St Andrew's Junior College

2017 JC1 Common Test – Examiner's Report

General Comments (Overall):

Most students could have done better given that for problems examined in this paper tested concepts and skills which were taught during lectures and tutorials. For most questions except for Question 1, gap in conceptual understanding and lack of mastery of skills and techniques are ubiquitous throughout the paper.

For certain questions which require the use of O-level Additional Math assumed knowledge such as differentiation techniques, it was obvious that there is still a lack of mastery. The lack of knowledge in the Additional Mathematics content was evident in the various questions throughout the paper (such as Q2, 4, 8, 9 and 10). These skills included algebraic manipulations and those related to specific content taught at O-level.

Most students did not do well for the Application Problem (Question 10) in this paper although there are students who made a fairly good attempt. It was observed that students are still inadequate in solving non-routine problems, observing patterns and deducing results.

Where time management is concerned, it was apparent that a number of students struggled to finish the paper within the stipulated time. That resulted in them losing a substantial number of marks due to the relatively heavier weightage carried by the questions at the end of the paper.

Q	Solution	Marker's Comments
1	Let the price per kg of D13, Golden Phoenix, Musang King be x, y and z dollars respectively. $4.2x+3.1y+1.5z=98.6$ --(1) $3.7x+4.8z=118.2$ --(2) $3.3x+3.7y+3z=131.6$ --(3) Using GC, solving (1), (2) and (3), $x=6, y=14, z=20$ Total amount Wayne Paid $=2.9(6)+4.2(14)+1.2(20)$ $=\$100.20$	Majority of the cohort scored full credit for this question. For those who did not, it was mainly due to the lack of definition or incorrect definition of variables. Students are to take note of the importance of accurate definition of variables used in the equations. There is a small percentage of students who had calculation error despite obtaining the correct values of all 3 variables. The GC is a tool when solving system of Linear

		Equations. After forming the equations correctly, students need to ensure that the values are accurately entered into the GC in order to obtain the required values/answer.
2	$S_n = 243 - 3^{5-n}$ $u_n = S_n - S_{n-1}$ $= \left(243 - 3^{5-n}\right) - \left(243 - 3^{5-(n-1)}\right)$ $= 3^{6-n} - 3^{5-n}$ $= 3^{5-n}(3-1)$ $\therefore u_n = 2\left(3^{5-n}\right)$ $\frac{u_n}{u_{n-1}} = \frac{2\left(3^{5-n}\right)}{2\left(3^{5-(n-1)}\right)} = \frac{3^{5-n}}{3^{6-n}} = \frac{1}{3}$ which is a constant independent of n Since $\frac{u_n}{u_{n-1}}$ is a constant, the series is a geometric series with common ratio $r = \frac{1}{3}$. Since $\lvert r \rvert < 1$, the geometric series is convergent. Method 1 : As $n \to \infty$, $3^{5-n} \to 0$ and $S_n \to 243$ $\therefore$ Sum to infinity = 243 Method 2 : $a = S_1 = 243 - 3^4 = 162$ and $r = \frac{1}{3}$	Some made algebraic and indices manipulation mistakes. Students are to be reminded that : (a) $S_{n-1} = 243 - 3^{5-(n-1)}$ ← Must put brackets! (b) $3^{6-n} - 3^{5-n} = 3^6 3^{-n} - 3^5 3^{-n}$ $= 3^{-n}(3^6 - 3^5) \neq 3^{-n}3$ (c) End answers must always be given in the most simplified form even if the question did not ask for it. To show that the series is a geometric series, it is insufficient to just show that $\frac{u_n}{u_{n-1}} = ... = \frac{1}{3}$. Students are expected to give a conclusion and explain how they arrive at the conclusion. To show that the series is convergent, students are expected to state the value of r first. Students who did not state the value of r and just state that $\lvert r \rvert < 1$ are not awarded the mark.

	$\therefore$ Sum to infinity $= \dfrac{162}{1-\frac{1}{3}} = 243$	
3 (a)	Given the graph of $y = \mathrm{f}(mx)$ and $y = \mathrm{f}(x)$ are identical for all real values of x, one possible equation of f is $\mathrm{f}(x) = k$, where k is a constant, $k \in \mathbb{R}$.	Majority did not realise that only a constant function is not affected by horizontal scaling. Only a few students were able to give a reasonable answer . Students also need to take note that the question specifically asked for an equation of f(x) instead of y .
3 (b)	$\mathrm{g}(x) = -3x + 3 + \dfrac{1}{x-1}$ ↓ ***D'*** **: Translation of 5 units in the negative direction parallel to the y-axis** $= (-3x + 3 + \dfrac{1}{x-1}) - 5$ $= -3x + \dfrac{1}{x-1} - 2$ ↓ ***C'*** **: Reflection in the** ***x*****-axis** $= -\,(-3x + \dfrac{1}{x-1} - 2)$	Many students made a good attempt for this question, except those who made careless mistakes with algebraic manipulations and not clear about the effect of the individual transformations. Students lost marks if they did not specify the transformation done clearly at each step. Some students had mistaken the reflection in the x-axis as the reciprocal of the function **or** mixed up with reflection in the y-axis Students are reminded about the importance of describing transformations precisely. Where descriptions of translations are concerned, students are advised to write as 'Translation of the graph by a units in the positive/negative direction parallel to the x/y-axis".

$$= 3x - \frac{1}{x-1} + 2$$

↓ ***B'*** **: Scaling parallel to the *x*-axis by a factor of 3**

$$= 3\left(\frac{x}{3}\right) - \frac{1}{\left(\frac{x}{3}\right) - 1} + 2$$

$$= x - \frac{3}{x-3} + 2$$

↓ ***A'*** **: Translation of 2 units in the positive direction parallel to the *x*-axis**

$$= (x-2) - \frac{3}{(x-2)-3} + 2$$

$$= x - \frac{3}{x-5} .$$

Ans: $y = x - \frac{3}{x-5}$

Alternative Method (1):
Combining the 4 transformations,

$$f(x) = -[g(\frac{x-2}{3}) - 5]$$

Since $g(x) = -3x + 3 + \frac{1}{x-1}$

$$g(\frac{x-2}{3}) = -3(\frac{x-2}{3}) + 3 + \frac{1}{(\frac{x-2}{3} - 1)} = -x + \frac{3}{x-5} + 5$$

$$\therefore f(x) = -[-x + \frac{3}{x-3} + 5 - 5] = x - \frac{3}{x-5} \text{. (Ans)}$$

Not many students attempted this approach. Those who used this approach did a fairly good attempt but some could not obtain the answer due to weak algebraic skills.

