‘A’ LEVEL H2 MATHS 2012 – PAPER 2

Question 1

[Ans: (a) $y = 8x^2 - \frac{3}{4}x^4 + Ax + B$ (b) $t = \frac{1}{24}\left(\ln\left|\frac{4+3u}{4-3u}\right| - \ln 7\right)$]

(a) $\frac{d^2y}{dx^2} = 16 - 9x^2$

$$\frac{dy}{dx} = \int 16 - 9x^2\,dx$$
$$= 16x - 3x^3 + A$$
$$y = \int 16x - 3x^3 + A\,dx$$
$$= 8x^2 - \frac{3}{4}x^4 + Ax + B$$

(b) $\frac{du}{dt} = 16 - 9u^2$

$$\frac{1}{16 - 9u^2}\frac{du}{dt} = 1$$
$$\int \frac{1}{4^2 - (3u)^2}\,du = \int dt$$
$$\frac{1}{3}\left(\frac{1}{2 \times 4}\right)\ln\left|\frac{4+3u}{4-3u}\right| + C = t$$
$$t = \frac{1}{24}\ln\left|\frac{4+3u}{4-3u}\right| + C$$

When $t = 0$, $u = 1$,

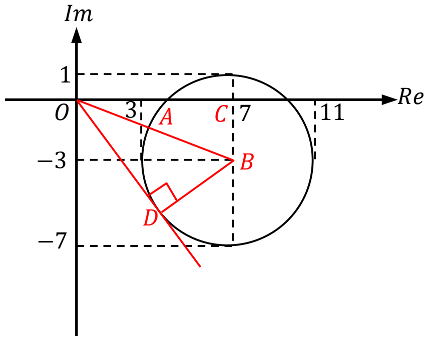
$$\frac{1}{24}\ln\left|\frac{4+3}{4-3}\right| + C = 0$$
$$C = -\frac{1}{24}\ln 7$$

$$\therefore t = \frac{1}{24}\ln\left|\frac{4+3u}{4-3u}\right| - \frac{1}{24}\ln 7$$
$$= \frac{1}{24}\left(\ln\left|\frac{4+3u}{4-3u}\right| - \ln 7\right)$$

Question 2

[Ans: (i) sketch (ii)(a) $\sqrt{58}-4$ (b) $\frac{7}{\sqrt{58}}\left(\sqrt{58}-4\right)-i\frac{3}{\sqrt{58}}\left(\sqrt{58}-4\right)$ (iii) -0.9579]

(i)



(ii) (a) $|z| = OB - AB = \sqrt{3^2+7^2} - 4 = \sqrt{58} - 4$

(b) $z = \left(\sqrt{58}-4\right)[\cos(-\angle BOC) + i\sin(-\angle BOC)]$

$z = \left(\sqrt{58}-4\right)[\cos(\angle BOC) - i\sin(\angle BOC)]$

$z = \left(\sqrt{58}-4\right)\left(\frac{7}{\sqrt{58}} - i\frac{3}{\sqrt{58}}\right)$

$z = \frac{7}{\sqrt{58}}\left(\sqrt{58}-4\right) - i\frac{3}{\sqrt{58}}\left(\sqrt{58}-4\right)$

(iii) $\angle BOC = \tan^{-1}\left(\frac{3}{7}\right)$

$\angle BOD = \sin^{-1}\left(\frac{4}{\sqrt{58}}\right)$

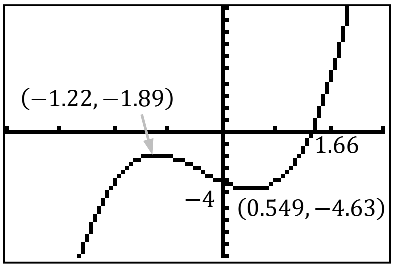
$\therefore \arg z = -\left[\tan^{-1}\left(\frac{3}{7}\right) + \sin^{-1}\frac{4}{\sqrt{58}}\right]$

$= -0.9579$

Question 3

[Ans: (i) sketch (ii) 2; prove (iii) -1 (iv) sketch (v) $x = -2,\ 0,\ 1$ or 2]

