CAMBRIDGE/2009/PAPER2

1. $\sin(A-B)=\frac{3}{8}$

$\sin A\cos B-\cos A\sin B=\frac{3}{8}$

(i) $\frac{5}{8}-\cos A\sin B=\frac{3}{8}$

$\cos A\sin B=\frac{5}{8}-\frac{3}{8}=\frac{1}{4}$

(ii) $\sin(A+B)$
$=\sin A\cos B+\cos A\sin B$
$=\frac{5}{8}+\frac{1}{4}=\frac{7}{8}$

(iii) $\frac{\tan A}{\tan B}$
$=\frac{\sin A/\cos B}{\sin B/\cos B}=\frac{\sin A\cos B}{\sin B\cos A}$
$=\frac{5/8}{1/4}=\frac{5}{2}$

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2. (i) $\frac{7}{2x^2-x-6}=\frac{7}{(2x+3)(x-2)}$

Let $\frac{7}{(2x+3)(x-2)}=\frac{A}{2x+3}+\frac{B}{x-2}$

$7=A(x-2)+B(2x+3)$

Let $x=-\frac{3}{2}$,

$7=A\left(-\frac{3}{2}-2\right)\Rightarrow A=-2$

Let $x=2$,

$7=B(4+3)\Rightarrow B=1$

$\therefore \frac{7}{2x^2-x-6}=\frac{1}{x-2}-\frac{2}{2x+3}$

(ii) $\int_3^9 \frac{7}{2x^2-x-6}dx$
$=\int_3^9 \frac{1}{x-2}-\frac{2}{2x+3}dx$
$=\left[\ln|x-2|-2\frac{\ln|2x+3|}{2}\right]_3^9$
$=(\ln 7-\ln 21)-(\ln 1-\ln 9)$
$=\ln\left[\frac{7}{21}(9)\right]=\ln 3$

3. (i) $8^x-2^{x+2}=15$
$(2^x)^3-(2^2)(2^x)-15=0$
For $u=2^x$,
$u^3-4u-15=0$

(ii) Let $f(u)=u^3-4u-15$
$f(3)=3^3-4(3)-15=0$
$\therefore u=3$ is a root of the equation.

$$
\begin{array}{r}
u^2+3u+5 \\
u-3\,\overline{)\,u^3+0u^2-4u-15} \\
\underline{-(u^3-3u^2)} \\
3u^2-4u \\
\underline{-(3u^2-9u)} \\
5u-15 \\
\underline{-(5u-15)} \\
0
\end{array}
$$

$\therefore u^3-4u-15=0$
$(u-3)(u^2+3u+5)=0$
$u-3=0$ or $u^2+3u+5=0$

$u=\frac{-3\pm\sqrt{9-20}}{2}$
$=\frac{-3\pm\sqrt{-11}}{2}$
(no real solution)

$\therefore u=3$ is the only real solution of the equation. (shown)

(iii) $8^x-2^{x+2}=15$
$\Rightarrow 2^x=3$
$x\ln 2=\ln 3\Rightarrow x=\frac{\ln 3}{\ln 2}=1.58$

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4. (i) Given $AC=BC$ and $PC=QC$
$\angle ACP=\angle BCQ$ (Common $\angle$s)
$\therefore \Delta ACP$ is congruent to ΔBCQ (SAS congruent)
$\Rightarrow \angle CAP=\angle CBQ$ (corr. angles of congruent Δs)

ΔABC is an isosceles triangle ($AC=BC$).
$\therefore \angle CAB=\angle CBA$ (Base $\angle$s of isosc. Δs)

$\angle XAB=\angle CAB-\angle CAP$
$=\angle CBA-\angle CBQ=\angle XBA$
$\therefore \Delta AXB$ is an isosceles triangle (Base $\angle$s of isosc. Δs)

(ii) $AP=BQ$ (corr. sides of congruent Δs)
$AX=BX$ (corr. sides of isosceles Δs)
$\therefore PX=QX$

5. (i) $\left(2-\frac{x}{4}\right)^n$

$= 2^n + (n)(2^{n-1})\left(-\frac{x}{4}\right) + \frac{n(n-1)}{2!}(2^{n-2})\left(-\frac{x}{4}\right)^2 + \cdots$