Alternative Method (2):

$$g(x) = -f(3x+2) + 5$$

$$\therefore -f(3x+2) + 5 = -3x + 3 + \frac{1}{x-1}$$

$$f(3x+2) = 3x + 2 - \frac{1}{x-1}$$

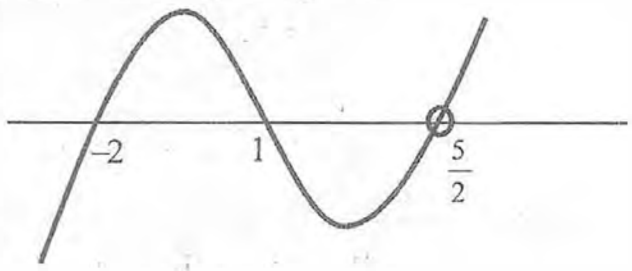
$$= 3x + 2 - \frac{1}{\frac{1}{3}(3x+2) - \frac{2}{3} - 1}$$

$$= (3x+2) - \frac{3}{(3x+2) - 5}$$

Replacing (3*x*+2) by *x*,

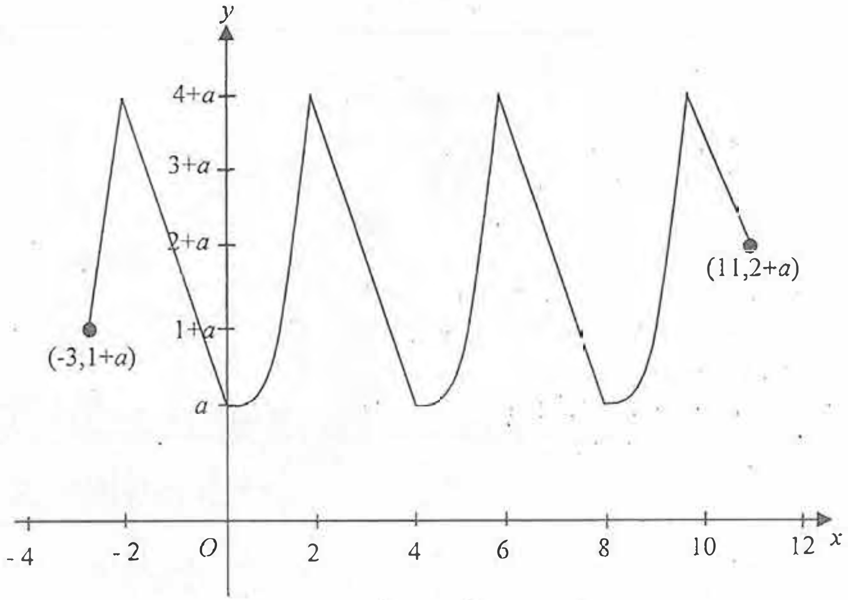
$$f(x) = x - \frac{3}{x-5} \text{ (Ans)}$$

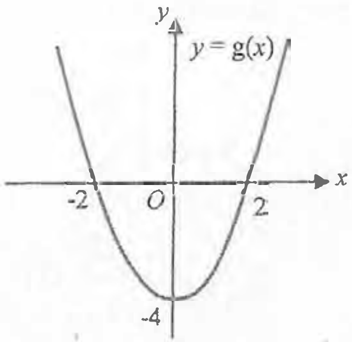
Students who attempted using this approach mostly did not get the answer due to poor algebraic skills.

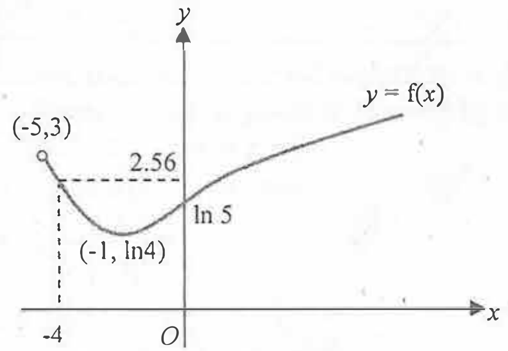
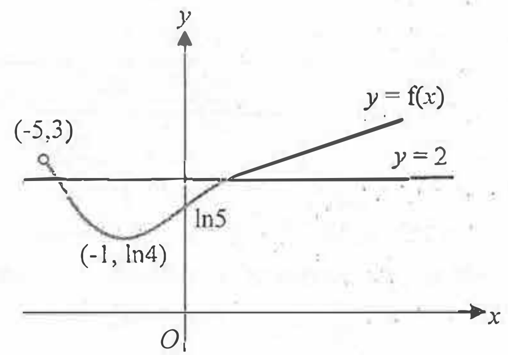
4 (i)	$\frac{x^2+7x-17}{2x-5} \geq 3,\ x \neq \frac{5}{2}$ $\frac{x^2+7x-17}{2x-5} - 3 \geq 0$ $\frac{x^2+7x-17-3(2x-5)}{2x-5} \geq 0$ $\frac{x^2+x-2}{2x-5} \geq 0$ $\frac{(x+2)(x-1)}{2x-5} \geq 0$ Multiplying by $(2x-5)^2$ $(x+2)(x-1)(2x-5) \geq 0$  Hence, $-2 \leq x \leq 1$ or $x \geq \frac{5}{2}$ Since $x \neq \frac{5}{2}$, $-2 \leq x \leq 1$ or $x > \frac{5}{2}$ (Ans)	Q4(i) is well attempted by majority of the students. However there were some students who made careless mistakes or used tedious method to solve the inequality, which is not advisable. Students are reminded not to multiply $(2x-5)$ throughout as this expression could be negative. On the other hand, those who started off by multiplying throughout by $(2x-5)^2$ ended up with much more tedious workings. It was observed that some students did not use a graphical approach (refer to worked solutions) to solve inequalities. It is recommended that students use this method (instead of using sign test method) to be consistent to what was taught. Most students were not able to express the end answer correctly due to the following reason(s): (i) Without checking that $x \neq \frac{5}{2}$ and /or (ii) Did not use the correct term 'or' to combine the two disjoint sets. Instead, 'and' or comma was incorrectly used at the end answer.

