A-LEVEL H2 MATHS 2016 – PAPER 1

Question 1

[Ans: $\dfrac{3x^2+5x-2}{x-4}$; $x<-2$ or $\dfrac{1}{3}<x<4$]

$$\frac{4x^2+4x-14}{x-4}-(x+3)$$

$$=\frac{4x^2+4x-14-(x+3)(x-4)}{x-4}$$

$$=\frac{4x^2+4x-14-(x^2-x-12)}{x-4}$$

$$=\frac{3x^2+5x-2}{x-4}$$

$$\frac{4x^2+4x-14}{x-4}<x+3, \qquad x\neq 4$$

$$\frac{4x^2+4x-14}{x-4}-(x+3)<0$$

$$\frac{3x^2+5x-2}{x-4}<0$$

$$\frac{(3x-1)(x+2)}{x-4}<0$$

$-$ $+$ $-$ $+$

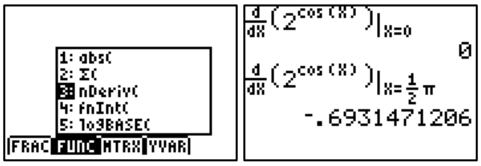
-2 $\frac{1}{3}$ 4

$x<-2$ or $\dfrac{1}{3}<x<4$

Question 2

[Ans: (i) 0; -0.693 (ii) $y=2$; $y=-0.693x+2.09$;$(0.128, 2)$]

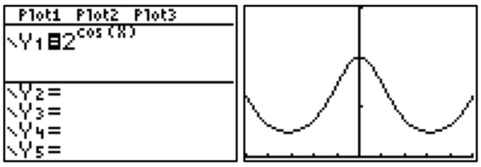
(i) From GC,



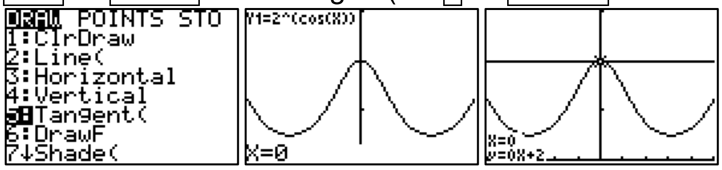
when $x=0$, gradient of the curve $=0$

when $x=\frac{1}{2}\pi$, gradient of the curve $=-0.693$

(ii) From GC,



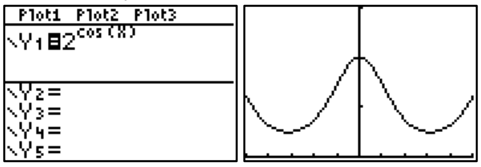
2ND → PRGM → "5:Tangent(" → 0 → ENTER



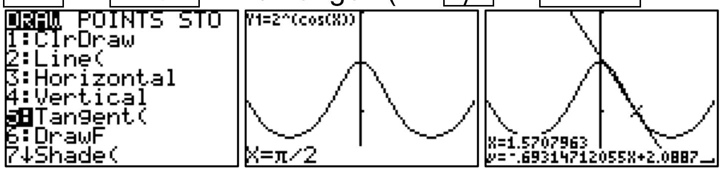
Equation of the tangent where $x=0$:

$y=(0)x+2=2$

From GC,



2ND → PRGM → "5:Tangent(" → $\pi/2$ → ENTER



Equation of the tangent where $x=\frac{1}{2}\pi$:

$y=-0.693x+2.09$

$y=2$ (1)

$y=-0.6932x+2.089$ (2)

Sub. (1) into (2),

$-0.6932x+2.089=2$

$x=0.128$

$\therefore$ the tangents meet at $(0.128, 2)$.

