O-LEVEL A-MATHS 2013 – PAPER 1

Question 1

Let $y = 0$,

$$|2x - 3| - 2 = 0$$
$$|2x - 3| = 2$$
$$2x - 3 = -2 \quad \text{or} \quad 2x - 3 = 2$$
$$x = \frac{1}{2} \qquad\qquad x = \frac{5}{2}$$

$\therefore$ x-intercepts are $\left(\frac{1}{2}, 0\right)$ and $\left(\frac{5}{2}, 0\right)$.

Let $x = 0$,

$$y = |-3| - 2 = 1$$

$\therefore$ y-intercept is $(0, 1)$.

Question 2

$$\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right)$$
$$= 2\sin\frac{1}{2}\left(\frac{7\pi}{12} + \frac{\pi}{12}\right)\cos\frac{1}{2}\left(\frac{7\pi}{12} - \frac{\pi}{12}\right)$$
$$= 2\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) = 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$
$$= \sqrt{\frac{3}{2}}$$
$$= \sqrt{\frac{3}{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$
$$= \frac{\sqrt{6}}{2}$$

Question 3

Given $y = x^3 + px^2 + qx + 10$

$$\frac{dy}{dx} = 3x^2 + 2px + q$$

Let $3x^2 + 2px + q = 3(x - 3)(x - 7)$

$$3x^2 + 2px + q = 3(x^2 - 10x + 21)$$
$$3x^2 + 2px + q = 3x^2 - 30x + 63$$

$\therefore 2p = -30 \Rightarrow p = -15$

$\therefore q = 63$

Question 4

(i) LHS

$= \frac{(\sin A - \cos A)(1 + \sin A \cos A)}{\cos^3 A}$

$= \frac{\sin A + \sin^2 A \cos A - \cos A - \sin A \cos^2 A}{\cos^3 A}$

$= \frac{\sin A + (1 - \cos^2 A)\cos A - \cos A - \sin A\,(1 - \sin^2 A)}{\cos^3 A}$

$= \frac{\sin A + \cos A - \cos^3 A - \cos A - \sin A + \sin^3 A}{\cos^3 A}$

$= \frac{\sin^3 A - \cos^3 A}{\cos^3 A}$

$= \frac{\sin^3 A}{\cos^3 A} - \frac{\cos^3 A}{\cos^3 A}$

$= \tan^3 A - 1$

= RHS (shown)

(ii) $(\sin A - \cos A)(1 + \sin A \cos A) = 2\cos^3 A$

$\frac{(\sin A - \cos A)(1 + \sin A \cos A)}{\cos^3 A} = 2$

$\tan^3 A - 1 = 2$

$\tan^3 A = 3$

$\tan A = \sqrt[3]{3}$

$A = \tan^{-1} \sqrt[3]{3} = 55.3°$

Question 5

(i) x^3 term of the expansion

$= \binom{5}{3}(a^2)(-x)^3 + \binom{6}{3}(2^3)x^3$

$= 10a^2(-1)^3x^3 + 160x^3$

$= (-10a^2 + 160)x^3$

$\therefore -10a^2 + 160 = 70$

$a^2 = 9 \Rightarrow a = 3\ (\because a > 0)$

(ii) x^2 term of the expansion

$= \binom{5}{2}(a^3)(-x)^2 + \binom{6}{2}(2^4)x^2$

$= 10a^3x^2 + 240x^2$

$= 10(3^3)x^2 + 240x^2 = 510x^2$

Coefficient of x^2 is 510.