$= 2^n - \frac{n2^{n-1}}{4}x + \frac{n(n-1)2^{n-2}}{32}x^2 + \cdots$

$= 2^n - n2^{n-3}x + n(n-1)2^{n-7}x^2 + \cdots$

(ii) $(1+x)\left(2-\frac{x}{4}\right)^n$

$= (1+x)(2^n - n2^{n-3}x + n(n-1)2^{n-7}x^2 + \cdots)$

$= 2^n - n2^{n-3}x + n(n-1)2^{n-7}x^2 + 2^n x - n2^{n-3}x^2 + \cdots$

$= 2^n + (2^n - n2^{n-3})x + [n(n-1)2^{n-7} - n2^{n-3}]x^2 + \cdots$

∴ By comparison,

$2^n - n2^{n-3} = 0$

$2^n\left(1-\frac{n}{8}\right) = 0$

$\frac{n}{8} = 1 \Rightarrow n = 8$

(iii) $a = 2^8 = 256$

$b = (8)(7)2^1 - (8)2^5 = -144$

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6. (i) Let $x = \frac{2\pi}{3}$,

$y = 1 + 2\cos\left(\frac{2\pi}{3}\right) = 1 + 2\left(-\frac{1}{2}\right) = 0$

∴ x-coordinate of A is $\frac{2\pi}{3}$ (shown)

Let $y = 0$

$1 + 2\cos x = 0$

$\cos x = -\frac{1}{2}$

$x = \pi - \cos^{-1}\frac{1}{2}, \pi + \cos^{-1}\frac{1}{2}$

$= \frac{2\pi}{3}, \frac{4\pi}{3}$

∴ x-coordinate of B is $\frac{4\pi}{3}$

(iii) Total area

$= \int_0^{\frac{2\pi}{3}} 1 + 2\cos x\,dx + \left|\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} 1 + 2\cos x\,dx\right|$

$= [x + 2\sin x]_0^{\frac{2\pi}{3}} + \left|[x + 2\sin x]_{\frac{2\pi}{3}}^{\frac{4\pi}{3}}\right|$

$= \left(\frac{2\pi}{3} + 2\sin\frac{2\pi}{3}\right) + \left|\left(\frac{4\pi}{3} + 2\sin\frac{4\pi}{3}\right) - \left(\frac{2\pi}{3} + 2\sin\frac{2\pi}{3}\right)\right|$

$= 5.20$ units^2

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7. (i) Let $x = 0$,

$y = |0-5| - 2 = 5 - 2 = 3$

∴ y-intercept is at $(0, 3)$

Let $y = 0$,

$|3x-5| - 2 = 0$

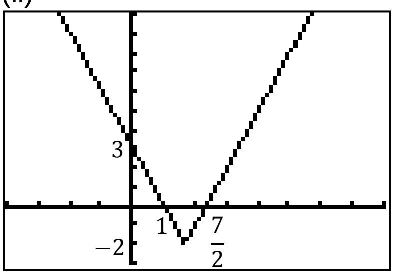
$|3x-5| = 2$

$3x - 5 = 2$ or $3x - 5 = -2$

$x = \frac{7}{2}$ $x = 1$

∴ x-intercepts are at $\left(\frac{7}{2}, 0\right)$ and $(1, 0)$.

(ii)



(iii) $x = |3x-5| - 2$

$|3x-5| = x + 2$

$3x - 5 = x + 2$ or $3x - 5 = -(x+2)$

$2x = 7$ $4x = 3$

$x = \frac{7}{2}$ $x = \frac{3}{4}$

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8. (i) $v = \frac{ds}{dt}$

$= 400\left(-e^{-\frac{t}{10}}\left(-\frac{1}{10}\right)\right) - 16$

$= 40e^{-\frac{t}{10}} - 16$

(ii) $a = \frac{dv}{dt}$

$= 40e^{-\frac{t}{10}}\left(-\frac{1}{10}\right) = -4e^{-\frac{t}{10}}$

(iii) When $t = 0$,

$v = 40e^0 - 16 = 24$ m/s

(iv) Let $v = 0$

$40e^{-\frac{t}{10}} - 16 = 0$

$e^{-\frac{t}{10}} = \frac{16}{40} = \frac{2}{5}$

$-\frac{t}{10} = \ln\frac{2}{5}$

$t = -10\ln\frac{2}{5} = 9.163$

(shown)

(v) When $t = 0$,

$s = 400(1 - e^0) - 16(0) = 0$

When $t = -10\ln\frac{2}{5}$,

$s = 400\left(1 - e^{\ln\frac{2}{5}}\right) - 16\left(-10\ln\frac{2}{5}\right)$

$= 93.3935$

Average speed

$= \frac{93.3935}{-10\ln\frac{2}{5}}$

$= 10.2$ m/s

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9. (i) Equation of circle:

