O-LEVEL A-MATHS 2012 – PAPER 1

Question 1

$$\frac{dy}{dx} = \frac{(x+3)(2)-(1)(2x+1)}{(x+3)^2}$$
$$= \frac{2x+6-2x-1}{(x+3)^2}$$
$$= \frac{5}{(x+3)^2}$$

When $x = 2$,

$$\frac{dx}{dt} = 0.4$$
$$\frac{dy}{dx} = \frac{5}{(2+3)^2} = \frac{1}{5}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$
$$= \left(\frac{1}{5}\right)(0.4)$$
$$= 0.08$$

Question 2

From ΔABC,

$$\cos\frac{\pi}{6} = \frac{AB}{8}$$
$$\Rightarrow AB = 8\cos\frac{\pi}{6}$$
$$= 8\left(\frac{\sqrt{3}}{2}\right) = 4\sqrt{3}$$

$$\sin\frac{\pi}{6} = \frac{BC}{8}$$
$$\Rightarrow BC = 8\sin\frac{\pi}{6}$$
$$= 8\left(\frac{1}{2}\right) = 4$$

From ΔBCM,

$$CM = \sqrt{BM^2 + BC^2}$$
$$= \sqrt{\left(2\sqrt{3}\right)^2 + 4^2}$$
$$= \sqrt{28} = 2\sqrt{7}$$

From ΔACM,

$$\frac{CM}{\sin\frac{\pi}{6}} = \frac{AM}{\sin\angle ACM}$$
$$\frac{2\sqrt{7}}{1/2} = \frac{2\sqrt{3}}{\sin\angle ACM}$$
$$\sin\angle ACM = \frac{\sqrt{3}}{2\sqrt{7}}$$
$$\sin\angle ACM = \frac{\sqrt{21}}{14}$$
$$\Rightarrow \angle ACM = \sin^{-1}\frac{\sqrt{21}}{14}, \qquad k = 21$$

Question 3

(i) $\frac{d}{dx}\left(x^2\ln x\right)$

$$= x^2\left(\frac{1}{x}\right) + 2x\ln x$$
$$= x + 2x\ln x$$

(ii) $\int x + 2x\ln x dx = x^2\ln x + a$

$$\int x\,dx + 2\int x\ln x\,dx = x^2\ln x + a$$
$$\frac{1}{2}x^2 + 2\int x\ln x\,dx = x^2\ln x + a$$
$$2\int x\ln x\,dx = x^2\ln x - \frac{1}{2}x^2 + a$$
$$\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + c$$

Question 4

$$\begin{pmatrix}3 & -5\\ 7 & 1\end{pmatrix}^{-1}$$
$$= \frac{1}{(3)(1)-(7)(-5)}\begin{pmatrix}1 & 5\\ -7 & 3\end{pmatrix}$$
$$= \frac{1}{38}\begin{pmatrix}1 & 5\\ -7 & 3\end{pmatrix}$$

$5a - 3b + 2 = 0 \Rightarrow 3b - 5a = 2$
$a + 7b - 11 = 0 \Rightarrow 7b + a = 11$

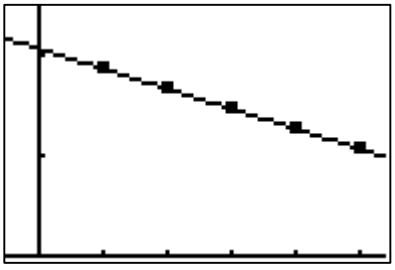
$$\Rightarrow \begin{pmatrix}3 & -5\\ 7 & 1\end{pmatrix}\begin{pmatrix}b\\ a\end{pmatrix} = \begin{pmatrix}2\\ 11\end{pmatrix}$$
$$\begin{pmatrix}b\\ a\end{pmatrix} = \begin{pmatrix}3 & -5\\ 7 & 1\end{pmatrix}^{-1}\begin{pmatrix}2\\ 11\end{pmatrix}$$
$$= \frac{1}{38}\begin{pmatrix}1 & 5\\ -7 & 3\end{pmatrix}\begin{pmatrix}2\\ 11\end{pmatrix}$$
$$= \begin{pmatrix}3/2\\ 1/2\end{pmatrix}$$