4(ii)	$\dfrac{e^{-2x}+7e^{-x}-17}{2e^{-x}-5} \geq 3$ Replacing x in (i) with e^{-x} , $-2 \leq e^{-x} \leq 1$ or $e^{-x} > \dfrac{5}{2}$ $0 < e^{-x} \leq 1$ or $e^{-x} > \dfrac{5}{2}$ $e^{-x} > 0$ and $e^{-x} \leq 1$ $-x > \ln\dfrac{5}{2}$ $x \in \mathbb{R}$ and $-x \leq \ln 1$ $x \in \mathbb{R}$ and $x \geq -\ln 1 = 0$ or $x < -\ln\dfrac{5}{2}$ Combining the solutions, $x < \ln\dfrac{2}{5}$ $x \geq 0$ Hence $x < \ln\dfrac{2}{5}$ or $x \geq 0$	4(ii) is poorly done. Students are advised to be Mathematically precise and accurate when writing their solutions. Instead of writing 'x' replaced by e^{-x} , quite a number of students wrote as let $x = e^{-x}$, which is wrong! While solving the inequality $e^{-x} \geq -2$, most students would just dismiss the inequatlity as undefined or no solution. However the correct answer should be since $e^{-x} > 0\ \forall x \in \mathbb{R}$, the solution set is $x \in \mathbb{R}$. As a result, while solving $-2 \leq e^{-x} \leq 1$, students are required to take the intersection of $x \in \mathbb{R}$ and $x \geq 0$, thus obtaining $x \geq 0$ as the resulting range of x for this double inequality. For the other inequality, there was quite a number of students who conveniently discarded the negative sign for $x < -\ln\dfrac{5}{2}$, thinking that once the inequality sign is changed, the negative sign would disappear as well!

5(i)	$f(21) = f(4\times5+1)$ $= f(21-4\times5)$ $= f(1)$ since $f(x) = f(x+4)$ for all x $= 1+a$ $f(39) = f(4\times9+3)$ $= f(39-4\times9)$ $= f(3)$ since $f(x) = f(x+4)$ for all x $= 2+a$ $\therefore f(21)+f(39) = 3+2a$	Most students did poorly for question 5 as a whole. A large number of students were not aware that this question involves a periodic function, represented by $f(x)=f(x+4)$ in the question. Marks were deducted if the working did not demonstrate the periodic nature of the function with a period of 4 units [e.g. : $f(21)=f(4\times5+1)=f(1)$]. There were a small number of cases where the student used $f(21) = f(2\times10+1)$. This is incorrect as it represents that the function is a periodic function with a period of 2 units instead of 4. Care needs to be taken with such expressions. A small number of students calculated f(21) and f(39) but not its sum which was the requirement of the question.

5(ii)		A moderate number of students drew two separate graphs, or drew piecewise but not periodic functions, or drew periodic but not piecewise functions, or did not provide a graph at all. Students need to understand that the definition of f for this question was that of both a piecewise and periodic function. Many students did not label the end points, or the maximum y-coordinate of the graph. End points of each component of the function are significant and should be labelled. [*Recall*: A function is defined by both its rule and domain.] One common error was that students assumed that the y-coordinate at $x=-3$ and $x=11$ are the same. The value of the function at each end point should be evaluated separately. Another common error made was that students did not draw the quadratic component of the curve which corresponds to its minimum point. Students also failed to note that the function is a continuous function at the end points of each segment which has the same value at each end point. Hence, many indicated the 'connecting' points with a hollow circle which was incorrect. One serious error seen that was not uncommon was that students reflected the curve for negative values of x.

6(i)	 Since $R_g = [-4, \infty) \subseteq D_f = (-5, \infty)$, fg exists. $fg(x) = f(x^2 - 4)$ $= \ln\left[(x^2 - 4 + 1)^2 + 4\right]$ $= \ln\left[(x^2 - 3)^2 + 4\right]$ $D_{fg} = D_g = (-\infty, \infty) = \mathbb{R}$	Majority of students were able to show the existence of fg. However, many students gave $R_g = (-4, \infty)$ which was a common mistake. Many students missed out finding the rule for fg(x) and only attempted to find the domain and range of fg. Fulfilling the requirement of the question is important. Many students are still weak in finding the range of fg using the mapping method. A common mistake seen from students who attempted to use this method was to find the range of fg by substituting the end point from R_g, that is, $x = -4$ into f(x) to obtain $R_{fg} = [\ln 13, \infty)$ without checking the graph of f. Students need to understand that substituting the end points of the domain does not necessarily give the minimum or maximum value of the range of

	 $D_g = (-\infty, \infty) \xrightarrow{g} R_g = [-4, \infty) \xrightarrow{f} [\ln 4, \infty) = R_{fg}$ $\therefore R_{fg} = [\ln 4, \infty)$	any function. Hence, a graph with its end points properly labelled (if any) is NECESSARY in finding the range of functions. There are also many students who gave the answer for R_{fg} without showing any workings or graphs. Justifications are important in Mathematics. Many students also did not give the **exact** range as required by the question and hence were penalised by their own failure to read the question and give what was required!
6(ii)	 Since the horizontal line $y = 2$ cuts the graph of $y = f(x)$ at more than one point, f is not a one-one function, thus the inverse does not exist.	Many students did not give the specific equation of the horizontal line but instead gave equations $y = k$ without specifying the value of k or gave the wrong range of k, for example, $k \geq \ln 4$, $k \in \mathbb{R}$, etc. With reference to the graph of f in the given domain $(-5, \infty)$, the correct value of k must be any constant in the range (ln 4, ln 20) for the horizontal line to cut the graph of f at more than one point.
6 (iii)	Least value of $k = -1$	Most students obtained the correct answer for this part.
6 (iv)	For f^{-1} to exist, $D_f = (-1, \infty)$, that is, $x > -1$.	This part was poorly attempted. Majority of students who attempted this part tried

$f^{-1}(x) \le 0$

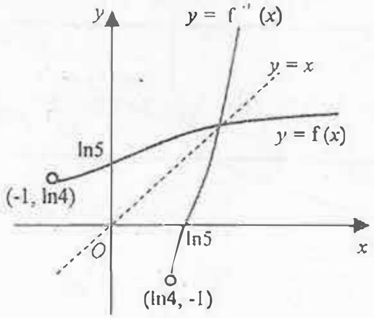
$\Rightarrow x \le f(0)$ since f is an increasing function for $x > -1$

$\therefore x \le \ln 5$

Since $D_{f^{-1}} = R_f = (\ln 4, \infty), x > \ln 4$

Hence, $\ln 4 < x \le \ln 5$

Alternative Method:



From the graph, $f^{-1}(x) \le 0$ for $\ln 4 < x \le \ln 5$.

to find the rule for $f^{-1}(x)$ and solve for the range of x using the given inequality $f^{-1}(x) \le 0$. Those who obtained the correct rule for $f^{-1}(x)$ only obtained $x \le \ln 5$.

