O-LEVEL A-MATHS 2014 – PAPER 1

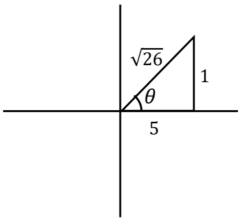
Question 1

$(2-kx)^5+(3+x)^6=860x^3+\cdots$
$\binom{5}{3}(2)^{5-3}(-kx)^3+\binom{6}{3}(3)^{6-3}(x)^3+\cdots=860x^3+\cdots$
$-40k^3x^3+540x^3+\cdots=860x^3+\cdots$
$(-40k^3+540)x^3+\cdots=860x^3+\cdots$

$\therefore -40k^3+540=860$
$-40k^3=320$
$k^3=-8$
$k=-2$

Question 2

$\tan(A+B)=8$
$\frac{\tan A+\tan B}{1-\tan A\tan B}=8$
$\frac{\tan A+3}{1-3\tan A}=8$
$\tan A+3=8-24\tan A$
$25\tan A=5$
$\tan A=\frac{1}{5}$



$\sin A=\frac{1}{\sqrt{26}}$
$=\frac{\sqrt{26}}{26}$

Question 3

$y = 2 - \frac{1}{x^2} = 2 - x^{-2}$

$\frac{dy}{dx} = -(-2)x^{-3} = \frac{2}{x^3}$

When $\frac{dx}{dt} = 0.12$,

$\frac{dy}{dt} = 0.03$ (given)

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$\Rightarrow 0.03 = \frac{2}{x^3}(0.12)$

$x^3 = 8 \Rightarrow x = 2$

$\Rightarrow y = 2 - \frac{1}{2^2} = \frac{7}{4}$

$\therefore$ y-coordinate of the particle at the instant is $\frac{7}{4}$.

Question 4

Let $\frac{(x+2)^2}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$

$\therefore (x+2)^2 = Ax(x-2) + B(x-2) + Cx^2$

Let $x = 0$,

$(0+2)^2 = 0 + B(-2) + 0$

$B = -2$

Let $x = 2$,

$(2+2)^2 = 0 + 0 + C(2)^2$

$C = 4$

Let $x = 1$,

$(1+2)^2 = A(1)(1-2) - 2(1-2) + 4(1)^2$

$9 = -A + 6$

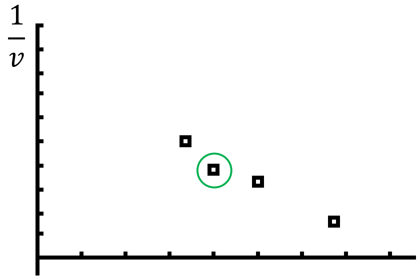
$A = -3$

$\therefore \frac{(x+2)^2}{x^2(x-2)} = -\frac{3}{x} - \frac{2}{x^2} + \frac{4}{x-2}$

Question 5

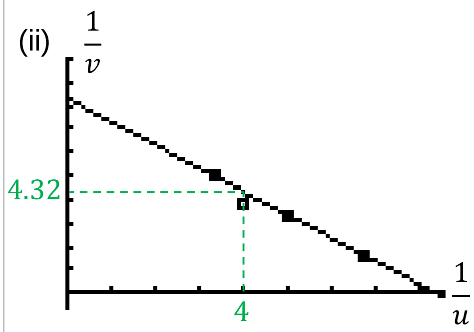
(i) Table:

$\frac{1}{u}$	6.67	5.00	4.00	3.33
$\frac{1}{v}$	1.66	3.34	3.80	4.98



$v = 0.263$ is the incorrect recording.