$a = \frac{1}{2}, b = \frac{3}{2}$

Question 5

(i)

t	1	2	3	4	5
$\ln x$	1.88	1.68	1.48	1.28	1.08



(ii) $x = Me^{-kt}$
$\ln x = \ln(Me^{-kt})$
$\ln x = \ln M - kt$

From graph,
$\ln M \approx$ vertical-intercept
$\ln M \approx 2.08$
$M \approx e^{2.08} = 8.00$

(iii) When $t = 0$,
$x = M = 8.00$
Let $x = 0.5M = 4.00$,
$\ln x = 1.39$

From graph, when $\ln x = 1.38$,
$t \approx 3.45$ min

Question 6

(i) $2\sin 2\theta\,(\sec\theta - \tan\theta)$
$= 4\sin\theta\cos\theta\left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)$
$= 4\sin\theta - 4\sin^2\theta$

(ii) $2\sin 2\theta\,(\sec\theta - \tan\theta) + 3 = 0$
$4\sin\theta - 4\sin^2\theta + 3 = 0$
$4\sin^2\theta - 4\sin\theta - 3 = 0$
$(2\sin\theta + 1)(2\sin\theta - 3) = 0$
$\sin\theta = -\frac{1}{2}$ or $\sin\theta = \frac{3}{2}$ (NA)

Basic ∠
$= \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

$\theta = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$
$= \frac{7\pi}{6}, \frac{11\pi}{6}$

Question 7

$(2x+q)(x+p)^6$
$= (2x+q)\left(x^6 + 6x^5p + \binom{6}{2}x^4p^2 + \cdots\right)$
$= (2x+q)(x^6 + 6px^5 + 15p^2x^4 + \cdots)$
$= 12px^6 + 30p^2x^5 + qx^6 + 6pqx^5 + \cdots$
$= (12p+q)x^6 + (30p^2 + 6pq)x^5 + \cdots$

$12p + q = -7 \quad (1)$

$30p^2 + 6pq = 0$
$6p(5p+q) = 0$
$q = -5p \quad (2)$

Sub. (2) into (1),
$12p - 5p = -7$
$p = -1$

Sub. $p = -1$ into (2),
$q = -5(-1) = 5$

Question 8

(i) At point $\left(\frac{2\pi}{3}, 2\right)$,

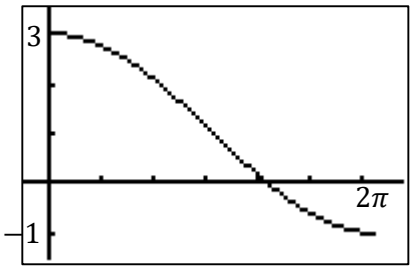
$$2\cos\left(\frac{2\pi}{3} \div 2\right) + c = 2$$