Question 3

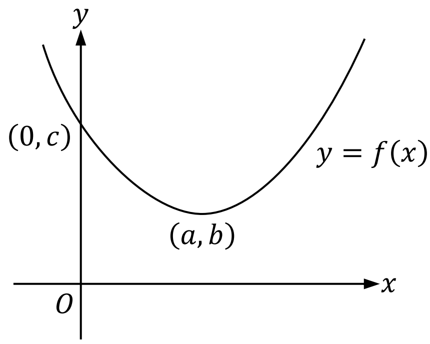
[Ans: $k = \dfrac{c-b}{a^4}$, $l = a$, $m = b$; sketch]

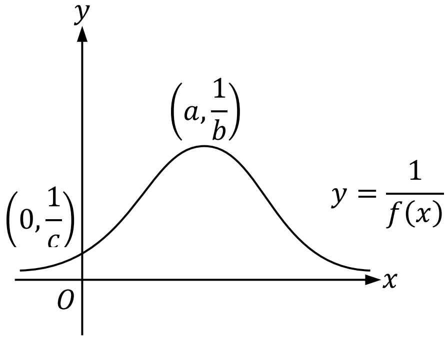
Given $f(x) = k(x-l)^4 + m$
From observation,
min. point on $f(x)$ is at (l, m)

$$\therefore l = a$$
$$m = b$$

At $(0, c)$,

$$f(0) = c$$
$$k(0-l)^4 + m = c$$
$$k = \frac{c-m}{l^4} = \frac{c-b}{a^4}$$





Question 4

[Ans: (i) show; $r = 0.74$ (ii) $\frac{50}{13}b(0.74)^n$]

(i)

	AP	GP	
(4)	$a+3d$	br^4	(5)
(9)	$a+8d$	br^7	(8)
(12)	$a+11d$	br^{14}	(15)

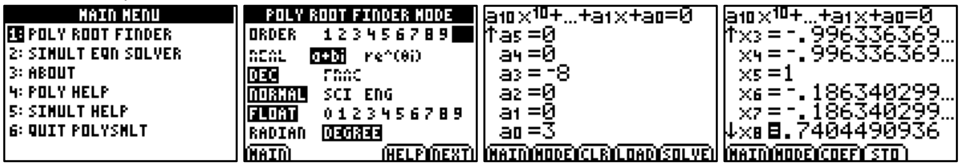
$a+3d = br^4 \Rightarrow a = br^4 - 3d \quad (1)$
$a+8d = br^7 \quad (2)$
$a+11d = br^{14} \quad (3)$

Sub. (1) into (2)
$(br^4 - 3d) + 8d = br^7$
$5d = br^4(r^3 - 1) \quad (4)$

Sub. (1) into (3)
$(br^4 - 3d) + 11d = br^{14}$
$8d = br^4(r^{10} - 1) \quad (5)$

$(4)/(5)$
$$\frac{5d}{8d} = \frac{br^4(r^3-1)}{br^4(r^{10}-1)}$$
$$\frac{5}{8} = \frac{r^3-1}{r^{10}-1}$$
$5r^{10} - 5 = 8r^3 - 8$
$5r^{10} - 8r^3 + 3 = 0$ (shown)

From GC,



$r = 0.74$

(ii) Sum
$= br^n + br^{n+1} + br^{n+2} + \cdots$
$$= \frac{br^n}{1-r}$$
$$= \frac{b(0.74)^n}{1-0.74}$$
$$= \frac{50}{13}b(0.74)^n$$

Queston 5

[Ans: (i) $\begin{pmatrix} 2b \\ 4b-4a \\ -2a \end{pmatrix}$ (ii) $\begin{pmatrix} -2a \\ -8a \\ -2a \end{pmatrix}$; $\pm\dfrac{1}{6\sqrt{2}}$ (iii) 3]