(ii)



From graph, when $\frac{1}{u} = 4$,

$$\frac{1}{v} \approx 4.32 \Rightarrow v \approx 0.231$$

(iii) $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow \frac{1}{v} = -\frac{1}{u} + \frac{1}{f}$

From the graph,

$$\frac{1}{f} = \text{vertical-intercept}$$

$$\frac{1}{f} = 8.30 \Rightarrow f \approx 0.120$$

Question 6

(i) LHS

$$= \frac{1}{(1+\operatorname{cosec}\theta)(\sec\theta - \tan\theta)}$$
$$= \frac{1}{\left(1+\frac{1}{\sin\theta}\right)\left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)}$$
$$= \frac{1}{\left(\frac{\sin\theta + 1}{\sin\theta}\right)\left(\frac{1-\sin\theta}{\cos\theta}\right)}$$
$$= \frac{1}{\left(\frac{1-\sin^2\theta}{\sin\theta\cos\theta}\right)}$$
$$= \frac{\sin\theta\cos\theta}{1-\sin^2\theta}$$
$$= \frac{\sin\theta\cos\theta}{\cos^2\theta}$$
$$= \frac{\sin\theta}{\cos\theta}$$
$= \tan\theta =$ RHS (proven)

(ii) Given

$$\frac{1}{(1+\operatorname{cosec}\theta)(\sec\theta - \tan\theta)} = 3\cot\theta$$

From (i),

$\tan\theta = 3\cot\theta$

$\tan\theta = \frac{3}{\tan\theta}$

$\tan^2\theta = 3$

$\tan\theta = \sqrt{3}$ ($\tan\theta > 0$ as θ is acute)

$\theta = \tan^{-1}\sqrt{3}$

$\quad = 1.05$

Question 7

(i) At A, $x = h$

$y = 2(h) = 2h$

$\therefore A(h, 2h)$

At C, $x = h$

$y = \frac{1}{2}(h)$

$\therefore C\left(h, \frac{1}{2}h\right)$

y-coordinate of $B = 2h$

Let $B(x, 2h)$,

Gradient of BC = Gradient of OA

$\frac{2h - \frac{1}{2}h}{x - h} = 2 \Rightarrow \frac{3}{2}h = 2x - 2h$

$2x = \frac{7}{2}h \Rightarrow x = \frac{7}{4}h$

$\therefore B\left(\frac{7}{4}h, 2h\right)$

(ii) For $h = 4$,

$A(4, 8)$, $B(7, 8)$, $C(4, 2)$

Area of trapezium $OABC$

$= \frac{1}{2}\begin{vmatrix} 0 & 4 & 7 & 4 & 0 \\ 0 & 2 & 8 & 8 & 0 \end{vmatrix}$

$= \frac{1}{2}[(0 + 32 + 56 + 0) - (0 + 14 + 32 + 0)]$

$= \frac{1}{2}(42) = 21 \text{ units}^2$

Question 8

$f'(x) = \sin 4x - \cos 2x$

$f(x) = \int \sin 4x - \cos 2x \, dx$

$= -\frac{1}{4}\cos 4x - \frac{1}{2}\sin 2x + c$

$f\left(\frac{\pi}{2}\right) = 0$

$-\frac{1}{4}\cos 4\left(\frac{\pi}{2}\right) - \frac{1}{2}\sin 2\left(\frac{\pi}{2}\right) + c = 0$

$-\frac{1}{4} + c = 0 \Rightarrow c = \frac{1}{4}$

$\therefore f(x) = -\frac{1}{4}\cos 4x - \frac{1}{2}\sin 2x + \frac{1}{4}$

$f''(x) = 4\cos 4x + 2\sin 2x$

$f''(x) + 4f(x)$

$= 4\cos 4x + 2\sin 2x + 4\left(-\frac{1}{4}\cos 4x - \frac{1}{2}\sin 2x + \frac{1}{4}\right)$

$= 4\cos 4x + 2\sin 2x - \cos 4x - 2\sin 2x + 1$

$= 3\cos 4x + 1$ (shown)

Question 9

(i) When $a = 2$,

$y = 2x^2 + 5x + 2 - 5$
$= 2x^2 + 5x - 3$

Let $2x^2 + 5x - 3 > 9$
$2x^2 + 5x - 12 > 0$
$(2x - 3)(x + 4) > 0$
$x < -4$ or $x > \frac{3}{2}$

