O-LEVEL A-MATHS 2016 – PAPER 1

Question 1

[Ans: (i) $(-2, 16)$, $(4, 4)$ (ii) show]

(i) $y = 2x^2 - 6x - 4 \quad (1)$
$y + 2x = 12 \quad (2)$

Sub. (1) into (2)
$(2x^2 - 6x - 4) + 2x = 12$
$2x^2 - 4x - 16 = 0$
$x^2 - 2x - 8 = 0$
$(x + 2)(x - 4) = 0$
$x = -2$ or $x = 4$

Sub. $x = -2$ into (2)
$y + 2(-2) = 12 \Rightarrow y = 16$

Sub. $x = 4$ into (2)
$y + 2(4) = 12 \Rightarrow y = 4$

∴ the line intersects with the curve at $(-2, 16)$ and $(4, 4)$.

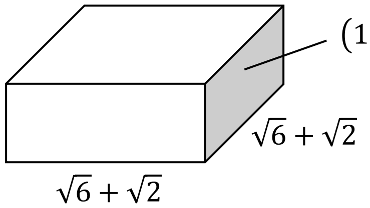
(ii) $y = 2x^2 - kx - 4 \quad (1)$
$y + 2x = 12 \quad (2)$

Sub. (1) into (2)
$(2x^2 - kx - 4) + 2x = 12$
$2x^2 + (2 - k)x - 16 = 0$
Discriminant
$= (2 - k)^2 - 4(2)(-16)$
$= (2 - k)^2 + 128 > 0$ for all real value of k

∴ the line intersects the curve at two distinct points for all real values of k. (shown)

Question 2

[Ans: $(5-2\sqrt{3})$cm]



Height

$$=\frac{16+4\sqrt{3}}{(\sqrt{6}+\sqrt{2})^2}$$

$$=\frac{16+4\sqrt{3}}{(\sqrt{6})^2+2\sqrt{6}\sqrt{2}+(\sqrt{2})^2}$$

$$=\frac{16+4\sqrt{3}}{8+2\sqrt{12}}=\frac{16+4\sqrt{3}}{8+2\sqrt{4\times 3}}=\frac{16+4\sqrt{3}}{8+4\sqrt{3}}=\frac{4+\sqrt{3}}{2+\sqrt{3}}$$

$$=\frac{4+\sqrt{3}}{2+\sqrt{3}}\left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right)$$

$$=\frac{8-4\sqrt{3}+2\sqrt{3}-(\sqrt{3})^2}{2^2-(\sqrt{3})^2}$$

$$=\frac{5-2\sqrt{3}}{4-3}$$

$$=5-2\sqrt{3}$$

Question 3

[Ans: (a)(i) $-\frac{\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}$ (ii) $0\le\cos^{-1}x\le\pi$ (b) $a=-1$, $b=6$, $c=2$]

(a) (i) $-\frac{\pi}{2}\le\sin^{-1}x\le\frac{\pi}{2}$

(ii) $0\le\cos^{-1}x\le\pi$

(b) From observation,

$a=-1$

$c=2$

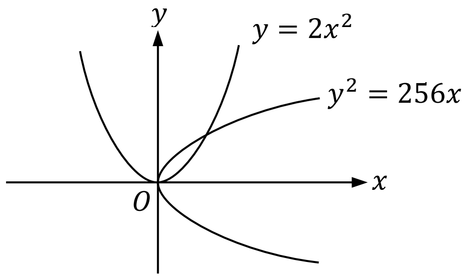
$$\frac{2\pi}{1/b}=12\pi$$

$$2b\pi=12\pi\Rightarrow b=6$$

Question 4

[Ans: (i) sketch (ii) $y = 8x$]

(i)



(ii) $y^2 = 256x \quad (1)$
$y = 2x^2 \quad (2)$

Sub. (2) into (1)
$(2x^2)^2 = 256x$
$4x^4 - 256x = 0$
$4x(x^3 - 64) = 0$
$x = 0$ or $x^3 = 64 \Rightarrow x = 4$

