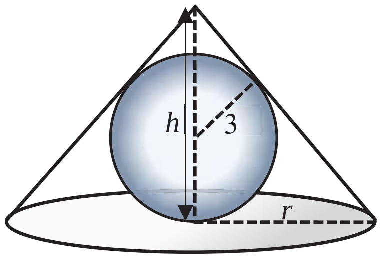
1	It is given that $y=\dfrac{x^2+x-1}{x+2}$, $x\in\mathbb{R}$, $x\neq -2$. Find algebraically the set of values that y can take. [5]
2	Without the use of the graphing calculator, solve the inequality $$\frac{2x^2-3x-1}{x^2-4x+1}<1 .$$ [4] Hence solve the inequality $\dfrac{2x+3\sqrt{x}-1}{x+4\sqrt{x}+1}<1$. [4]
3	Functions f and h are defined by $$\mathrm{f}:x\mapsto\frac{1}{(x+1)(x-3)},\quad x\in\mathbb{R},\ k\le x<3$$ $$\mathrm{h}:x\mapsto x+\frac{1}{x},\quad x\in\mathbb{R},\ x<0 .$$ **(i)** Given that f has an inverse, state the smallest possible value of k. [1] For the rest of this question, use the value of k obtained in **(i)**. **(ii)** Write down the equation of the line in which the graph of $y=\mathrm{f}(x)$ must be reflected in order to obtain the graph of $y=\mathrm{f}^{-1}(x)$. Sketch the graphs of f and f^{-1} on a single diagram, clearly indicating the equations of asymptotes and coordinates of endpoints if any. [5] **(iii)** Determine, with a reason, the number of solution(s) of the equation $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$. [1] **(iv)** Give a reason why the composite function hf exists and find its range. [3]
4	**(a)** Find $\displaystyle\int\frac{x^2+x+1}{x^2-x+1}\,\mathrm{d}x$. [4] **(b)** By using the substitution $x=2\cos u$, find the exact value of $\displaystyle\int_0^1\sqrt{4-x^2}\,\mathrm{d}x$. [5]
5	The curve C has equation $y=\ln x$, $x>0$. The tangent to the curve at $x=1$ is denoted by L. **(i)** Find the equation of L . [2] **(ii)** The region P is bounded by C, L and the line $y=-1$. Find the area of P. [3] **(iii)** The region Q is bounded by C, L and the line $x=6$. Find the exact volume

of the solid obtained when Q is rotated completely about the x-axis. [5]

6 The diagram shows a sphere of radius 3 metres inscribed in a right circular cone of radius r metres and height h metres. The sphere is in contact with the base and the inner surface of the cone.

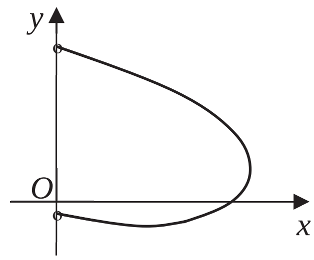


(i) Using similar triangles, or otherwise, show that $r = \dfrac{3h}{\sqrt{h^2 - 6h}}$. [2]

(ii) Find the minimum volume of the cone in terms of π, proving that it is a minimum. [Volume of a cone $V = \dfrac{1}{3}\pi r^2 h$] [5]

7 The diagram shows the curve with parametric equations

$$x = \mathrm{e}^t \cos t \ , \quad y = \mathrm{e}^t \sin t \quad \text{where} \quad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$



(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. [2]

P is a point on the curve such that the normal at P is parallel to the y-axis.

(ii) Find the equation of this normal. [3]

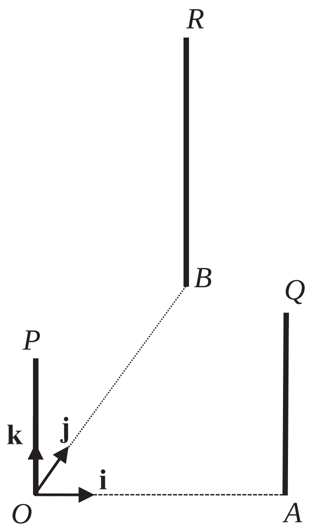
(iii) Given that this normal cuts the curve again at point N, find the coordinates of N correct to 3 decimal places. [3]

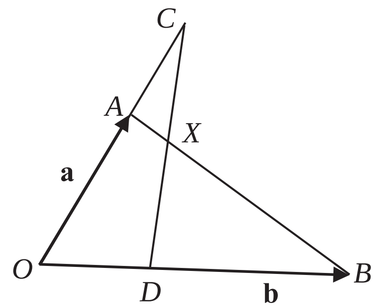
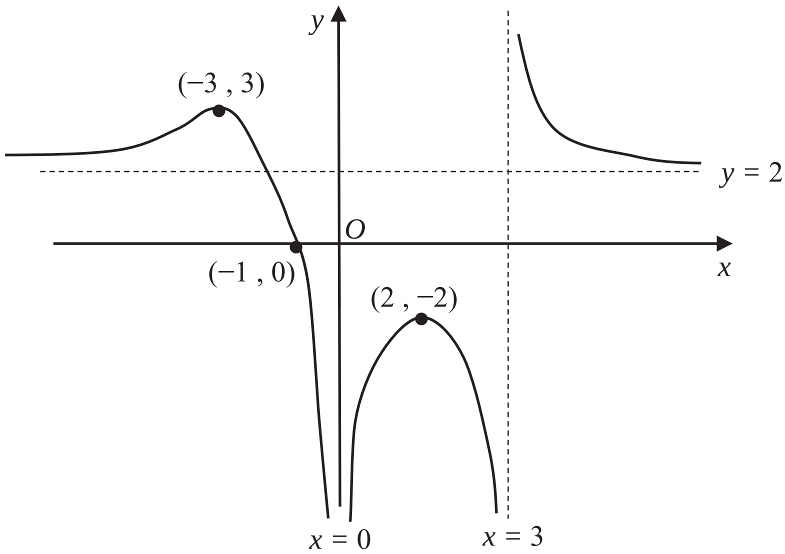
(iv) Hence find the area of the triangle OPN, where O is the origin. [2]

8 In order to construct a temporary shelter, three vertical poles OP, AQ and BR of heights 6 m, 8 m and 10 m respectively are planted firmly with their bases on the horizontal ground.