(i) From GC,



(ii) $f(x) = 4$

$x^3 + x^2 - 2x - 4 = 4$

From GC,

$x = 2$

By long division,

$f(x) = 4$

$\Rightarrow (x - 2)(x^2 + 3x + 4) = 0$

For $x^2 + 3x + 4 = 0$

Discriminant

$= 3^2 - 4(1)(4) = -7 < 0$

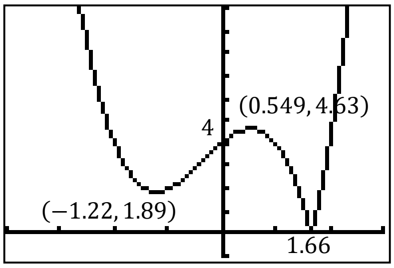
$\Rightarrow x^2 + 3x + 4 = 0$ has no solution

$\therefore f(x) = 4$ has only one solution $x = 2$ and no other real solutions.

(iii) Let $x + 3 = 2$

$x = -1$

(iv) From GC,



(v) First cubic equation:

$f(x) = 4$

$x^3 + x^2 - 2x - 4 = 4$

$x = 2$

Second cubic equation:

$-f(x) = 4$

$-(x^3 + x^2 - 2x - 4) = 4$

$x^3 + x^2 - 2x = 0$

$x(x^2 + x - 2) = 0$

$x(x + 2)(x - 1) = 0$

$x = -2,\ 0$ or 1

Roots of $|f(x)| = 4$ are $-2,\ 0,\ 1$ and 2

Question 4

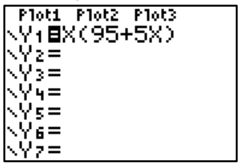
[(i) 1 December 2002 (ii) 45th (iii) 1.80%]

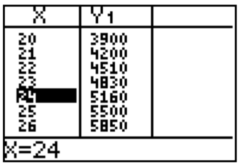
(i) $100 + 110 + 120 + \cdots > 5000$

$\frac{n}{2}[2(100) + (n-1)(10)] > 5000$

$n(95 + 5n) > 5000$

From GC,





Least $n = 24$

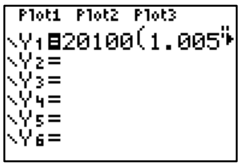
∴ Mrs A's account first become greater than $\$5000$ on 1 December 2002.

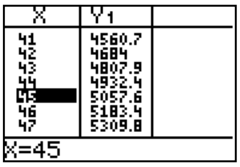
(ii)

Month	Total value at end of month
1	$100(1.005)$
2	$[100 + 100(1.005)](1.005)$ $= 100(1.005) + 100(1.005)^2$
3	$[100 + 100(1.005) + 100(1.005)^2](1.005)$ $= 100(1.005) + 100(1.005)^2 + 100(1.005)^3$
⋮	
n	$\frac{100(1.005)(1.005^n - 1)}{1.005 - 1}$ $= 20100(1.005^n - 1)$

Let $20100(1.005^n - 1) > 5000$

From GC,





Least $n = 45$

∴ During the end of the 45th month, the value of Mr B's account first became greater than $\$5000$.

(iii) Let the interest rate to achieve the result be i.

The number of months from 1 January 2001 to 2 December 2003 where an interest is paid

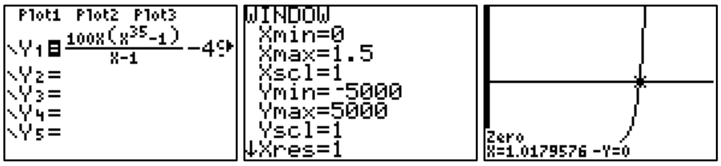
$= 12 \times 3 - 1 = 35$

Let $x = 1 + \frac{i}{100}$.

$\therefore \frac{100x(x^{35} - 1)}{x - 1} + 100 = 5000$

$\frac{100x(x^{35} - 1)}{x - 1} - 4900 = 0$

From GC,



$x = 1.0179576$

$$1 + \frac{i}{100} = 1.0179576$$
$i = 1.80\%$

Question 5

[Ans: (i)(a) 0.00599 (b) 0.166 (ii) 0.999666]

(i) (a) Required probability

$$= 0.001p + 0.999(1 - p)$$
$$= 0.001(0.995) + 0.999(1 - 0.995)$$
$$= 0.00599$$

(b) Required probability

$$= P(has\ disease | test\ positive)$$
$$= \frac{P(has\ disease\ \cap\ test\ positive)}{P(test\ positive)}$$
$$= \frac{0.001(0.995)}{0.00599} = 0.166$$