(i) $\boldsymbol{u} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$, $\boldsymbol{v} = \begin{pmatrix} a \\ 0 \\ b \end{pmatrix}$

$$\begin{aligned}(\boldsymbol{u}+\boldsymbol{v}) \times (\boldsymbol{u}-\boldsymbol{v}) &= \boldsymbol{u}\times\boldsymbol{u} - \boldsymbol{u}\times\boldsymbol{v} + \boldsymbol{v}\times\boldsymbol{u} - \boldsymbol{v}\times\boldsymbol{v} \\ &= \boldsymbol{0} + \boldsymbol{v}\times\boldsymbol{u} + \boldsymbol{v}\times\boldsymbol{u} - \boldsymbol{0} \\ &= 2\boldsymbol{v}\times\boldsymbol{u} \\ &= 2\begin{pmatrix} a \\ 0 \\ b \end{pmatrix} \times \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = 2\begin{pmatrix} b \\ 2b-2a \\ -a \end{pmatrix} = \begin{pmatrix} 2b \\ 4b-4a \\ -2a \end{pmatrix}\end{aligned}$$

or

$$\boldsymbol{u}+\boldsymbol{v} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \begin{pmatrix} a \\ 0 \\ b \end{pmatrix} = \begin{pmatrix} 2+a \\ -1 \\ 2+b \end{pmatrix}$$

$$\boldsymbol{u}-\boldsymbol{v} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} - \begin{pmatrix} a \\ 0 \\ b \end{pmatrix} = \begin{pmatrix} 2-a \\ -1 \\ 2-b \end{pmatrix}$$

$$\begin{aligned}(\boldsymbol{u}+\boldsymbol{v}) \times (\boldsymbol{u}-\boldsymbol{v}) &= \begin{pmatrix} 2+a \\ -1 \\ 2+b \end{pmatrix} \times \begin{pmatrix} 2-a \\ -1 \\ 2-b \end{pmatrix} \\ &= \begin{pmatrix} (-1)(2-b)-(2+b)(-1) \\ (2+b)(2-a)-(2+a)(2-b) \\ -2-a+2-a \end{pmatrix} \\ &= \begin{pmatrix} -2+b+2+b \\ 4-2a+2b-ab-4+2b-2a+ab \\ -2a \end{pmatrix} = \begin{pmatrix} 2b \\ 4b-4a \\ -2a \end{pmatrix}\end{aligned}$$

(ii) $2b = -2a \Rightarrow b = -a$

$$(\boldsymbol{u}+\boldsymbol{v}) \times (\boldsymbol{u}-\boldsymbol{v}) = \begin{pmatrix} 2(-a) \\ 4(-a)-4a \\ -2a \end{pmatrix} = \begin{pmatrix} -2a \\ -8a \\ -2a \end{pmatrix}$$

$$|(\boldsymbol{u}+\boldsymbol{v}) \times (\boldsymbol{u}-\boldsymbol{v})| = 1$$

$$\left|\begin{pmatrix} -2a \\ -8a \\ -2a \end{pmatrix}\right| = 1$$

$$\sqrt{(-2a)^2+(-8a)^2+(-2a)^2} = 1$$

$$72a^2 = 1 \Rightarrow a = \pm\frac{1}{\sqrt{72}} = \pm\frac{1}{6\sqrt{2}}$$

(iii) $(\boldsymbol{u}+\boldsymbol{v})\cdot(\boldsymbol{u}-\boldsymbol{v})=0$

$\boldsymbol{u}\cdot\boldsymbol{u}-\boldsymbol{u}\cdot\boldsymbol{v}+\boldsymbol{v}\cdot\boldsymbol{u}-\boldsymbol{v}\cdot\boldsymbol{v}=0$

$|\boldsymbol{u}|^2-\boldsymbol{u}\cdot\boldsymbol{v}+\boldsymbol{u}\cdot\boldsymbol{v}-|\boldsymbol{v}|^2=0$

$|\boldsymbol{v}|^2=|\boldsymbol{u}|^2$

$|\boldsymbol{v}|=|\boldsymbol{u}|$

$=\sqrt{(2)^2+(-1)^2+(2)^2}$

$=3$

or

$(\boldsymbol{u}+\boldsymbol{v})\cdot(\boldsymbol{u}-\boldsymbol{v})=0$

$\begin{pmatrix}2+a\\-1\\2+b\end{pmatrix}\cdot\begin{pmatrix}2-a\\-1\\2-b\end{pmatrix}=0$