Question 6

Let $\dfrac{7x+2}{(x^2+4)(x-2)}=\dfrac{Ax+B}{x^2+4}+\dfrac{C}{x-2}$

$7x+2=(Ax+B)(x-2)+C(x^2+4)$

Let $x=2$,

$7(2)+2=0+C(2^2+4)$

$8C=16 \Rightarrow C=2$

Let $x=0$,

$0+2=(0+B)(0-2)+2(0+4)$

$2=-2B+8 \Rightarrow B=3$

Let $x=1$,

$7+2=(A+3)(1-2)+2(1^2+4)$

$9=-(A+3)+10 \Rightarrow A=-2$

$\therefore \dfrac{7x+2}{(x^2+4)(x-2)}=\dfrac{-2x+3}{x^2+4}+\dfrac{2}{x-2}$

Question 7

$\alpha+\beta=-\dfrac{p}{2},\ \alpha\beta=-\dfrac{1}{2}$

$\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}=-\left(\dfrac{-5}{1}\right)=5$

$\dfrac{\beta^2+\alpha^2}{\alpha^2\beta^2}=5$

$\dfrac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}=5$

$\dfrac{\left(-\frac{p}{2}\right)^2-2\left(-\frac{1}{2}\right)}{\left(-\frac{1}{2}\right)^2}=5$

$4\left(\dfrac{p^2}{4}+1\right)=5$

$\dfrac{p^2}{4}+1=\dfrac{5}{4}$

$p^2=1$

$p=1\ (\because p>0)$

$\left(\dfrac{1}{\alpha^2}\right)\left(\dfrac{1}{\beta^2}\right)=\dfrac{q}{1}$

$q=\dfrac{1}{(\alpha\beta)^2}$

$\quad=\dfrac{1}{\left(-\frac{1}{2}\right)^2}=4$

Question 8

(i) Let $t = 0$,

$$T = 20 - 38e^{0} = -18$$

The chicken is kept at -18℃ in the freezer.

(ii) When $t = 2$,

$$T = 20 - 38e^{-0.6(2)} = 8.55$$

The temperature of the chicken when $t = 2$ is 8.55℃.

(iii) $T = 20 - 38e^{-0.6t}$

$38e^{-0.6t} = 20 - T$

$e^{-0.6t} = \frac{20 - T}{38}$

$-0.6t = \ln\left(\frac{20 - T}{38}\right)$

$t = -\frac{5}{3}\ln\left(\frac{20 - T}{38}\right)$

(iv) For all real values of t,

$e^{-0.6t} > 0$

$-38e^{-0.6t} < 0$

$20 - 38e^{-0.6t} < 20$

$T < 20$

∴ The temperature of the chicken can never reach 20℃.

Question 9

(i) $y + 3 > 0$

$(2x^2 + 3x - 5) + 3 > 0$

$2x^2 + 3x - 2 > 0$

$(2x - 1)(x + 2) > 0$

$x < -2$ or $x > \frac{1}{2}$

(ii) Let $P(x, y)$

$\frac{dy}{dx} = 4x + 3$

At P,

$\frac{dx}{dt} = 0.04$, $\frac{dy}{dt} = 0.2$ (given)

$\frac{dy}{dx} = 4x + 3$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$\Rightarrow 0.2 = (4x + 3)(0.04)$

$x = \frac{1}{2}$

When $x = \frac{1}{2}$,

$$y = 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) - 5 = -3$$

$\therefore P\left(\frac{1}{2}, -3\right)$

Question 10

(i) $L_1: 4y + x = 48 \Rightarrow x = 48 - 4y \quad (1)$
$L_2: 3y = 4x - 40 \quad (2)$

Sub. (1) into (2)
$3y = 4(48 - 4y) - 40$
$19y = 152 \Rightarrow y = 8$

Sub. $y = 8$ into (1)
$x = 48 - 4(8) = 16$

$\therefore A(16, 8)$ and $M(8, 4)$

For L_2, let $y = 0$
$4x - 40 = 0 \Rightarrow x = 10$

$\therefore C(10, 0)$

Gradient of OM, m_{OM}
$= \frac{4}{8} = \frac{1}{2}$

Gradient of CM, m_{CM}
$= \frac{4 - 0}{8 - 10} = -2$

$m_{OM} m_{CM} = \left(\frac{1}{2}\right)(-2) = -1$

$\therefore \angle OMC$ is $90°$.