The base O is taken as the origin and the other bases are such that $OA = OB = 8$ m.

The mutually perpendicular unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are defined with $\mathbf{i}$ along OA, $\mathbf{j}$ along OB and $\mathbf{k}$ vertically upwards as shown in the diagram.



	(i) Verify that the plane with equation $\mathbf{r} \cdot \begin{pmatrix} -1 \\ -2 \\ 4 \end{pmatrix} = 24$ passes through P, Q and R. [3] **(ii)** Find the angle between the plane PQR and the horizontal, giving your answer correct to the nearest 0.1°. [2] **(iii)** Find the position vector of the point A', which is obtained when point A is reflected about the plane PQR. [5] **(iv)** A point N lying on the horizontal ground has coordinates $(\alpha, 9, 0)$ where α is a positive constant. Given that the shortest distance from point N to the plane PQR is 12 m, find the value of α. [3]
9	With reference to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are not parallel. C lies on OA produced with $OC : AC = 3 : 1$ and D divides OB in a ratio of $2 : 3$. X is the point of intersection of AB and CD.  **(i)** Show that $\overrightarrow{OX} = \frac{9}{11}\mathbf{a} + \frac{2}{11}\mathbf{b}$. [5] **(ii)** Hence, find the area of the triangle ADX, giving your answer in the form of $k\lvert \mathbf{a} \times \mathbf{b} \rvert$, where k is a constant to be determined. [4]
10	**(a)**  The diagram above shows the graph of $y = \mathrm{f}(x)$. The curve crosses the

	x-axis at $(-1, 0)$ and has maximum points at $(-3, 3)$ and $(2, -2)$. The lines $x = 0$, $x = 3$ and $y = 2$ are asymptotes of the curve. Sketch the graph of $y = f'(x)$, stating the equations of the asymptotes and the coordinates of the points of intersections with the axes. [4] **(b)** A cubic curve is defined by the equation $y = ax^3 + bx^2 + cx + \frac{32}{3}$, where a, b and c are real constants. The curve passes through the point $(1, 1)$ and has a stationary point at $x = 2$. The curve undergoes the following sequence of transformations: I: A reflection in the x-axis. II: A scaling by a factor of $\frac{1}{2}$ parallel to the x-axis. The resulting curve passes through the point $(2, -16)$. Find the values of a, b and c. [6]
11	For this question, both a and b are real numbers and $0 < b < a$. **(i)** Sketch the graph of $y = \lvert x - a \rvert - b$, showing clearly the coordinates of the intersections with the axes in terms of a and b. [2] **(ii)** Describe a sequence of transformations that map the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to the graph of $\frac{(x-a)^2}{a^2} + \frac{(y+b)^2}{b^2} = 1$. Hence, sketch on the diagram drawn in part **(i)**, the graph of $$\frac{(x-a)^2}{a^2} + \frac{(y+b)^2}{b^2} = 1.$$ [4] **(iii)** It is given that the solution set to the inequality $$\left(\lvert x-a \rvert - b \right)\left(-b + \frac{b}{a}\sqrt{a^2 - (x-a)^2} \right) < 0$$ is $\{ x \in \mathbb{R} : 0 \le x < 1 \text{ or } 5 < x \le 2a \}$. With the help of the diagram, find the values of a and b. [3]

ANNEX B

AJC H2 Math JC1 Promo Examination Paper

QN	Topic Set	Answers
1	Equations and Inequalities	$y \le -5$ or $y \ge -1$
2	Equations and Inequalities	$-2 < x < 2-\sqrt{3}$ or $1 < x < 2+\sqrt{3}$; $0 \le x < 4$
3	Functions	(i) 1 (ii) $y = x$ (iii) zero (iv) $R_{hf} = (-\infty, -2]$
4	Integration techniques	(a) $x + \ln\left\|x^2 - x + 1\right\| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + c$ (b) $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$
5	Application of Integration	(i) $y = x - 1$ (ii) 0.132 (iii) $\frac{95}{3}\pi - 6\pi\ln 6(\ln 6 - 2)$
6	Differentiation & Applications	(ii) 72π
7	Differentiation & Applications	(i) $\frac{\cos t + \sin t}{\cos t - \sin t}$ (ii) $\frac{\sqrt{2}}{2}e^{-\frac{\pi}{4}}$ (iii) (0.322, 4.464) (iv) 0.771
8	Vectors	(ii) 29.2 degrees (iii) $\frac{8}{21}\begin{pmatrix} 13 \\ -16 \\ 32 \end{pmatrix}$ (iv) 13.0
9	Vectors	(ii) $\frac{3}{55}\left\|\mathbf{a} \times \mathbf{b}\right\|$
10	Graphs and Transformation	(b) $a = \frac{1}{3}$, $b = 2$, $c = -12$
11	Graphs and Transformation	(ii) A translation of magnitude a units in the positive direction of the x-axis. A translation of magnitude b units in the negative direction of the y-axis. (iii) $a = 3,\ b = 2$.