$$2\left(\frac{1}{2}\right) + c = 2 \Rightarrow c = 1$$

(ii) Amplitude $= 2$

Period $= \frac{2\pi}{1/2} = 4\pi$

(iii) $y = 2\cos\left(\frac{x}{2}\right) + 1$



Question 9

(i) $v = 12(t+1)^{-2} - 3$

$a = \frac{dv}{dt} = 12(-2)(t+1)^{-3} = -\frac{24}{(t+1)^3}$

(ii) At instantaneous rest,

$v = 0$

$\frac{12}{(t+1)^2} - 3 = 0$

$(t+1)^2 = 4$

$t + 1 = \pm 2$

$t = -3$ (NA) or $t = 1$

$s = \int 12(t+1)^{-2} - 3\,dt$

$= 12\frac{(t+1)^{-1}}{-1} - 3t + c$

$= -\frac{12}{t+1} - 3t + c$

When $t = 0$,

$s = 0$

$-\frac{12}{0+1} - 0 + c = 0$

$c = 12$

$\therefore s = -\frac{12}{t+1} - 3t + 12$

At instantaneous rest,

$t = 1$

$s = -\frac{12}{1+1} - 3(1) + 12 = 3$

Distance travelled

$= 3 - 0$

$= 3$

Question 10

(i) $\frac{dy}{dx} = 3x^2 - 6x - 24$

Let $\frac{dy}{dx} = 0$,

$3x^2 - 6x - 24 = 0$
$x^2 - 2x - 8 = 0$
$(x + 2)(x - 4) = 0$
$x = -2$ or $x = 4$

(ii) $\frac{d^2y}{dx^2} = 6x - 6$

When $x = -2$,

$\frac{d^2y}{dx^2} = -12 - 6 = -18 < 0$

When $x = 4$,

$\frac{d^2y}{dx^2} = 24 - 6 = 18 > 0$

∴ The minimum point of the curve happens when $x = 4$.

When $x = 4$,

$y = 0$
$4^3 - 3(4^2) - 24(4) + c = 0$
$c = 80$

Question 11

$(\alpha + 3\beta) + (3\alpha + \beta) = 16$
$\Rightarrow \alpha + \beta = 4$

$(\alpha + 3\beta)(3\alpha + \beta) = 52$
$3\alpha^2 + \alpha\beta + 9\alpha\beta + 3\beta^2 = 52$
$3(\alpha^2 + \beta^2) + 10\alpha\beta = 52$
$3[(\alpha + \beta)^2 - 2\alpha\beta] + 10\alpha\beta = 52$
$3(\alpha + \beta)^2 + 4\alpha\beta = 52$
$3(4)^2 + 4\alpha\beta = 52$
$\alpha\beta = 1$

∴ New equation:

$x^2 - 4x + 1 = 0$

Question 12

(i) $9\left(3^{x}\right)^{3}=\sqrt{\dfrac{1}{27^{x}}}$

$9(3^{3x}) = (3^{-3x})^{\frac{1}{2}}$

$9(3^{3x}) = 3^{-\frac{3x}{2}}$

$3^{3x}3^{\frac{3x}{2}} = \dfrac{1}{9}$

$3^{\frac{9x}{2}} = 3^{-2}$

$\dfrac{9x}{2} = -2$

$x = -\dfrac{4}{9}$

(ii) $\dfrac{a+b\sqrt{3}}{5+2\sqrt{3}}=\dfrac{5+2\sqrt{3}}{2+\sqrt{3}}$

$a + b\sqrt{3} = \dfrac{\left(5 + 2\sqrt{3}\right)^{2}}{2 + \sqrt{3}}$

$= \dfrac{25 + 20\sqrt{3} + 12}{2 + \sqrt{3}}$

$= \dfrac{37 + 20\sqrt{3}}{2 + \sqrt{3}}\left(\dfrac{2 - \sqrt{3}}{2 - \sqrt{3}}\right)$

$= \dfrac{74 - 37\sqrt{3} + 40\sqrt{3} - 60}{4 - 3}$

$= 14 + 3\sqrt{3}$

$\therefore a = 14$ and $b = 3$

Question 13

(i) $x^2 + y^2 - 6x + 8y - 375 = 0$
$(x-3)^2 - 3^2 + (y+4)^2 - 4^2 - 375 = 0$
$(x-3)^2 + (y+4)^2 = 400$

$\therefore C(3, -4), r = 20$

(ii) Gradient of CP
$= \dfrac{12-(-4)}{-9-3} = -\dfrac{4}{3}$

Equation of CP:
$y + 4 = -\dfrac{4}{3}(x-3)$
$y = -\dfrac{4}{3}x$

When $x = 0$, $y = 0$

$\therefore CP$ passes through the origin O.

(iii) $OP = \sqrt{9^2 + 12^2} = 15$

Equation of circle:
$\left[x - \left(-\dfrac{9}{2}\right)\right]^2 + \left(y - \dfrac{12}{2}\right)^2 = \left(\dfrac{15}{2}\right)^2$
$\left(x + \dfrac{9}{2}\right)^2 + (y-6)^2 = \dfrac{225}{4}$