Some students substituted $x = -1$ and $x = 0$ into f(x) without indicating that f(x) is an increasing function for $x > -1$.

Very few students attempted to use the graphical method to solve this part. Some who did gave very poor/wrong sketches of the graphs of f(x) and $f^{-1}(x)$.

Again, many students did not give the **exact** range for x.

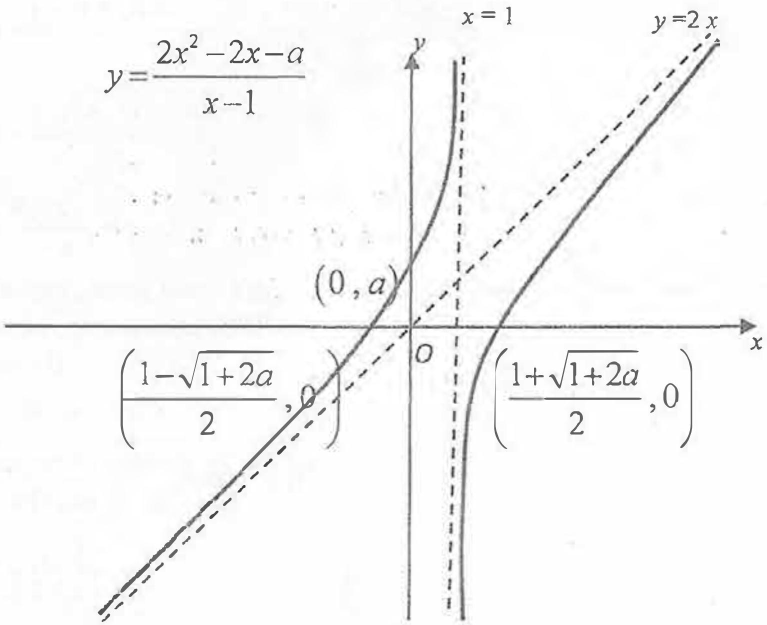
7(i)	$\text{h}(x) = a + \dfrac{b-x}{(x-c)^2}$ Observe that the horizontal asymptote is given by $y = -1$ and vertical asymptote is given by $x = 2$. Therefore $a = -1$ and $c = 2$. Substitute $A(0,0)$, we have $0 = -1 + \dfrac{b}{(-2)^2}$ $\Rightarrow b = 4$. Ans: $a = -1$, $b = 4$ and $c = 2$	Part (i) was well attempted. However, quite a number of students used the coordinates of the points $A(0, 0)$, $B(3, 0)$ and $C\left(6, -\dfrac{9}{8}\right)$ to form 3 equations which made the solving process a tedious one. It was especially so for those who used differentiation. Mistakes were often found in the use of quotient rule. Given the characteristics of the graph, students should be able to relate it to the corresponding components in the equation of h(x). (For example, relating the value of c to the vertical asymptote.)
7(ii)	$\text{h}(x) = -1 + \dfrac{4-x}{(x-2)^2}$ $\text{k}(x) = -1 + \dfrac{x+3}{(x+1)^2}$ Comparing the two expressions, the two transformations in sequence are Step 1: Reflection of graph of h(x) in the y-axis, followed by $\text{h}(-x) = -1 + \dfrac{4-(-x)}{(-x-2)^2}$ $= -1 + \dfrac{x+4}{(x+2)^2}$ Step 2: Translation of graph by 1 unit in the positive x-axis. $\text{h}(-(x-1)) = -1 + \dfrac{(x-1)+4}{(x-1+2)^2}$	Students need to read the question requirement carefully. It was stated clearly that the total number of transformations was two. It should not be any more or less. Many students did not attempt this part or simply wrote only one transformation 'translation of 3 units in the positive x-direction' which was a common mistake. Some inappropriate terms like 'transform', 'shift', 'flip' were seen on the answer scripts. Students should learn the proper description for transformation. Common mistakes were 'reflect parallel to y-axis' , 'translate towards x-axis by

	$= -1 + \frac{x+3}{(x+1)^2} = \mathrm{k}(x)$. Alternative Method: Translation of the graph by 1 unit in the negative x-direction followed by the reflection in the y-axis. $\mathrm{h}(x) = -1 + \frac{4-x}{(x-2)^2} \longrightarrow \mathrm{h}(x+1) = -1 + \frac{4-(x+1)}{(x+1-2)^2} = -1 + \frac{3-x}{(x-1)^2}$ $\longrightarrow \mathrm{h}(-x+1) = -1 + \frac{3-(-x)}{(-x-1)^2} = -1 + \frac{3+x}{(x+1)^2} = \mathrm{k}(x)$	scale factor 1' . Where descriptions of translations are concerned, students are advised to write as 'Translation of the graph by a units in the positive/negative direction parallel to the x/y-axis". The sequence of transformation should be indicated clearly by writing 'Step 1….Step 2' or use the phrase '…followed by….'. In addition, the two transformations should be supported by justifying how the equation of $y = \mathrm{h}(x)$ being transformed into the equation $y = \mathrm{k}(x)$. Many students failed to do so.
8(i)	$y = \frac{2x^2 - 2x - a}{x-1}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(4x-2)(x-1) - (2x^2 - 2x - a)}{(x-1)^2}$ $= \frac{4x^2 - 4x - 2x + 2 - 2x^2 + 2x + a}{(x-1)^2}$ $= \frac{2x^2 - 4x + 2 + a}{(x-1)^2}$ To find stationary points, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow 2x^2 - 4x + 2 + a = 0$	The question asked for set of values of a for which there would be no stationary points. There was a significant number of students who equated the equation of the curve to zero. However, whether there is a solution for $\frac{2x^2 - 2x - a}{x-1} = 0$ only signifies whether or not there is an intersection point of the curve with the x – axis. Students need to understand that this question needs to begin with differentiating the equation of the graph since that is how stationary points are obtained. Moreover, from O-level Additional Mathematics knowledge, it is known that we need to be able to solve or find real roots for the equation $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ so

