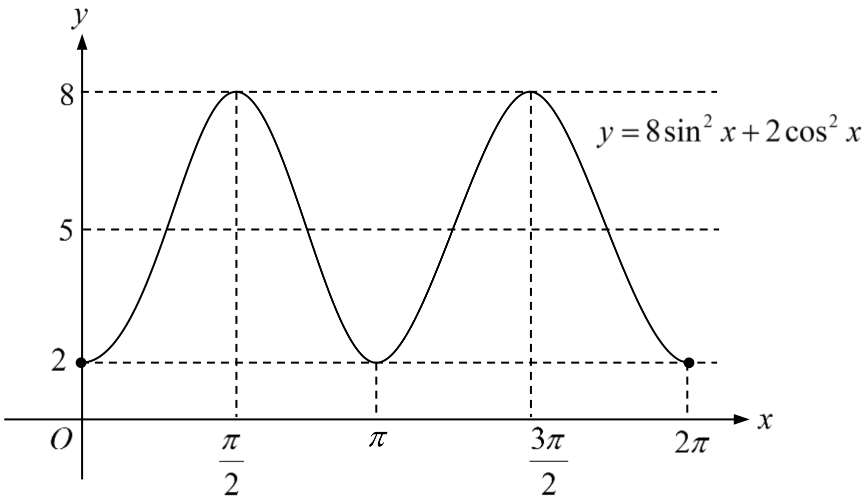
$$= 4 - 4\cos 2x + 1 + \cos 2x$$

$$= 5 - 3\cos 2x$$

(ii) Period $= \frac{2\pi}{2} = \pi$

Amplitude $= 3$

(iii)



Question 9

[Ans: (i) $y=\frac{1}{2}x$ (ii) $B(8,4)$]

(i) Gradient of AC

$=\frac{10-0}{0-5}=-2$

Gradient of OB

$=\frac{1}{2}$

Equation of OB: $y=\frac{1}{2}x$

(ii) Equation of AC:

$y=-2x+10$

$y=\frac{1}{2}x$ (1) $y=-2x+10$ (2)

(1) = (2)

$\frac{1}{2}x=-2x+10$

$\frac{5}{2}x=10\Rightarrow x=4$

Sub. $x=4$ into (1)

$y=\frac{1}{2}(4)=2$

$\therefore$ coordinates of midpoint of OB is $(4,2)$

Let coordinates of B be (p,q).

$\frac{0+p}{2}=4\Rightarrow p=8$

$\frac{0+q}{2}=2\Rightarrow q=4$

$\therefore B(8,4)$

Question 10

[Ans: (i) $r=\dfrac{20-3x}{2\pi}$ (ii) show (iii) 4.15 m (iv) minimum value; explain]

(i) Perimeter of the triangle bed $=3x$
Circumference of the circle bed $=2\pi r$

$$3x+2\pi r=20$$
$$r=\frac{20-3x}{2\pi}$$

(ii) Area of triangle bed $=\dfrac{1}{2}(x)(x)\sin\left(\dfrac{\pi}{3}\right)=\dfrac{\sqrt{3}x^2}{4}$

Area of circle bed $=\pi r^2=\pi\left(\dfrac{20-3x}{2\pi}\right)^2=\dfrac{(20-3x)^2}{4\pi}$

$$A=\frac{\sqrt{3}x^2}{4}+\frac{(20-3x)^2}{4\pi}$$
$$=\frac{\sqrt{3}\pi x^2+(20-3x)^2}{4\pi}\text{ (shown)}$$

(iii) $\dfrac{dA}{dx}$

$$=\frac{\sqrt{3}\pi(2x)+2(20-3x)(-3)}{4\pi}=\frac{2\sqrt{3}\pi x-120+18x}{4\pi}$$
$$=\frac{\left(2\sqrt{3}\pi+18\right)x-120}{4\pi}$$

Let $\dfrac{dA}{dx}=0$

$$\frac{\left(2\sqrt{3}\pi+18\right)x-120}{4\pi}=0$$
$$\left(2\sqrt{3}\pi+18\right)x-120=0$$
$$x=\frac{120}{2\sqrt{3}\pi+18}=4.15$$

(iv) $\dfrac{d^2A}{dx^2}=\dfrac{2\sqrt{3}\pi+18}{4\pi}>0$

$\therefore$ the stationary point is a minimum point.

The gardener might be disappointed because with this total area he would be able to grow the least number of flowers.

Question 11

[Ans: (i) $5+\dfrac{6}{2x-3}$ (ii) increasing (ii) decreasing (iv) $f(x)=5x+3\ln|2x-3|-2$]

(i) Given $f'(x)=\dfrac{10x-9}{2x-3}$

$$\begin{array}{r} 5 \\ 2x-3\overline{)10x-9} \\ \underline{-(10x-15)} \\ 6 \end{array}$$

$\therefore f'(x)=5+\dfrac{6}{2x-3}$

(ii) Given $x>\dfrac{3}{2}\Rightarrow 2x-3>0$

$\therefore f'(x)=5+\dfrac{6}{2x-3}>0$

$\Rightarrow f(x)$ is increasing.

(iii) $f'(x)=5+\dfrac{6}{2x-3}=5+6(2x-3)^{-1}$

$f''(x)=-6(2x-3)^{-2}(2)=-\dfrac{12}{(2x-3)^2}$

Since $(2x-3)^2>0$ for all real $x>\dfrac{3}{2}$, $f''(x)=-\dfrac{12}{(2x-3)^2}<0$

$\Rightarrow f'(x)$ is decreasing.

(iv) $f(x)$

$=\int 5+\dfrac{6}{2x-3}dx$

$=5x+6\left(\dfrac{\ln|2x-3|}{2}\right)+c$

$=5x+3\ln|2x-3|+c$

$f(2)=8$

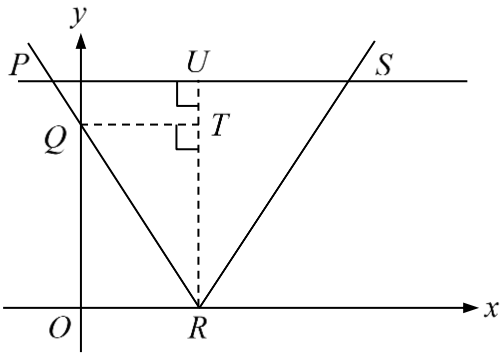
$5(2)+3\ln|2(2)-3|+c=8\Rightarrow 10+c=8\Rightarrow c=-2$

$\therefore f(x)=5x+3\ln|2x-3|-2$

Question 12

[Ans: (i) 18 (ii) $a=\frac{3}{2}$ and $b=12$]

(i) Given $P(-2,15)$



$\frac{QT}{PU}=\frac{QR}{PR}$ (similar triangles)

$\frac{QT}{QT+2}=\frac{4}{5}$

$5QT=4QT+8$

$QT=8=OR$

$\therefore R(8,0)$ and $U(8,15)$

Let coordinates of S be $(h,15)$.

Since U is the midpoint of P and S,

$\frac{-2+h}{2}=8 \Rightarrow h=18$

$\therefore x$-coordinate of S is 18.

(ii) $\therefore S(18,15)$

Gradient of $RS=\frac{15-0}{18-8}=\frac{3}{2}$

Equation of RS : $y-0=\frac{3}{2}(x-8) \Rightarrow y=\frac{3}{2}x-12=ax-b$

$\therefore a=\frac{3}{2}$ and $b=12$