$4-a^2+1+4-b^2=0$

$a^2+b^2=9$

$\sqrt{a^2+0^2+b^2}=\sqrt{9}$

$|v|=3$

Question 6

[Ans: (i) prove (ii) $u_1=4,\ u_2=16,\ u_3=52$ (iii) $2+\frac{1}{4}n(n+1)(n^2+n+2)$]

(i) Let P_n: $\sum_{r=1}^{n} r(r^2+1)=\frac{1}{4}n(n+1)(n^2+n+2),\ n\in\mathbb{Z}^+$

$P_1: LHS=\sum_{r=1}^{1} r(r^2+1)=(1)(1^2+1)=2$

$RHS=\frac{1}{4}(1)(1+1)(1^2+1+2)=2$

$\therefore P_1$ is true.

Assume P_k is true for some $k\in\mathbb{Z}^+$. i.e. $\sum_{r=1}^{k} r(r^2+1)=\frac{1}{4}k(k+1)(k^2+k+2)$

[To prove P_{k+1} is true. i.e.

$\sum_{r=1}^{k+1} r(r^2+1)=\frac{1}{4}(k+1)[(k+1)+1][(k+1)^2+(k+1)+2]=\frac{1}{4}(k+1)(k+2)(k^2+3k+4)$]

$\sum_{r=1}^{k+1} r(r^2+1)$

$=\sum_{r=1}^{k} r(r^2+1)+(k+1)[(k+1)^2+1]$

$=\frac{1}{4}k(k+1)(k^2+k+2)+(k+1)(k^2+2k+2)$

$=\frac{1}{4}(k+1)(k^3+k^2+2k+4k^2+8k+8)$

$=\frac{1}{4}(k+1)(k^3+2k^2+10k+8)$

$=\frac{1}{4}(k+1)(k+2)(k^2+3k+4)$

$\therefore P_{k+1}$ is true if P_k is true.

Since P_1 is true, by mathematical induction, P_n is true for $n\in\mathbb{Z}^+$. (proven)

(ii) Given $u_n = u_{n-1} + n^3 + n$

$u_1 = u_0 + 1^3 + 1 = 2 + 1^3 + 1 = 4$

$u_2 = u_1 + 2^3 + 2 = 4 + 2^3 + 2 = 14$

$u_3 = u_2 + 3^3 + 3 = 14 + 3^3 + 3 = 44$

(iii) $\sum_{r=1}^{n}\left(u_r - u_{r-1}\right) = \sum_{r=1}^{n}\left(r^3 + r\right)$

$$\sum_{r=1}^{n}(u_r - u_{r-1}) = \sum_{r=1}^{n} r(r^2 + 1)$$

$$\begin{aligned} u_1 - u_0 &= \frac{1}{4}n(n+1)(n^2+n+2) \\ +u_2 - u_1 & \\ +u_3 - u_2 & \\ \vdots & \\ +u_{n-1} - u_{n-2} & \\ +u_n - u_{n-1} & \end{aligned}$$

$$u_n - u_0 = \frac{1}{4}n(n+1)(n^2+n+2)$$

$$u_n = u_0 + \frac{1}{4}n(n+1)(n^2+n+2)$$

$$= 2 + \frac{1}{4}n(n+1)(n^2+n+2)$$

Question 7

[Ans: (a) verify; $2+3i$ (b) $a=3$, $k=-30$]

(a) Let $f(w)=w^2+(-1-8i)w+(-17+7i)$

$$\begin{aligned} f(-1+5i) &= (-1+5i)^2+(-1-8i)(-1+5i)+(-17+7i) \\ &= (1-10i-25)+(1-5i+8i+40)+(-17+7i) \\ &= 0 \end{aligned}$$

$\therefore -1+5i$ is a root of $f(w)$.