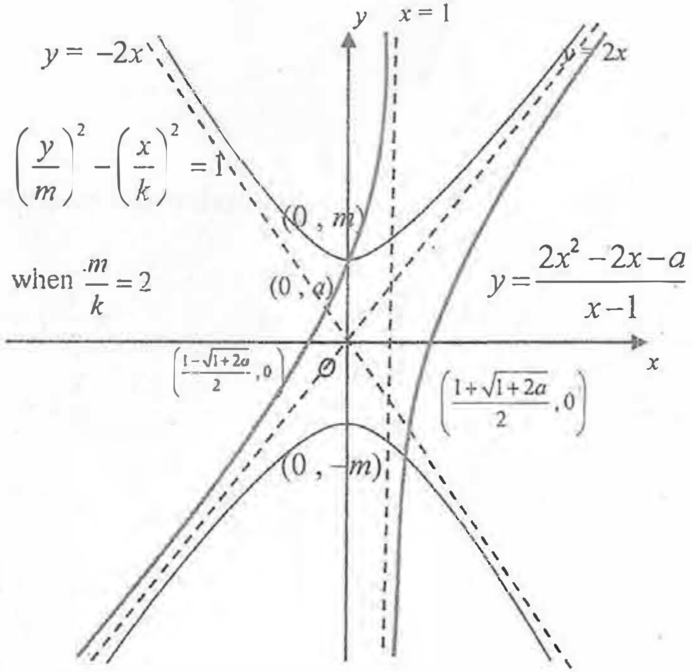
	Since C has no stationary points, there must be no real roots for $2x^2 - 4x + 2 + a = 0$ $\therefore D < 0$ $(-4)^2 - 4(2)(2+a) < 0$ $16 - 16 - 8a < 0$ $a > 0$ set of values for C to have no stationary point: $\{a \in \mathbb{R} : a > 0\}$	that there are stationary points for the graph. Since the question has stated clearly that there are no stationary points for the curve C, it means that there are no solutions for $\frac{dy}{dx} = 0$. It so happens that when the gradient function is equated to zero, we obtain the equation $2x^2 - 4x + 2 + a = 0$. Note that this is a quadratic equation. Since we say that there is no solution for $2x^2 - 4x + 2 + a = 0$ (which implies that there is no real roots for the equation), we make use of the discriminant to be less than 0 to solve for the set of values of a. A common error at this step was that many only stated that there was no stationary points which did not make clear connections to the fact that it means that there is no real roots/solutions for the quadratic equation. Students need to understand that for discriminant (i.e. $b^2 - 4ac$) is only used when determining the nature of roots for a quadratic equation in the form $ax^2 + bx + c = 0$. It makes absolutely no meaning when the discriminant is applied on a quadratic expression or inequality. Finally, students failed to read the requirement of the question. A handful of the candidates did not write the final answer as a set of values. Instead,

		the answer was left only as an inequality.
	Alternative solution: To find stationary points, $\frac{dy}{dx}=0 \Rightarrow 2x^2-4x+2+a=0$ $x=\frac{-(-4)\pm\sqrt{16-4(2)(2+a)}}{2(2)}$ $=\frac{4\pm\sqrt{16-16-8a}}{4}$ $=\frac{4\pm\sqrt{-8a}}{4}$ However, since there are no stationary points, x is not a real number. Hence, there are no real roots for x. Hence, $-8a<0$. $\Rightarrow a>0$ $\{a\in\mathbb{R}:a>0\}$	A few students gave this as an alternative. However, there was a lack of clear connection between the solution of x with the need for it to be a real solution. This was where credit was lost.
8 (ii)	C has no stationary point, i.e. $a>0$ Performing long division, $x-1\overline{)2x^2-2x-a}$ quotient $2x$ $2x^2-2x$ $-a$ $y=2x-\frac{a}{x-1}$ Vertical Asymptote: $x=1$ Oblique Asymptote: $y=2x$	For this part of the question, there was a handful of students who ignored the requirement that the graph has no stationary points. The graph provided by students was that where two stationary points were present for the curve. Hence, this resulted in the loss of marks for the characteristics of the graph. A common MAJOR mistake was that students substituted a value for a on their own and continued the question with this FIXED value. This was unacceptable as the objective of this question was to sketch the curve in general with the unknown a, where any values of a satisfying the range of values found in (i), would result in the same curve drawn.

	x – intercept: when $y = 0$ $0 = 2x - \frac{a}{x-1}$ $2x(x-1) = a$ $2x^2 - 2x = a$ $2x^2 - 2x - a = 0$ $x = \frac{2 \pm \sqrt{4 - 4(2)(-a)}}{4}$ $= \frac{2 \pm \sqrt{4+8a}}{4}$ $= \frac{1 \pm \sqrt{1+2a}}{2}$, since $a > 0$ y – intercept: when $x = 0$ $y = 0 - \frac{a}{0-1}$ $y = a$	In this question, it was apparent that most candidates need to work on their algebraic manipulations as many did not know how to perform arithmetic operations on rational expressions as well as operations under square roots. **Common ERROR:** 1. $\frac{2 \pm \sqrt{4+8a}}{4} = \frac{1 \pm \sqrt{4+8a}}{2}$ It **should be**: $\frac{2 \pm \sqrt{4+8a}}{4} = \frac{1}{2} \pm \frac{\sqrt{4+8a}}{4}$ $= \frac{1}{2} \pm \frac{2\sqrt{1+2a}}{4}$ $= \frac{1}{2} \pm \frac{\sqrt{1+2a}}{2}$ Many students failed to calculate the x – intercept which only required students to find the values of x as a root of a quadratic equation. Even though a is an unknown which carried a range of values, it was to be treated as an **unknown** constant which fitted the range found.