Sub. $x = 4$ into (2)
$y = 2(4)^2 = 32$

Equation of line:
$y = \dfrac{32}{4}x$
$\Rightarrow y = 8x$

Queston 5

[Ans: $2 + \dfrac{5}{x+3} - \dfrac{3}{x-2}$]

$$\begin{array}{r} 2 \\ x^2 + x - 6 \overline{\big)\, 2x^2 + 4x - 31} \\ \underline{-(2x^2 + 2x - 12)} \\ 2x - 19 \end{array}$$

$$\frac{2x^2 + 4x - 31}{x^2 + x - 6} = 2 + \frac{2x - 19}{x^2 + x - 6} = 2 + \frac{2x - 19}{(x+3)(x-2)}$$

Let $\dfrac{2x-19}{(x+3)(x-2)} = \dfrac{A}{x+3} + \dfrac{B}{x-2}$
$2x - 19 = A(x-2) + B(x+3)$

When $x = -3$,
$2(-3) - 19 = A(-3 - 2)$
$-5A = -25 \Rightarrow A = 5$

When $x = 2$,
$2(2) - 19 = B(2 + 3)$
$5B = -15 \Rightarrow B = -3$

$$\therefore \frac{2x^2 + 4x - 31}{x^2 + x - 6} = 2 + \frac{5}{x+3} - \frac{3}{x-2}$$

Question 6

[Ans: (i) explain (ii) $a = \frac{3}{2}$, $b = -12$, $c = 18$ (iii) $\{q \in \mathbb{R}: 0 < q < 6\}$]

(i) Since x-coordinate of M is the midpoint of the x-coordinates of A and B,

$$\frac{p+6}{2} = 4$$

$$\therefore p = 2$$

(ii) Let $ax^2 + bx + c = a(x-2)(x-6)$

Since $a > 0$, curve of $y = a(x-2)(x-6)$ passes through $(4, -6)$.

$$-6 = a(4-2)(4-6)$$

$$-4a = -6$$

$$a = \frac{3}{2}$$

$$\begin{aligned} \therefore ax^2 + bx + c &= \frac{3}{2}(x-2)(x-6) \\ &= \frac{3}{2}(x^2 - 6x - 2x + 12) \\ &= \frac{3}{2}(x^2 - 8x + 12) \\ &= \frac{3}{2}x^2 - 12x + 18 \end{aligned}$$

$$\therefore b = -12, c = 18$$

(iii) From observation, $\{q \in \mathbb{R}: 0 < q < 6\}$

Question 7

[Ans: (i) show (ii) $\frac{125(t-8)}{\sqrt{125(t^2-16t+80)}}$ (iii) 44.7m]

(i) $OP = 5t$
$OQ = 100 - 10t$
$s = \sqrt{OP^2 + OQ^2}$
$= \sqrt{(5t)^2 + (100-10t)^2}$
$= \sqrt{25t^2 + 10000 - 2000t + 100t^2}$
$= \sqrt{125t^2 - 2000t + 10000}$
$= \sqrt{125(t^2 - 16t + 80)}$
(shown)

(ii) $s = [125(t^2 - 16t + 80)]^{\frac{1}{2}}$
$\frac{ds}{dt} = \frac{1}{2}[125(t^2 - 16t + 80)]^{-\frac{1}{2}}[125(2t - 16)]$
$= \frac{125(2t-16)}{2\sqrt{125(t^2-16t+80)}}$
$= \frac{125(t-8)}{\sqrt{125(t^2-16t+80)}}$

(iii) Let $\frac{ds}{dt} = 0$
$\frac{125(t-8)}{\sqrt{125(t^2-16t+80)}} = 0$
$t - 8 = 0$
$t = 8$

t	$(8)^-$	8	$(8)^+$
$\frac{ds}{dt}$	$-$	0	$+$
Shape	\	_	/

$\therefore$ least s
$= \sqrt{125[(8)^2 - 16(8) + 80]}$
$= 44.7$

Question 8

[Ans: (i) $B(7,12)$ (ii) $M\left(\frac{13}{2},3\right)$, $D(6,-6)$]