(ii) For L_1, let $x = 0$,
$4y = 48 \Rightarrow y = 12$

$\therefore B(0, 12)$

Ratio
$= \frac{1}{2}(12)(16) : \frac{1}{2}(10)(8) = 12 : 5$

Question 11

(i) Given $y = \frac{x^2}{x+2}$

$$\frac{dy}{dx} = \frac{(x+2)(2x) - x^2(1)}{(x+2)^2} = \frac{x^2+4x}{(x+2)^2}$$

$$\frac{d^2y}{dx^2} = \frac{(x+2)^2(2x+4) - (x^2+4x)(2)(x+2)}{[(x+2)^2]^2}$$
$$= \frac{(2x+4)[(x+2)^2 - (x^2+4x)]}{(x+2)^4}$$
$$= \frac{2(x+2)(x^2+4x+4-x^2-4x)}{(x+2)^4}$$
$$= \frac{8}{(x+2)^3}$$

(ii) Let $\frac{dy}{dx} = 0$

$$\frac{x^2+4x}{(x+2)^2} = 0$$
$$x^2+4x = 0$$
$$x(x+4) = 0$$
$x = 0$ or $x = -4$

When $x = 0$,
$$\frac{d^2y}{dx^2} = \frac{8}{(0+2)^3} = 1 > 0$$

∴ When $x = 0$, it is a minimum point.

When $x = -4$,
$$\frac{d^2y}{dx^2} = \frac{8}{(-4+2)^3} = -1 < 0$$

∴ When $x = -4$, it is a maximum point.

Question 12

(i) $\frac{2x}{2x+3}=\frac{2x+3-3}{2x+3}=1-\frac{3}{2x+3}$

(ii) Let $y = x\ln(2x+3)$

$$\frac{dy}{dx} = x\left(\frac{1}{2x+3}\right)(2) + (1)\ln(2x+3)$$
$$= \frac{2x}{2x+3} + \ln(2x+3)$$

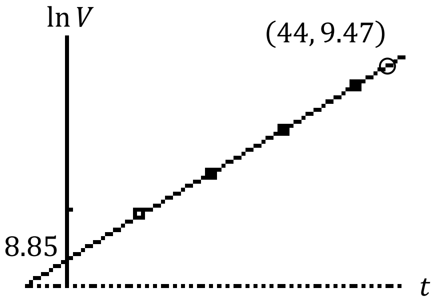
(iii) From (ii),

$$\int \frac{2x}{2x+3} + \ln(2x+3)\,dx = x\ln(2x+3) + A$$
$$\int \frac{2x}{2x+3}\,dx + \int \ln(2x+3)\,dx = x\ln(2x+3) + A$$
$$\int \ln(2x+3)\,dx$$
$$= x\ln(2x+3) - \int \frac{2x}{2x+3}\,dx + A$$
$$= x\ln(2x+3) - \int 1 - \frac{3}{2x+3}\,dx + A$$
$$= x\ln(2x+3) - x + \frac{3}{2}\ln|2x+3| + B$$
$$= \left(x + \frac{3}{2}\right)\ln|2x+3| - x + B$$

Question 13

(i)

t (years)	10	20	30	40
$\ln V$	8.99	9.13	9.27	9.41



(ii) $V = ae^{kt}$
$\ln V = \ln\left(ae^{kt}\right)$
$\ln V = \ln a + kt$

From graph,
$\ln a =$ vertical-intercept
$\ln a \approx 8.85$
$a \approx e^{8.85} = 6970$

$k =$ gradient
$k \approx 0.0141$

(iii) In year 2014,
$t = 44$

From graph, when $t = 44$
$\ln V = 9.47$
$V = e^{9.47} = 12900$

The value of the diamond will be approximately \$12900 in 2014.