(ii) $P(has\ disease | test\ positive) = 0.75$

$$\frac{0.001p}{0.001p + 0.999(1 - p)} = 0.75$$
$$\frac{0.001p}{0.999 - 0.998p} = 0.75$$
$$0.001p = 0.74925 - 0.7485p$$
$$p = 0.999666$$

Question 6

[Ans: (i) $H_0: \mu = 14.0$, $H_1: \mu \neq 14.0$ (ii) $\{x \in \mathbb{R}: 12.3 < \bar{x} < 15.7\}$ (iii) insufficient evidence]

(i) Let μ be the mean tail lengths of the squirrels population.
$H_0: \mu = 14.0$
$H_1: \mu \neq 14.0$

(ii) $n = 20$
$\bar{x} = k$
$\sigma^2 = 3.8^2$

Test statistics,

$$\overline{X} \sim N\left(14.0, \frac{3.8^2}{20}\right)$$

To not reject H_0,

p-value > 0.05

$2P(\bar{X} < k) > 0.05$
$P(\bar{X} < k) > 0.025$
$k > 12.335$

or

$2P(\bar{X} > k) > 0.05$
$P(\bar{X} > k) > 0.025$
$1 - P(\bar{X} < k) > 0.025$
$P(\bar{X} < k) < 0.975$
$k < 15.665$

$\therefore \{x \in \mathbb{R}: 12.3 < \bar{x} < 15.7\}$

(iii) Since $\bar{x} = 15.8$ is outside the non-critical region in (ii), there is sufficient evidence to reject H_0 at 5% level of significance. i.e. the mean tail length may not be the same as what she has known.

Question 7

[Ans: (i) $\frac{2}{15}$ (ii) $\frac{34}{35}$ (iii) $\frac{2}{455}$ (iv) $\frac{43}{273}$ (v) $\frac{1}{7}$]

(i) [SS]XXXXXXXXXXXXX

Required probability

$$=\frac{14!\,2!}{15!}=\frac{2}{15}$$

(ii) [BBB]XXXXXXXXXXXX

Required probability

$$=1-\frac{13!\,3!}{15!}=\frac{34}{35}$$

(iii) [SS]XX[BBB]XXXXXXXX

Required probability

$$=\frac{12!\,2!\,3!}{15!}=\frac{2}{455}$$

(iv) Probability for brothers to be next to each other, XXXX[BBB]XXXXXXXX

$$=\frac{13!\,3!}{15!}=\frac{1}{35}$$

Required probability

$$=\frac{2}{15}+\frac{1}{35}-\frac{2}{455}=\frac{43}{273}$$

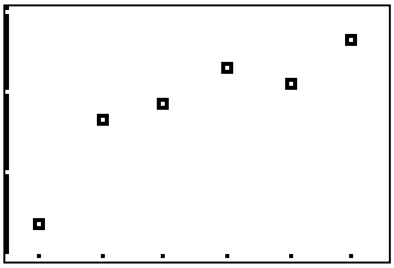
(v) Required probability

$$=\frac{(14-1)!\,2!}{(15-1)!}=\frac{1}{7}$$

Question 8

[Ans: (i) sketch (ii) reason (iii) explain (iv) 0.929744 (v) 92, reason (vi) $a = 4.10$, $b = -0.280$, 13^{th} week (vii) interpretation]

(i) From GC,

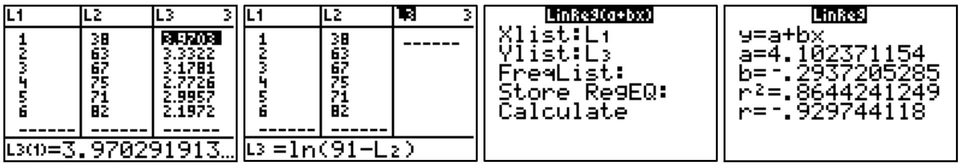


(ii) Amy may be sick during that week.

(iii) A linear model is inappropriate as it would predict Amy's marks to increase indefinitely as she practices for more weeks.

A quadratic model is also inappropriate as it may either fail in the same manner as a linear model or it may predict negative marks for after a few more weeks.

(iv) From GC,



$r = -0.929744$

(v) 92 is the most appropriate value for L as the value of $|r|$ generated is closest to 1.