(i) Equation of BC:

$$2y+3x=45$$
$$2y=-3x+45$$
$$y=-\frac{3}{2}x+\frac{45}{2} \quad (1)$$

$\therefore$ gradient of AB $=\frac{2}{3}$

Equation of AB:

$$y-6=\frac{2}{3}[x-(-2)]$$
$$y=\frac{2}{3}x+\frac{4}{3}+6=\frac{2}{3}x+\frac{22}{3} \quad (2)$$

$(1)=(2)$

$$-\frac{3}{2}x+\frac{45}{2}=\frac{2}{3}x+\frac{22}{3}$$
$$\frac{13}{6}x=\frac{91}{6}\Rightarrow x=7$$

Sub. $x=7$ into (1)

$$y=-\frac{3}{2}(7)+\frac{45}{2}=12$$

$\therefore B(7,12)$

(ii) For equation BC at C, $y=0$

$$0+3x=45\Rightarrow x=15$$

$\therefore C(15,0)$

$$M\left(\frac{-2+15}{2},\frac{6+0}{2}\right)\Rightarrow M\left(\frac{13}{2},3\right)$$

Let coordinates of D be (p,q).

$$\frac{p+7}{2}=\frac{13}{2}\Rightarrow p=6$$
$$\frac{q+12}{2}=6\Rightarrow q=-9$$

$\therefore D(6,-6)$

Question 9

[Ans: (i) $(-2,-6)$ and $(2,-6)$ (ii) both are maximum points]

(i) $y = 2 - x^2 - 16x^{-2}$

$\frac{dy}{dx} = -2x - 16(-2x^{-3}) = -2x + \frac{32}{x^3}$

Let $\frac{dy}{dx} = 0$

$-2x + \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x^4 = 16$

$x = \pm 2$

When $x = -2$,

$y = 2 - (-2)^2 - \frac{16}{(-2)^2} = -6$

When $x = 2$,

$y = 2 - (2)^2 - \frac{16}{(2)^2} = -6$

$\therefore$ the stationary points of the curve are at $(-2,-6)$ and $(2,-6)$.

(ii) $\frac{dy}{dx} = -2x + 32x^{-3}$

$\frac{d^2y}{dx^2} = -2 + 32(-3x^{-4}) = -2 - \frac{96}{x^4}$

When $x = -2$,

$\frac{d^2y}{dx^2} = -2 - \frac{96}{(-2)^2} = -8 < 0$

$\therefore$ $(-2,-6)$ is a maximum point.

When $x = 2$,

$\frac{d^2y}{dx^2} = -2 - \frac{96}{(2)^2} = -8 < 0$

$\therefore$ $(2,-6)$ is a maximum point.

Question 10

[Ans: (i) show (ii) $-\frac{1}{4x^2}-\frac{\ln x}{2x^2}+c$ (iii) $-\frac{1}{4x^2}-\frac{\ln x}{2x^2}+1$]

(i) $\frac{d}{dx}\left(\frac{\ln x}{x^2}\right)$

$=\frac{x^2\left(\frac{1}{x}\right)-(\ln x)(2x)}{(x^2)^2}=\frac{x-2x\ln x}{x^4}$

$=\frac{1}{x^3}-\frac{2\ln x}{x^3}$

(shown)

(ii) $\int\frac{1}{x^3}-\frac{2\ln x}{x^3}dx=\frac{\ln x}{x^2}$

$\int x^{-3}\,dx-2\int\frac{\ln x}{x^3}\,dx=\frac{\ln x}{x^2}$

$\frac{x^{-2}}{-2}-2\int\frac{\ln x}{x^3}\,dx=\frac{\ln x}{x^2}$

$2\int\frac{\ln x}{x^3}\,dx=-\frac{1}{2x^2}-\frac{\ln x}{x^2}$

$\int\frac{\ln x}{x^3}\,dx=-\frac{1}{4x^2}-\frac{\ln x}{2x^2}+c$

(iii) $f(x)=\int\frac{\ln x}{x^3}dx$

$f(x)=-\frac{1}{4x^2}-\frac{\ln x}{2x^2}+c$

$f(1)=\frac{3}{4}$

$-\frac{1}{4(1)^2}-\frac{\ln 1}{2(1)^2}+c=\frac{3}{4}$

$c=1$

$\therefore f(x)=-\frac{1}{4x^2}-\frac{\ln x}{2x^2}+1$

Question 11

[Ans: (a) prove (b)(i) $\frac{\pi}{30}$ (ii) 40min]