Let the second root be a.

$$w^2+(-1-8i)w+(-17+7i)=[w-(-1+5i)](w-a)$$

From observation,

$$\begin{aligned} -(-1+5i)(-a) &= -17+7i \\ a &= \frac{-17+7i}{-1+5i} \\ &= \frac{-17+7i}{-1+5i}\left(\frac{-1-5i}{-1-5i}\right) \\ &= \frac{17+85i-7i+35}{(-1)^2+(5)^2} \\ &= \frac{52+78i}{26} \\ &= 2+3i \end{aligned}$$

(b) $(1+ai)^3-5(1+ai)^2+16(1+ai)+k=0$

$[1+3ai+3(ai)^2+(ai)^3]-5(1+2ai-a^2)+16+16ai+k=0$

$[(1-3a^2)+(3a-a^3)i]+[(5a^2-5)-10ai]+[(16+k)+16ai]=0$

$(1-3a^2+5a^2-5+16+k)+(3a-a^3-10a+16a)i=0$

$(12+2a^2+k)+(9a-a^3)i=0$

$\therefore 9a-a^3=0$

$a(9-a^2)=0$

$a=0$ (NA) or $a=-3$ (NA) or $a=3$

$12+2a^2+k=0$

$12+2(3)^2+k=0$

$k=-30$

Question 8

[Ans: (i) show; $f''(x) = 2a^2y + 2a^2y^3$; $f'''(x) = 2a^3 + 8a^3y^2 + 6a^3y^4$

(ii) $1 + 2ax + 2a^2x^2 + \frac{8}{3}a^3x^3 + \cdots$ (iii) $2x + \frac{8}{3}{}^3\ x^3 + \cdots$]

(i) $f(x) = \tan(ax + b)$

$$f'(x) = a\sec^2(ax + b)$$
$$= a[1 + \tan^2(ax + b)]$$
$$= a + ay^2$$

(shown)

$$f''(x) = 2ayf'(x)$$
$$= 2ay(a + ay^2)$$
$$= 2a^2y + 2a^2y^3$$

$$f'''(x) = 2a^2f'(x) + 6a^2y^2f'(x)$$
$$= 2a^2(a + ay^2) + 6a^2y^2(a + ay^2)$$
$$= 2a^3 + 2a^3y^2 + 6a^3y^2 + 6a^3y^4$$
$$= 2a^3 + 8a^3y^2 + 6a^3y^4$$

(ii) When $b = \frac{1}{4}\pi$,

$$f(x) = \tan\left(ax + \frac{1}{4}\pi\right)$$

$$f(0) = \tan\left(0 + \frac{1}{4}\pi\right) = 1 = y$$
$$f'(0) = a + a(1)^2 = 2a$$
$$f''(0) = 2a^2(1) + 2a^2(1)^3 = 4a^2$$
$$f'''(0) = 2a^3 + 8a^3(1)^2 + 6a^3(1)^4 = 16a^3$$

$$\therefore f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$
$$= 1 + 2ax + \frac{4a^2}{2!}x^2 + \frac{16a^3}{3!}x^3 + \cdots$$
$$= 1 + 2ax + 2a^2x^2 + \frac{8}{3}a^3x^3 + \cdots$$

(iii) Let $a = 2$ and $b = 0$,

$$\tan(ax + b) = \tan(2x)$$

$$f(0) = \tan(0) = 0 = y$$
$$f'(0) = a + a(0)^2 = a = 2$$
$$f''(0) = 2a^2(0) + 2a^2(0)^3 = 0$$
$$f'''(0) = 2a^3 + 8a^3(0)^2 + 6a^3(0)^4 = 2a^3 = 2(2)^3 = 16$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$
$$= 0 + 2x + 0 + \frac{16}{3!}x^3 + \cdots$$
$$= 2x + \frac{8}{3}x^3 + \cdots$$

