O-LEVEL A-MATHS 2010 – PAPER 1

Question 1

[Ans: (i) -2 (ii) 24]

(i) Given $f(x) = x^4 - x^3 + kx - 4$

$f(2) = 0$
$(2)^4 - (2)^3 + k(2) - 4 = 0$
$2k = -4$
$k = -2$

(ii) $f(x) = x^4 - x^3 - 2x - 4$

Reminder
$= f(-2) = (-2)^4 - (-2)^3 - 2(-2) - 4$
$= 24$

Question 2

[Ans: (i) show (ii) $\frac{\pi}{2} + 1$]

(i) LHS
$= (\sin x + \cos x)^2$
$= \sin^2 x + 2\sin x \cos x + \cos^2 x$
$= (\sin^2 x + \cos^2 x) + 2\sin x \cos x$
$= 1 + \sin 2x =$ RHS (shown)

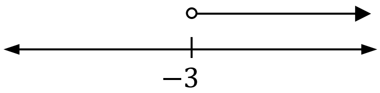
(ii) $\int_0^{\frac{\pi}{2}} (\sin x + \cos x)^2 \, dx$

$$= \int_0^{\frac{\pi}{2}} 1 + \sin 2x \, dx$$
$$= \left[x - \frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}}$$
$$= \left(\frac{\pi}{2} - \frac{1}{2}\cos \pi\right) - \left(0 - \frac{1}{2}\cos 0\right)$$
$$= \left[\frac{\pi}{2} - \left(-\frac{1}{2}\right)\right] - \left[0 - \frac{1}{2}\right]$$
$$= \frac{\pi}{2} + 1$$

Question 3

[Ans: (i) $x > -3$ (ii) $x < -2$ or $x > \frac{7}{3}$; $-3 < x < -2$ or $x > \frac{7}{3}$]

(i) $3(2-x) < x + 18$
$6 - 3x < x + 18$
$4x > -12$
$x > -3$



(ii) $3(x^2 - 5) > x - 1$
$3x^2 - x - 14 > 0$
$(3x - 7)(x + 2) > 0$
$x < -2$ or $x > \frac{7}{3}$

To satisfy (i) & (ii), values of x:
$-3 < x < -2$ or $x > \frac{7}{3}$

Question 4

[Ans: (i) $\frac{5}{2}$ (ii) 0.15 units per second]

(i) $\frac{dy}{dx} = 2\cos 2x + 3\sin x$
Gradient
$= 2\cos\frac{\pi}{3} + 3\sin\frac{\pi}{6} = 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)$
$= \frac{5}{2}$

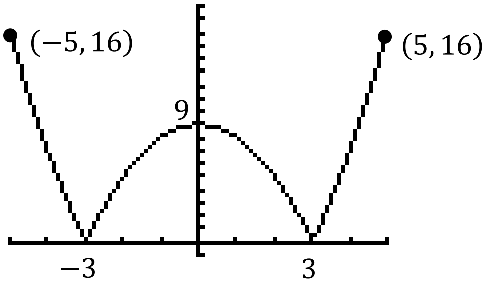
(ii) When $x = \frac{\pi}{6}$,
$\frac{dy}{dx} = \frac{5}{2}$
$\frac{dx}{dt} = 0.06$

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$
$= \left(\frac{5}{2}\right)(0.06)$
$= 0.15$

Question 5

[Ans: (i) sketch (ii) ± 6]

(i)



(ii) Let $|9 - x^2| = 27$

$9 - x^2 = 27$ or $9 - x^2 = -27$

$x^2 = -18$ (NA) $\quad$ $x^2 = 36$

$x = \pm 6$

Question 6

[Ans: (i) $2xe^{2x} + e^{2x}$ (ii) show]

(i) $\frac{d}{dx}\left(xe^{2x}\right)$

$= x(e^{2x})(2) + (1)e^{2x}$

$= 2xe^{2x} + e^{2x}$

(ii) $\frac{d}{dx}\left(xe^{2x}\right) = 2xe^{2x} + e^{2x}$

$[xe^{2x}]_0^1 = \int_0^1 2xe^{2x} + e^{2x}\, dx$

$\int_0^1 2xe^{2x}\, dx + \left[\frac{1}{2}e^{2x}\right]_0^1 = (e^2 - 0)$

$2\int_0^1 xe^{2x}\, dx + \frac{1}{2}e^2 - \frac{1}{2} = e^2$

$2\int_0^1 xe^{2x}\, dx = \frac{1}{2} + \frac{1}{2}e^2$

$\int_0^1 xe^{2x}\, dx = \frac{1 + e^2}{4}$

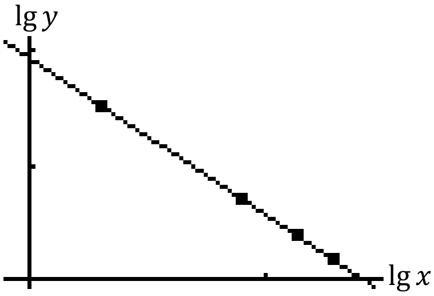
(shown)

Question 7

[Ans: (i) plot (ii) $k \approx 84.1$, $n \approx 1.35$]

(i)

$\lg x$	0.301	0.903	1.146	1.301
$\lg y$	1.519	0.705	0.377	0.167



(ii) $yx^n = k$

$\lg(yx^n) = \lg k$

$\lg y + n \lg x = \lg k$

$\lg y = -n \lg x + \lg k$

From graph,

Gradient $= -1.351$

$\therefore n = 1.35$

$\lg y$-intercept $= 1.925$

$\therefore \lg k = 1.925$

$k = 10^{1.925} = 84.1$

Question 8

[Ans: (i) $-3 \le x \le 1$ (ii) $k = -27$ or 5]