$(x-2)^2 + (y+1)^2 = 5^2$

$x^2 - 4x + 4 + y^2 + 2y + 1 - 25 = 0$

$x^2 + y^2 - 4x + 2y - 20 = 0 \quad (1)$

$g = -2, f = 1, c = -20$

(ii) $A(-3, -1)$

(iii) Gradient of AB

$= \frac{-1-0}{-3-0} = \frac{1}{3}$

∴ equation of AB:

$y = \frac{1}{3}x \quad (2)$

(iv) Sub. (2) into (1)

$x^2 + \left(\frac{1}{3}x\right)^2 - 4x + 2\left(\frac{1}{3}x\right) - 20 = 0$

$\frac{10}{9}x^2 - \frac{10}{3}x - 20 = 0$

$x^2 - 3x - 18 = 0$

$(x-6)(x+3) = 0$

$x = -3$ or $x = 6$

When $x = 6$,

$y = \frac{1}{3}(6) = 2$

$\therefore B(6, 2)$

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10. $\frac{d^2y}{dx^2} = 6x - 6$

$\frac{dy}{dx} = \int 6x - 6\,dx = 3x^2 - 6x + c$

At $(3, 10)$,

$\frac{dy}{dx} = 12$

$3(3)^2 - 6(3) + c = 12 \Rightarrow c = 3$

$\therefore \frac{dy}{dx} = 3x^2 - 6x + 3$

$y = \int 3x^2 - 6x + 3\,dx$

$= x^3 - 3x^2 + 3x + d$

At $(3, 10)$,

$(3)^3 - 3(3)^2 + 3(3) + d = 10$

$d = 1$

$\therefore y = x^3 - 3x^2 + 3x + 1$

Let $\frac{dy}{dx} = 0$

$3x^2 - 6x + 3 = 0$

$x^2 - 2x + 1 = 0 \Rightarrow (x-1)^2 = 0$

$x = 1$

x	1^-	1	1^+
$\frac{dy}{dx}$	$+$	0	$+$
Shape	/	–	/

When $x = 1$,

$y = 1 - 3 + 3 + 1 = 2$

∴ Coordinates of the stationary point is $(1, 2)$ and it is a stationary point of inflexion.

11. (i) $\angle OAB = \theta$

$\cos\theta = \frac{AB}{17} \Rightarrow AB = 17\cos\theta$

$\sin\theta = \frac{OB}{17} \Rightarrow OB = 17\sin\theta$

$\cos\theta = \frac{OC}{31} \Rightarrow OC = 31\cos\theta$

$\sin\theta = \frac{CD}{31} \Rightarrow CD = 31\sin\theta$

$\therefore AB + BC + CD$
$= AB + (OC - OB) + CD$
$= 17\cos\theta + (31\cos\theta - 17\sin\theta)$
$+ 31\sin\theta$
$= 48\cos\theta + 14\sin\theta$ (shown)

(ii) Let
$48\cos\theta + 14\sin\theta$
$= R\sin(\theta + \alpha)$
$= R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

$R\cos\alpha = 14 \quad (1)$
$R\sin\alpha = 48 \quad (2)$

$(2)/(1)$

$\tan\alpha = \frac{48}{14}\alpha = \tan^{-1}\frac{24}{7}$

$(1)^2 + (2)^2$

$R^2 = 14^2 + 48^2 \Rightarrow R = 50$

$\therefore 48\cos\theta + 14\sin\theta$
$= 50\sin\left(\theta + \tan^{-1}\frac{24}{7}\right)$

$AB + BC + CD = 49$
$48\cos\theta + 14\sin\theta = 49$
$50\sin\left(\theta + \tan^{-1}\frac{24}{7}\right) = 49$
$\sin\left(\theta + \tan^{-1}\frac{24}{7}\right) = \frac{49}{50}$
$\theta + \tan^{-1}\frac{24}{7} = \sin^{-1}\frac{49}{50}, 180 - \sin^{-1}\frac{49}{50}$
$\theta = 4.8°, 27.7°$

(iii) Maximum value of $AB + BC + CD$
$= 50$ cm

This occur when:
$\sin\left(\theta + \tan^{-1}\frac{24}{7}\right) = 1$
$\theta + \tan^{-1}\frac{24}{7} = 90°$
$\theta = 16.3°$