Question 9

[Ans: (i)(a) show (b) $y = 5 - 5e^{-2t}$; $x = 5t + \frac{5}{2}e^{-2t} - \frac{5}{2}$ (ii) $x = 5t^2 + 20\sin\frac{1}{2}t - 10t$

(iii) 1.47s; 1.05s]

(i) (a) $y = \frac{dx}{dt}$

$\Rightarrow \frac{dy}{dt} = \frac{d^2x}{dt^2}$

$\therefore \frac{d^2x}{dt^2} + 2\frac{dx}{dt} = 10$

$\frac{dy}{dt} + 2y = 10$

$\Rightarrow \frac{dy}{dt} = 10 - 2y$ (shown)

(b) $\frac{dy}{dt} = 10 - 2y$

$\frac{1}{10-2y}\frac{dy}{dt} = 1$

$\int \frac{1}{10-2y}\,dy = \int dt$

$-\frac{1}{2}\ln|10-2y| = t + A$

$\ln|10-2y| = -2t - 2A$

$|10-2y| = e^{-2A}e^{-2t}$

$10 - 2y = \pm e^{-2A}e^{-2t}$

$y = 5 \pm \frac{1}{2}e^{-2A}e^{-2t}$

$y = 5 + Be^{-2t}$

$\frac{dx}{dt} = 5 + Be^{-2t}$

When $t = 0$,

$\frac{dx}{dt} = 0$

$5 + B = 0 \Rightarrow B = -5$

$\therefore y = 5 - 5e^{-2t}$

$\frac{dx}{dt} = 5 - 5e^{-2t}$

$x = \int 5 - 5e^{-2t}\,dt$

$= 5t + \frac{5}{2}e^{-2t} + C$

When $t = 0$,

$x = 0$

$\frac{5}{2} + C = 0 \Rightarrow C = -\frac{5}{2}$

$\therefore x = 5t + \frac{5}{2}e^{-2t} - \frac{5}{2}$

(ii) $\dfrac{d^2x}{dt^2} = 10 - 5\sin\dfrac{1}{2}t$

$\dfrac{dx}{dt} = \int 10 - 5\sin\dfrac{1}{2}t\,dt$

$= 10t + 10\cos\dfrac{1}{2}t + A$

When $t = 0$,

$\dfrac{dx}{dt} = 0$

$10 + A = 0 \Rightarrow A = -10$

$\dfrac{dx}{dt} = 10t + 10\cos\dfrac{1}{2}t - 10$

$x = \int 10t + 10\cos\dfrac{1}{2}t - 10\,dt$

$= 5t^2 + 20\sin\dfrac{1}{2}t - 10t + B$

When $t = 0$,

$x = 0$

$B = 0$

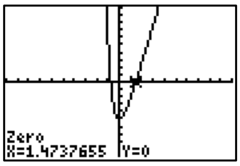
$\therefore x = 5t^2 + 20\sin\dfrac{1}{2}t - 10t$

(iii) For model in (i),

$x = 5$

$5t + \dfrac{5}{2}e^{-2t} - \dfrac{5}{2} = 5 \Rightarrow 5t + \dfrac{5}{2}e^{-2t} - \dfrac{15}{2} = 0$

From GC,



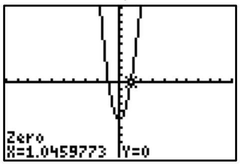
$t = 1.47$

For model in (ii),

$x = 5$

$5t^2 + 20\sin\dfrac{1}{2}t - 10t = 5 \Rightarrow 5t^2 + 20\sin\dfrac{1}{2}t - 10t - 5 = 0$

From GC,

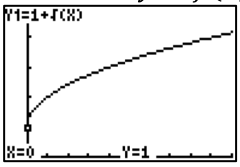


$t = 1.05$

Question 10

[Ans: (a)(i) $f^{-1}(x) = (x-1)^2, x \geq 1$ (ii) show; $x = 2.62$; explain
(b)(i) $g(4) = 6$; $g(7) = 8$; $g(12) = 9$ (ii) no; justify]