	Hence,  	Note the following for sketches of graphs: 1. All elements in the graphs should be sketched in pencil. 2. Axes should be perpendicular to each other. 3. The origin should be labelled. 4. All elements in the graph should be as long as the axes and as wide as the axes. 5. In marking out points, a cross should be used. Make it a habit to mark out essential points (such as stationary points and intersection with axes) as coordinates. 6. Asymptotes MUST be drawn in dotted lines. 7. Equations of asymptotes need to be written. 8. Graphs MUST be labelled with its equations, not the name (ie C). 9. The graph should be at least TWO-THIRDS a page. A common mistake was that the elements in a coordinate was swapped. For example, the y – intercept was written as $(a , 0)$ instead of $(0 , a)$. This is a very serious mathematical error. To draw a proper graph, students should note the following: 1. Calculate the (S)tationary points, (I)ntercepts with axes and (A)symptotes in a systematic manner. 2. Emplace the characteristics of the graphs in the order of A, I followed by S. This will allow the general positions of important features to be fixed, relative to one another.

8 (iii)	$x = k\tan t$ $\frac{x}{k} = \tan t$ $y = m\sec t$ $\frac{y}{m} = \sec t$ Using Trigometry Identity, $\sec^2 t = 1 + \tan^2 t$ $\left(\frac{y}{m}\right)^2 - \left(\frac{x}{k}\right)^2 = 1$ $\left(\frac{y}{m}\right)^2 = 1 + \left(\frac{x}{k}\right)^2$ As $x \to \pm\infty, \left(\frac{y}{m}\right)^2 \to \left(\frac{x}{k}\right)^2$ $\frac{y}{m} \to \pm\frac{x}{k}$ $y \to \pm\frac{m}{k}x$ Hyperbola with asymptotes $y = \pm\frac{m}{k}x$	Students who did not attempt this question generally was due to the lack of knowledge of the common methods which a parametric equation could be converted into a Cartesian equation. For students who tried, many did not recall the correct trigonometry identity. The three trigonometry identities are: 1. $\sin^2\theta + \cos^2\theta = 1$ 2. $\tan^2\theta + 1 = \sec^2\theta$ 3. $1 + \cot^2\theta = \text{cosec}^2\theta$, where 2 and 3 could be derived from 1. Students were also not familiar with equations of conic sections. Hence, many were not able to provide the standard form of the hyperbola. Note: To find the Cartesian equation of a given parametric equation, it just requires you to find the equation in terms of *x* and *y*, eliminating the parameter. Students need to make it a habit to derive the asymptotes of a hyperbola. Memorising might cause more confusion.



When $\frac{m}{k}=2$,the asymptotes for the hyperbola are $y=\pm 2x$ and note that $y=2x$ is the oblique asymptote for both C and C_1.

Hence when $0<\frac{m}{k}<2$, the curves intersect more than twice.

When $\frac{m}{k}\geq 2$, the curves intersects twice exactly.

In the question, it was stated clearly that the graph of the parametric graph should be added on to the graph in (ii). However, there were students who DID NOT read the questions and drew it on a separate diagram. This resulted in the graph not credited.

Also, there were students who redrew the entire graph from (ii) and then added on the hyperbola in (iii). This was just a waste of time during the exams.

For the parametric graph, it was important to make use of/reference to the asymptotes of the original graph. Students should state the value of $\frac{m}{k}$ either in writing or on the graph for the full credit of the addition of the hyperbola to be accorded.

Also, students had to reason out why the minimum value of $\frac{m}{k}$ is 2 with some form of explanation as it was not easily seen.

	$\therefore$ Least $\dfrac{m}{k} = 2$	
9(i)	$\dfrac{2r+3}{r(r+1)} = \dfrac{A}{r} + \dfrac{B}{r+1}$ $2r+3 = A(r+1) + Br$ When $r = 0$, $A = 3$ When $r = -1$, $B = -1$ $\dfrac{2r+3}{r(r+1)} = \dfrac{3}{r} - \dfrac{1}{r+1}$ $\dfrac{2r+3}{r(r+1)}\left(\dfrac{1}{3}\right)^r = \left(\dfrac{3}{r} - \dfrac{1}{r+1}\right)\left(\dfrac{1}{3}\right)^r$ $= \dfrac{1}{r3^{r-1}} - \dfrac{1}{3^r(r+1)}$	Most students were able to obtain the correct partial fraction and show the given expression. Despite the question being a "Show" question, there were still students who provided the wrong expression in the final step despite having the correct partial fraction. The error was further compounded when students insisted on using their incorrect expression for part (ii) instead of using what was given in the question in (i). Note: When a question provides an expression/identity to be shown, it is usually to assist students in carrying on with the subsequent parts.

	$\sum_{r=2}^{N} \frac{2r+3}{r(r+1)} \left(\frac{1}{3}\right)^r$ $= \sum_{r=2}^{N} \left(\frac{1}{r3^{r-1}} - \frac{1}{3^r(r+1)} \right)$ $= \frac{1}{2(3)} - \frac{1}{3^2(3)}$ $+ \frac{1}{3(3^2)} - \frac{1}{3^3(4)}$ $+ \cdots$ $+ \frac{1}{(N-1)3^{N-2}} - \frac{1}{3^{N-1}(N)}$ $+ \frac{1}{(N)3^{N-1}} - \frac{1}{3^N(N+1)}$ $= \frac{1}{6} - \frac{1}{3^N(N+1)}$	Most students were able to see the need to use MOD for this part. It is important to show $\sum_{r=2}^{N} \frac{2r+3}{r(r+1)} \left(\frac{1}{3}\right)^r = \sum_{r=2}^{N} \left(\frac{1}{r3^{r-1}} - \frac{1}{3^r(r+1)} \right)$ as this is the step that determined the need for Method of Difference (MOD). The common mistakes made were: 1.) Using 'n' instead of 'N' when listing out the terms in the MOD. 2.) Did not list out sufficient rows of terms. 3.) Did not show the cancellation of terms. 4.) Writing $\sum_{r=2}^{N} \frac{1}{r3^{r-1}} - \frac{1}{3^r(r+1)}$ instead of $\sum_{r=2}^{N} \left(\frac{1}{r3^{r-1}} - \frac{1}{3^r(r+1)} \right)$. Students need to take note of the presentation when doing MOD. The terms should be listed out in rows and cancellation of terms needs to be seen. There are students who changed the lower limit of the series to 1 when it was not necessary.
9 (ii)	As $N \to \infty$, $\frac{1}{6} - \frac{1}{3^N(N+1)} \to \frac{1}{6} - 0 = \frac{1}{6}$. Since $\frac{1}{6}$ is a finite number, the series converges to $\frac{1}{6}$.	Most students have problem with 9(ii). It is important to note that the reason of convergence must be explained well before evaluating the sum to infinity. This is because the