(a) LHS

$= (\sec\theta - \tan\theta)^2$

$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$

$= \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$

$= \frac{(1-\sin\theta)^2}{\cos^2\theta}$

$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$

$= \frac{(1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$

$= \frac{1-\sin\theta}{1+\sin\theta}$

= RHS (proven)

(b) (i) When $t = 15$,

$d = 80$

$|80\sin 15k| = 80$

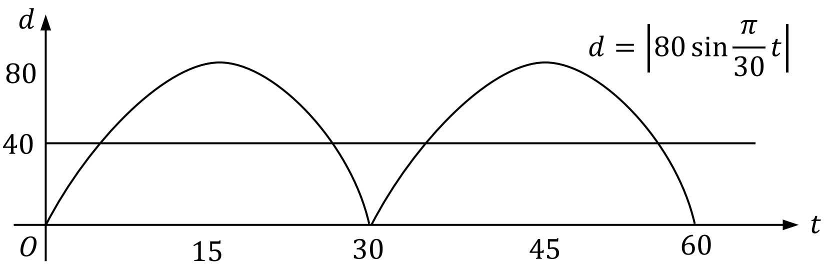
$|\sin 15k| = 1$

$\sin 15k = 1$

$15k = \frac{\pi}{2}$

$k = \frac{\pi}{30}$

(ii) $d > 40$



For $0 < t < 30$,

$\left|80\sin\frac{\pi}{30}t\right| = 40$

$80\sin\frac{\pi}{30}t = 40$

$\sin\frac{\pi}{30}t = \frac{1}{2}$

Basic angle $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

$\frac{\pi}{30}t = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$\frac{\pi}{30}t = \frac{\pi}{6}, \frac{5\pi}{6}$

$t = 5$ or 25

$\therefore$ period in each hour when $d > 40$

$= 2(25-5) = 40$

Question 12

[Ans: (i) shown (ii) $\cos x + \sin 2x$ (iii) $y = -2x + \frac{\pi}{3} + \sqrt{3}$]

(i) $\int_0^{\frac{\pi}{6}} f(x)dx = \frac{3}{4}$

$[\sin x + k\cos 2x]_0^{\frac{\pi}{6}} = \frac{3}{4}$

$\left(\sin\frac{\pi}{6} + k\cos\frac{\pi}{3}\right) - (\sin 0 + k\cos 0) = \frac{3}{4}$

$\left(\frac{1}{2} + \frac{1}{2}k\right) - (0 + k) = \frac{3}{4}$

$-\frac{1}{2}k = \frac{1}{4}$

$k = -\frac{1}{2}$ (shown)

(ii) $\int f(x)dx = \sin x + k\cos 2x + c$

$f(x) = \frac{d}{dx}\left(\sin x - \frac{1}{2}\cos 2x + c\right)$

$= \cos x - \frac{1}{2}(-\sin 2x)(2)$

$= \cos x + \sin 2x$

(iii) $\frac{dy}{dx} = -\sin x + 2\cos 2x$

When $x = \frac{\pi}{6}$,

$y = \cos\frac{\pi}{6} + \sin\left(2 \times \frac{\pi}{6}\right)$

$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \sqrt{3}$

$\frac{dy}{dx} = -\sin\frac{\pi}{6} + 2\cos\left(2 \times \frac{\pi}{6}\right)$

$= -\frac{1}{2} + 2\left(\frac{1}{2}\right) = \frac{1}{2}$

Equation of normal:

$y - \sqrt{3} = -2\left(x - \frac{\pi}{6}\right)$

$y = -2x + \frac{\pi}{3} + \sqrt{3}$