(ii) When $a = 4$,
$y = 4x^2 + 5x + 4 - 5$
$= 4x^2 + 5x - 1$

Let $4x^2 + 5x - 1 = x - 2$
$4x^2 + 4x + 1 = 0$

Discriminant
$= (4)^2 - 4(4)(1) = 0$

∴ The line is tangent to the curve. (shown)

(iii) Let $ax^2 + 5x + a - 5 = x - 2$
$ax^2 + 4x + a - 3 = 0$

Let Discriminant $= 0$
$(4)^2 - 4(a)(a - 3) = 0$
$16 - 4a^2 + 12a = 0$
$(a - 4)(a + 1) = 0$
$a = 4$ or $a = -1$

∴ The other value of $a = -1$.

Question 10

(i) Perimeter $= 130$

$4x + 2l = 130$

$l = \frac{130 - 4x}{2} = 65 - 2x$

Area

$= 2\left(\frac{1}{2}\right)(x)(x)\sin 60° + xl$

$= \frac{\sqrt{3}}{2}x^2 + x(65 - 2x)$

$= \frac{\sqrt{3}}{2}x^2 + 65x - 2x^2 = \frac{130x}{2} - \frac{4x^2}{2} + \frac{\sqrt{3}}{2}x^2$

$= \frac{130x - (4 - \sqrt{3})x^2}{2}$

(shown)

(ii) $\frac{dA}{dx} = \frac{1}{2}\left[130 - 2\left(4 - \sqrt{3}\right)x\right]$

Let $\frac{dA}{dx} = 0$

$\frac{1}{2}\left(130 - 2(4 - \sqrt{3})x\right) = 0$

$x = \frac{130}{2(4 - \sqrt{3})} = 28.7$

(iii) $\frac{d^2A}{dx^2} = -\left(4 - \sqrt{3}\right) < 0$

Since $\frac{d^2A}{dx^2} < 0$, $\therefore$ this value of x will give the largest area possible.

Question 11

(i) $y = x \ln x$

$$\frac{dy}{dx} = x\left(\frac{1}{x}\right) + (1)\ln x = 1 + \ln x$$

Let $\frac{dy}{dx} = 2$,

$1 + \ln x = 2$

$\ln x = 1 \Rightarrow x = e$

$\therefore y = e \ln e = e$

$\therefore P(e, e)$

(ii) Gradient of normal at $P = -\frac{1}{2}$

Equation of normal:

$y - e = -\frac{1}{2}(x - e)$

$y = -\frac{1}{2}x + \frac{3}{2}e \quad (1)$

$y = 2x - 3 \quad (2)$

$(1) = (2)$

$-\frac{1}{2}x + \frac{3}{2}e = 2x - 3$

$\frac{5}{2}x = 3 + \frac{3}{2}e$

$x = \frac{6}{5} + \frac{3}{5}e = \frac{3}{5}(e + 2)$

$\therefore$ x-coordinate of $Q = \frac{3}{5}(e+2)$ (shown), where $k = \frac{3}{5}$.

Question 12

(i) Since $(2x-3)^2 \geq 0$, the smallest value of $y=-4$ when $(2x-3)^2=0$

i.e. $x=\frac{3}{2}$

$\therefore$ Lowest point on the curve has coordinates of $\left(\frac{3}{2},-4\right)$.

(ii) Let $y=0$

$(2x-3)^2-4=0$

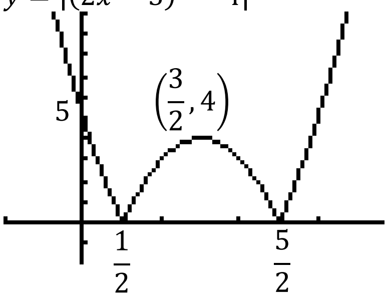
$(2x-3)^2=4$

$2x-3=\pm 2$

$x=\frac{1}{2}$ or $x=\frac{5}{2}$

Coordinates required are $\left(\frac{1}{2},0\right)$ and $\left(\frac{5}{2},0\right)$.

(iii) $y=|(2x-3)^2-4|$



(iv) From observation,

(a) 2

(b) 4

(c) 0