	Sum to infinity is $\frac{1}{6}$	sum to infinity cannot exist without determining whether the series converges or not. Many students used the wrong notation when dealing with limits. For example, instead of writing as $N \to \infty$, many students instead used $r \to \infty$ or even $N \to \pm\infty$ which shows the lack of understanding on the concept of series. There are also many students who tried to use concept of convergence of GP in solving this question. If the series is not a GP in the first place, there shouldn't be any common ratio to begin with and hence using |r|<1 as reason for convergence is a conceptual mistake. One must not assume that the given series is an AP or a GP unless it was stated explicitly or you have been told to prove it as it was in Q2.
9 (iii)	$\sum_{r=4}^{N+2} \frac{2r+1}{r(r-1)}\left(\frac{1}{3}\right)^{r-2} = \sum_{r=4}^{N+2} \frac{2r+1}{(r-1)r}\left(\frac{1}{3}\right)^{r-2}$ $\sum_{a=2}^{N} \frac{2a+3}{a(a+1)}\left(\frac{1}{3}\right)^{a} = \frac{1}{6} - \frac{1}{3^N(N+1)}$ Replace $r-1$ as a Let $r-1=a \Rightarrow r=a+1$	Most students were able to identify the correct substitution required but unable to manipulate the limits of the summation correctly after substituting. There are students who used MOD to solve the question despite the question clearly state the need to use (i). Majority of the students missed out on stating the value of *k* and f(N) at the end, which results in loss

$$\sum_{r=4}^{N+2}\frac{2r+1}{(r-1)r}\left(\frac{1}{3}\right)^{r-2}=\sum_{a+1=4}^{a+1=N+2}\frac{2(a+1)+1}{(a+1-1)(a+1)}\left(\frac{1}{3}\right)^{a+1-2}$$

$$=\sum_{a=3}^{N+1}\frac{2a+3}{a(a+1)}\left(\frac{1}{3}\right)^{a-1}$$

$$=3\sum_{a=3}^{N+1}\frac{2a+3}{a(a+1)}\left(\frac{1}{3}\right)^{a}$$

$$=3\left[\frac{1}{6}-\frac{1}{3^{N+1}(N+1+1)}-\frac{2(2)+3}{2(3)}\left(\frac{1}{3}\right)^{2}\right]$$

$$=3\left[\frac{1}{6}-\frac{1}{3^{N+1}(N+2)}-\frac{7}{54}\right]$$

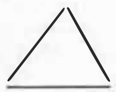
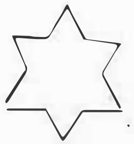
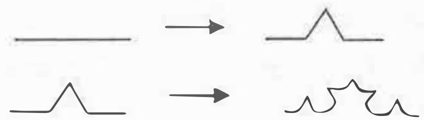
$$=3\left[\frac{1}{27}-\frac{1}{3^{N+1}(N+2)}\right]$$

$$=\frac{1}{9}-\frac{1}{3^{N}(N+2)}$$

$$k=\frac{1}{9}$$

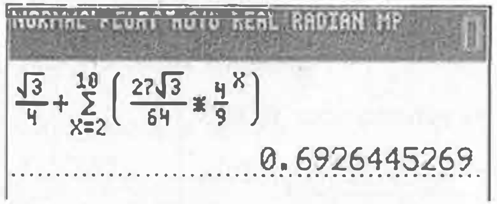
$$\mathrm{f}(N)=-\frac{1}{3^{N}(N+2)}$$

of credit.

<table>
<tr>
<td>10
(i)</td>
<td>
<table>
<tr><td>n</td><td>L_n</td><td></td><td>P_n</td></tr>
<tr><td>1</td><td>3</td><td></td><td>3</td></tr>
<tr><td>2</td><td>$3\times4=12$</td><td></td><td>$3\times4\times\frac{1}{3}=4$</td></tr>
<tr><td>3</td><td>$3\times4\times4=48$</td><td></td><td>$3\times4\times4\times\left(\frac{1}{3}\right)^2=\left(\frac{4}{3}\right)^2(3)$</td></tr>
<tr><td>⋮</td><td>⋮</td><td></td><td>⋮</td></tr>
<tr><td>n</td><td>$3\times4^{n-1}$</td><td></td><td>$\left(\frac{4}{3}\right)^{n-1}(3)$</td></tr>
</table>
</td>
<td>Most students were able to recognise that at each new stage, each length segment evolves to 4 smaller length segments and the length of each of the smaller length segments is $\frac{1}{3}$ of the previous length segment it was evolved from.
</td>
</tr>
<tr>
<td>10
(ii)</td>
<td>As $n\to\infty$, $\left(\frac{4}{3}\right)^{n-1}\to\infty$. Therefore $P_n\to\infty$
(or perimeter is infinite) .
Any reasonable explanation from (i)</td>
<td>For this part, marks are awarded based on the expression of P_n <u>from part (i).</u> Students should see that $\left(\frac{4}{3}\right)^{n-1}\to\infty$ as $n\to\infty$. If students did not attempt part (i), no marks are awarded even if the conclusion is correct.
Use of phrases such as increasing /large will not be credited.</td>
</tr>
</table>

10 (iii)	<u>Method 1 (easy method)</u> Since each triangle added is also an equilateral triangle, then using similar triangle (quote SSS/SAS etc) $\frac{\text{length of side of first triangle}}{\text{length of side of second triangle}}=\frac{1}{1/3}=\frac{3}{1}$ $\frac{\text{area of first triangle}}{\text{area of second triangle}}=\left(\frac{3}{1}\right)^2=\frac{9}{1}$ (shown) <u>Method 2</u> Area of the first triangle $=\frac{1}{2}(1)^2\sin 60=\frac{1}{2}\times\sin 60$ Added triangle has length $=\left(\frac{1}{3}\right)$ Added triangle has area $=\frac{1}{2}\left(\frac{1}{3}\right)^2\times\sin 60°=\frac{1}{9}\left(\frac{1}{2}\times\sin 60°\right)$ $=\left(\frac{1}{9}\right)$(Area of the first triangle) (shown)	Students need to quote the use of similar triangles and a relevant test if they wish to express the ratio of their lengths/ratio in proving the given ratio. Otherwise they need to express the formula for areas of triangles to show that the area of a triangle added in stage 2 is 1/9 the area of the first triangle. For this question, if students remembered that the area of a triangle is $\frac{1}{2}ab\sin\theta$, where $\theta=60°$ in this case, then there is **no need** to find the perpendicular height to obtain the area of the triangle (which is tedious since it is only worth 1 mark).