(a) (i) From GC, $y = f(x)$



$D_{f^{-1}} = R_f = [1, \infty)$

Let $y = f(x)$

$y = 1 + \sqrt{x}, x \geq 0$

$\sqrt{x} = y - 1$

$x = (y-1)^2$

$\therefore f^{-1}(x) = (x-1)^2, \quad x \geq 1$

(ii) $ff(x) = x$

$1 + \sqrt{1 + \sqrt{x}} = x, \quad x \in D_{ff} \cap R_{ff} = [2, \infty)$

$\sqrt{1 + \sqrt{x}} = x - 1$

$1 + \sqrt{x} = (x-1)^2$

$1 + \sqrt{x} = x^2 - 2x + 1$

$\sqrt{x} = x^2 - 2x$

$x = (x^2 - 2x)^2$

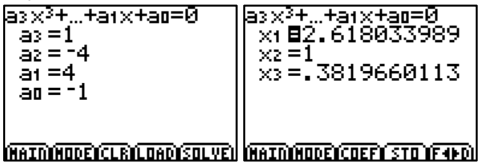
$x = x^4 - 4x^3 + 4x^2$

$x^4 - 4x^3 + 4x^2 - x = 0$

$x(x^3 - 4x^2 + 4x - 1) = 0$

$x = 0$ (NA) or $x^3 - 4x^2 + 4x - 1 = 0$ (shown)

From GC,



$x = 0.382$ (NA) or $x = 1$ (NA) or $x = 2.62$

$f(x) = f^{-1}(x)$ and $x = 2.62 \in (D_f \cap D_{f^{-1}}) = [1, \infty)$

$ff(x) = ff^{-1}(x)$

$ff(x) = x$

$\therefore$ the value of $x = 2.62$ obtained from the previous part satisfies the equation $f(x) = f^{-1}(x)$.

(b) (i) $g(4)$

$= 2 + g\left(\frac{1}{2} \times 4\right) = 2 + g(2)$

$= 2 + \left[2 + g\left(\frac{1}{2} \times 2\right)\right] = 4 + g(1)$

$= 4 + [1 + g(1-1)] = 5 + g(0)$

$= 5 + 1 = 6$

$g(7)$

$= 1 + g(7-1) = 1 + g(6)$

$= 1 + \left[2 + g\left(\frac{1}{2} \times 6\right)\right] = 3 + g(3)$

$= 3 + [1 + g(3-1)] = 4 + g(2)$

$= 4 + \left[2 + g\left(\frac{1}{2} \times 2\right)\right] = 6 + g(1)$

$= 6 + [1 + g(1-1)] = 7 + g(0)$

$= 7 + 1 = 8$

→ or

$g(7) = 4 + g(2)$

$= 4 + [g(4) - 2]$

$= 4 + (6 - 2) = 8$

$g(12)$

$= 2 + g\left(\frac{1}{2} \times 12\right) = 2 + g(6)$

$= 2 + \left[2 + g\left(\frac{1}{2} \times 6\right)\right] = 4 + g(3)$

$= 4 + [1 + g(3-1)] = 5 + g(2)$

$= 5 + \left[2 + g\left(\frac{1}{2} \times 2\right)\right] = 7 + g(1)$

$= 7 + [1 + g(1-1)] = 8 + g(0)$

$= 8 + 1 = 9$

→ or

$g(12) = 2 + g(6)$

$= 2 + [g(7) - 1]$

$= 2 + (8 - 1) = 9$

(ii) $g(5) = 1 + g(4) = 1 + 6 = 7$

$g(6) = 2 + g(3)$

$= 2 + [1 + g(2)] = 3 + g(2)$

$= 3 + [2 + g(1)] = 5 + g(1)$

$= 5 + [1 + g(0)] = 6 + 1 = 7$

Since $g(5) = g(6)$, g does not have an inverse.