(vi) From GC,

$a = 4.10$, $b = -0.280$

$\ln(92 - y) = 4.1045 - 0.27960x$

When $y = 90$,

$\ln(92 - 90) = 4.1045 - 0.27960x$

$x = 12.2$

Amy will obtain her first mark of at least 90% during the 13^{th} week.

(vii) L could be the highest mark Amy has ever obtained.

Question 9

[Ans: (i) assumption (ii) 0.373 (iii)(a) possible (b) not possible (iv) $k = 0.238$, $p = 0.39$]

(i) The probability of each voters supporting Alliance Party must be independent of each other, and it must be the same for each of them.

(ii) $A \sim B(30, 0.15)$

$P(A = 3 \; or \; 4)$

$= P(A = 3) + P(A = 4) = 0.373$

(iii) $A \sim B(30, 0.55)$

$n = 30$ (large)

$np = 16.5 > 5$

$n(1 - p) = 13.5 > 5$

(a) It is possible to approximate A with a normal distribution.

(b) It is not possible to approximate A with a Poisson distribution.

(iv) $A \sim B(30, p)$

$P(A = 15) = 0.06864$

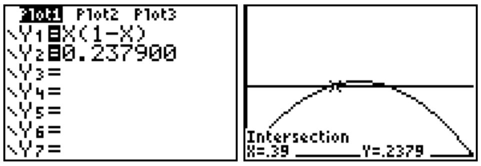
$\binom{30}{15} p^{15}(1 - p)^{15} = 0.06864$

$$[p(1 - p)]^{15} = \frac{0.06864}{\binom{30}{15}}$$

$$p(1 - p) = \left[\frac{0.06864}{\binom{30}{15}}\right]^{\frac{1}{15}} = 0.23790$$

$\therefore k = 0.238$

From GC,



$p = 0.39$

Question 10

[Ans: (i) conditions (ii) 0.0474 (iii) $x = 0.324$ (iv) 0.144 (v) 0.245 (vi) 0.912]

(i) The number of gold coins found must be independent of another, and the probability of finding two gold coins at the same location is negligible.

(ii) Let A be the number of gold coins found in 1 square metre.

$$A \sim Po(0.8)$$

Required probability
$= P(A \geq 3) = 1 - P(A \leq 2) = 0.0474$

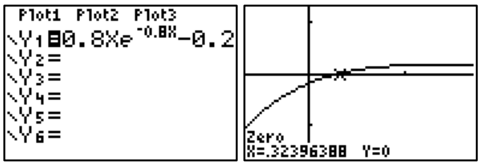
(iii) Let B be the number of gold coins in x square metres.

$$B \sim Po(0.8x)$$

$P(B = 1) = 0.2$

$$e^{-0.8x}\frac{(0.8x)^1}{1!} = 0.2 \Rightarrow 0.8xe^{-0.8x} - 0.2 = 0$$

From GC,



$x = 0.324$

(iv) Let C be the number of gold coins in 100 square metres.

$$C \sim Po(0.8 \times 100) \Rightarrow C \sim Po(80)$$

$\lambda = 80 > 10$

$$\therefore C \sim N(80, 80) \text{ approx.}$$

Required probability
$= P(C \geq 90)$
$= P(C > 89.5)$ (c.c.)
$= 0.144$

(v) Let D be the number of gold coins in 50 square metres.

$$D \sim Po(0.8 \times 50) \Rightarrow D \sim Po(40)$$

$\lambda = 40 > 10$

$$\therefore D \sim N(40, 40) \text{ approx.}$$

Let E be the number of pottery shards found in 50 square metres.

$$E \sim Po(3 \times 50) \Rightarrow E \sim Po(150)$$

$\lambda = 150 > 10$
$\therefore E \sim N(150, 150)$ approx.

$E(D + E) = 40 + 150 = 190$
$Var(D + E) = 40 + 150 = 190$

$$\therefore D + E \sim N(190, 190) \text{ approx.}$$

Required probability
$= P(D + E \geq 200)$
$= P(D + E > 199.5)$ (c.c.)
$= 0.245$

(vi) $E(E-3D) = 150 - 3(40) = 30$
$Var(E-3D) = 150 + 3^2(40) = 510$

$\therefore E - 3D \sim N(30, 510)$ approx.

Required probability
$= P(E \geq 3D)$
$= P(E - 3D \geq 0)$
$= P(E - 3D > -0.5)$ (c.c.)
$= 0.912$