10 (iv)	<table><tr><td>n</td><td>L_n</td><td>A_n = area of **all** triangles added</td></tr><tr><td>1</td><td>3</td><td>$=\frac{1}{2}(1)^2\sin 60=\frac{\sqrt{3}}{4}$</td></tr><tr><td>2</td><td>12</td><td>Notice that 3 triangles are added and the area of each triangle is $\frac{1}{9}$ of the previous triangle. $=3\left[\left(\frac{1}{9}\right)\frac{\sqrt{3}}{4}\right]=\frac{\sqrt{3}}{12}$</td></tr><tr><td>3</td><td>48</td><td>Notice that $3\times 4=12$ triangles are added and the area of each triangle is $\frac{1}{9}$ of the previous triangle. $=12\left[\left(\frac{1}{9}\right)^2\frac{\sqrt{3}}{4}\right]$</td></tr><tr><td>4</td><td>192</td><td>Notice that $3\times 4\times 4=48$ triangles are added and the area of each triangle is $\frac{1}{9}$ of the previous triangle. $=48\left[\left(\frac{1}{9}\right)^3\frac{\sqrt{3}}{4}\right]$</td></tr><tr><td>...</td><td>...</td><td>...</td></tr><tr><td>n</td><td>$3\times 4^{n-1}$</td><td>$=(3\times 4^{n-2})\left[\left(\frac{1}{9}\right)^{n-1}\frac{\sqrt{3}}{4}\right]$</td></tr></table>	Students need to deduce that each area of triangle added is 1/9 the area of a triangle from the previous stage and in each stage, 4 sides are formed. Therefore the number of triangles added are $3\times 4^{n-2}$ Many students thought the number of triangles added are multiples of 3 and so are unable to formulate the correct expression for A_n.

	Observe that $n \geq 2$, $A_n = \left(3\times 4^{n-2}\right)\left[\left(\frac{1}{9}\right)^{n-1}\frac{\sqrt{3}}{4}\right]$ (or $A_n = \frac{27\sqrt{3}}{64}\left(\frac{4}{9}\right)^n$ or any equivalent form)	
10 (v)	Area of fractal at stage 10 $= \frac{\sqrt{3}}{4} + \sum_{n=2}^{10} A_n$ Using GC, $\frac{\sqrt{3}}{4} + \sum_{n=2}^{10} \frac{27\sqrt{3}}{64}\left(\frac{4}{9}\right)^n = 0.69264$  **Or using GP (Alternative):** $= \frac{\sqrt{3}}{4} + \sum_{n=2}^{10} A_n$ $= \frac{\sqrt{3}}{4} + \sum_{n=2}^{10} \frac{27\sqrt{3}}{64}\left(\frac{4}{9}\right)^n$ $= \frac{\sqrt{3}}{4} + \frac{27\sqrt{3}}{64}\left(\left(\frac{4}{9}\right)^2 + \ldots \left(\frac{4}{9}\right)^{10}\right)$ $= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{12}\left(\frac{1-(4/9)^9}{1-(4/9)}\right)$ $= 0.69264 \text{ m}^2$ $=0.693 \text{ m}^2$ (to 3 sig fig)	This part was very badly attempted. Students need to understand the area of the fractal consists of the **sum of all the triangles from stage 1 to 10.** Many students thought that the question only required the area of the fractal at stage 10. Once $\frac{\sqrt{3}}{4} + \sum_{n=2}^{10} \frac{27\sqrt{3}}{64}\left(\frac{4}{9}\right)^n$ is written students can use GC to obtain the answer.

Recommended Follow-up Actions

Foundational Knowledge:

H2 Math requires a mastery of both O-level Additional Mathematics (Assumed knowledge) as well as the A-level H2 Math content and skills. It is noteworthy that H2 Math problems infuse knowledge across topics, and problems are set based on the assumption that there is a perfect mastery of both O and A level Mathematics knowledge. Students are therefore strongly advised to revise and master all these fundamental knowledge in order to achieve good performance in A-level H2 Mathematics.

It is recommended that students should to seek clarifications from their tutors as early as possible whenever they have doubts as a strong understanding of A level concepts are essential to solving problems in H2 Mathematics. More practice should be devoted to Application Questions, which is part of the H2 Math 9758 examination requirements.

Accuracy & Precision in Working Steps:

Students are reminded that careful attention should be given to their working steps and graphs, which should be clear, concise and logical. More effort is required to improve accuracy of the statements and graphs as the quality of presentation and graphs are still far away from the required standard of the A Levels.

Problem Solving Skills:

It was evident that some students have not mastered fundamental problem solving skills and the ability to make connections within parts of questions. Some questions were not well read enough for the problem to be well answered. Students need to realise that questions for H2 Mathematics are not as direct and straightforward as compared to those seen in O-level Additional Mathematics.

Time Management:

The Revision Sets as well as the Timed Practice papers should be well utilized to help one practice the time management for the examination. The recommended time is set at 1.5 min per mark allocated for the question. Nonetheless, students should also extend this practice to completing any questions be it for tutorials or questions in the Revision Package.

Use of Graphing Calculator as an Examination Tool:

Graphing calculator is a learning tool as well as an examination tool, which students are required to be proficient with all the functions and apps. It was alarming to note that some students still have difficulty with reset of memory resulting in a mass wipe out of the GC Apps installed.

Students should also know how to make use of their GCs to help them solve the questions in an examination efficiently and effectively.