(i) $y = x^3 + 3x^2 - 9x + k$

$\frac{dy}{dx} = 3x^2 + 6x - 9$

Let $\frac{dy}{dx} \le 0$,

$3x^2 + 6x - 9 \le 0$

$x^2 + 2x - 3 \le 0$

$(x+3)(x-1) \le 0$

$-3 \le x \le 1$

(ii) Let $\frac{dy}{dx} = 0$

$3x^2 + 6x - 9 = 0$

$x = -3$ or $x = 1$

When $x = -3$,

let $y = 0$

$(-3)^3 + 3(-3)^2 - 9(-3) + k = 0$

$k = -27$

When $x = 1$,

let $y = 0$

$(1)^3 + 3(1)^2 - 9(1) + k = 0$

$k = 5$

Question 9

[Ans: $9x^2 - 18x + 11 = 0$]

Given $3x^2 - 2x + 1 = 0$

$\alpha + \beta = \frac{2}{3}, \quad \alpha\beta = \frac{1}{3}$

Sum of new roots

$= (\alpha + 2\beta) + (2\alpha + \beta)$

$= 3(\alpha + \beta)$

$= 3\left(\frac{2}{3}\right) = 2$

Product of new roots

$= (\alpha + 2\beta)(2\alpha + \beta)$

$= 2\alpha^2 + 5\alpha\beta + 2\beta^2$

$= 2(\alpha^2 + \beta^2) + 5\alpha\beta$

$= 2[(\alpha + \beta)^2 - 2\alpha\beta] + 5\alpha\beta$

$= 2\left[\left(\frac{2}{3}\right)^2 - 2\left(\frac{1}{3}\right)\right] + 5\left(\frac{1}{3}\right) = \frac{11}{9}$

New equation:

$x^2 - 2x + \frac{11}{9} = 0$

$9x^2 - 18x + 11 = 0$

Question 10

[Ans: (i) show (ii) show]

(i) $\tan 75°$

$= \tan(30° + 45°)$

$= \dfrac{\tan 30° + \tan 45°}{1 - \tan 30° \tan 45°}$

$= \dfrac{\frac{1}{\sqrt{3}} + 1}{1 - \left(\frac{1}{\sqrt{3}}\right)(1)}$

$= \dfrac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}}$

$= \dfrac{\frac{1+\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$

$= \dfrac{1+\sqrt{3}}{\sqrt{3}-1}\left(\dfrac{\sqrt{3}+1}{\sqrt{3}+1}\right)$

$= \dfrac{\sqrt{3}+1+3+\sqrt{3}}{\left(\sqrt{3}\right)^2 - (1)^2}$

$= \dfrac{4+2\sqrt{3}}{2}$

$= 2 + \sqrt{3}$ (shown)

(ii) $\sec^2 75°$

$= 1 + \tan^2 75° = 1 + \left(2+\sqrt{3}\right)^2$

$= 1 + 4 + 4\sqrt{3} + 3$

$= 8 + 4\sqrt{3}$

$= 4\left(2+\sqrt{3}\right)$

$= 4\tan 75°$ (shown)

Question 11

[Ans: (i) $y=-\frac{8}{x}-2x+15$ (ii) ± 2 (iii) when $x=2$, max. point; when $x=-2$, min. point]

(i) Given $\frac{dy}{dx}=\frac{8}{x^2}-2$

$y=\int \frac{8}{x^2}-2\,dx$

$y=-\frac{8}{x}-2x+c$

At $(1,5)$,

$5=-\frac{8}{1}-2+c \Rightarrow c=15$

$\therefore y=-\frac{8}{x}-2x+15$

(ii) Let $\frac{dy}{dx}=0$

$\frac{8}{x^2}-2=0$

$x^2=4$

$x=\pm 2$

(iii) $\frac{d^2y}{dx^2}=-\frac{16}{x^3}$

When $x=-2$,

$\frac{d^2y}{dx^2}=2>0$

This is a minimum point.

When $x=2$,

$\frac{d^2y}{dx^2}=-2<0$

This is a maximum point.

Question 12

[Ans: (i) $(x+3)^2+(y-2)^2=25$ (ii) 8 units (iii) 2 (iv) 8 (v) $g=-8$, $f=-2$, $c=-12$]

(i) Equation:

$(x+3)^2+(y-2)^2=5^2$

$(x+3)^2+(y-2)^2=25$

(ii) When $x=0$,

$(0+3)^2+(y-2)^2=25$

$(y-2)^2=25-9=16$

$y-2=\pm 4 \Rightarrow y=-2$ or 6

$\therefore PQ=6+2=8$

(iii) y-coordinate of B

$=\dfrac{6-2}{2}=2$

(iv) Let centre of second circle be $B(a,2)$.

Distance $BP=\sqrt{80}$

$\sqrt{(a-0)^2+(2-6)^2}=\sqrt{80}$

$a^2+16=80$

$a^2=64$

$a=-8$ (NA) or $a=8$

$\therefore$ x-coordinate of B is 8

(v) Equation of second circle:

$(x-8)^2+(y-2)^2=80$

$x^2-16x+64+y^2-4y+4=80$

$x^2+y^2-16x-4y-12=0$

$x^2+y^2+2(-8)x+2(-2)y+(-12)=0$

$\therefore g=-8$, $f=-2$ and $c=-12$