Question 11

[Ans: (i)(a) show; $\lambda=-\dfrac{8}{9}$, $\mu=\dfrac{19}{18}$, $t=-\dfrac{5}{9}$ (b) $-2x+y+2z=-37$ and $-2x+y+2z=35$ (ii) $\dfrac{9}{2}$]

(i) Where $a=0$,

$$p:\mathbf{r}=\begin{pmatrix}1\\-3\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\0\end{pmatrix}+\mu\begin{pmatrix}0\\4\\-2\end{pmatrix},\qquad l:\mathbf{r}=\begin{pmatrix}-1\\0\\1\end{pmatrix}+t\begin{pmatrix}-2\\1\\2\end{pmatrix}$$

(a) $\begin{pmatrix}1\\2\\0\end{pmatrix}\cdot\begin{pmatrix}-2\\1\\2\end{pmatrix}=-2+2+0=0$

$\begin{pmatrix}0\\4\\-2\end{pmatrix}\cdot\begin{pmatrix}-2\\1\\2\end{pmatrix}=0+4-4=0$

$\therefore$ l is perpendicular to p.

At the intersection of l and p,

$$\begin{pmatrix}1\\-3\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\0\end{pmatrix}+\mu\begin{pmatrix}0\\4\\-2\end{pmatrix}=\begin{pmatrix}-1\\0\\1\end{pmatrix}+t\begin{pmatrix}-2\\1\\2\end{pmatrix}$$

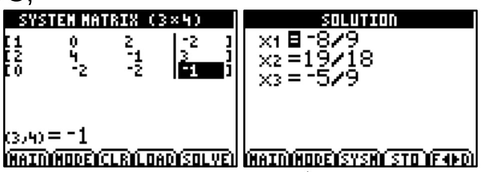
$$\begin{pmatrix}\lambda+2t\\2\lambda+4\mu-t\\-2\mu-2t\end{pmatrix}=\begin{pmatrix}-2\\3\\-1\end{pmatrix}$$

$\therefore\ \lambda+2t=-2$

$2\lambda+4\mu-t=3$

$-2\mu-2t=-1$

From GC,



$\lambda=-\dfrac{8}{9},\mu=\dfrac{19}{18},t=-\dfrac{5}{9}$

(b) $p:\mathbf{r}\cdot\begin{pmatrix}-2\\1\\2\end{pmatrix}=\begin{pmatrix}1\\-3\\2\end{pmatrix}\cdot\begin{pmatrix}-2\\1\\2\end{pmatrix}=-1$

Let the equation of the plane be $\mathbf{r}\cdot\begin{pmatrix}-2\\1\\2\end{pmatrix}=d$

Distance from the plane to $p=12$

$$\left|\frac{d}{\left|\begin{pmatrix}-2\\1\\2\end{pmatrix}\right|}-\frac{-1}{\left|\begin{pmatrix}-2\\1\\2\end{pmatrix}\right|}\right|=12$$

$$\left|\frac{d+1}{\sqrt{2^2+1^2+2^2}}\right|=12$$

$|d+1|=36$

$d+1=-36$ or $d+1=36$

$d=-37$ or $d=35$

∴ equation of plane:

$\boldsymbol{r}\cdot\begin{pmatrix}-2\\1\\2\end{pmatrix}=-37$ or $\boldsymbol{r}\cdot\begin{pmatrix}-2\\1\\2\end{pmatrix}=d$

$-2x+y+2z=-37$ $\quad$ $-2x+y+2z=35$

(ii) Normal vector of p

$=\begin{pmatrix}1\\2\\0\end{pmatrix}\times\begin{pmatrix}a\\4\\-2\end{pmatrix}=\begin{pmatrix}-4\\2\\4-2a\end{pmatrix}$

For l and p not to meet at a unique point,

$\begin{pmatrix}-4\\2\\4-2a\end{pmatrix}\cdot\begin{pmatrix}-2\\1\\2\end{pmatrix}=0$

$8+2+8-4a=0$

$a=\frac{9}{2}$