CAMBRIDGE/2008/PAPER1

1. (i) $\angle ABX = 180° - 120° = 60°$

$\sin\angle ABX = \dfrac{AX}{4} \Rightarrow AX = 4\sin 60°$

$= 4\left(\dfrac{\sqrt{3}}{2}\right) = 2\sqrt{3}$

(ii) $\cos 60° = \dfrac{BX}{4} \Rightarrow BX = 4\cos 60° = 2$

$\tan\angle ACB = \dfrac{AX}{CX} = \dfrac{2\sqrt{3}}{2+2} = \dfrac{\sqrt{3}}{2}$

$\angle ACB = \tan^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

(shown)

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2. $9^x(27)^y = 1$

$3^{2x}3^{3y} = 3^0$

$3^{2x+3y} = 3^0$

$\Rightarrow 2x + 3y = 0 \Rightarrow y = -\dfrac{2}{3}x \quad (1)$

$8^y \div \left(\sqrt{2}\right)^x = 16\sqrt{2}$

$2^{3y} \div 2^{\frac{1}{2}x} = 2^4 2^{\frac{1}{2}}$

$2^{3y-\frac{1}{2}x} = 2^{\frac{9}{2}}$

$\Rightarrow 3y - \dfrac{1}{2}x = \dfrac{9}{2} \Rightarrow 6y - x = 9 \quad (2)$

Sub. (1) into (2)

$6\left(-\dfrac{2}{3}x\right) - x = 9 \Rightarrow -5x = 9$

$x = -\dfrac{9}{5}$

Sub. $x = -\dfrac{9}{5}$ into (1)

$y = -\dfrac{2}{3}\left(-\dfrac{9}{5}\right) = \dfrac{6}{5}$

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3. $\boldsymbol{A} = \begin{pmatrix} 7 & -8 \\ 1 & 6 \end{pmatrix}$

$\boldsymbol{A}^{-1} = \dfrac{1}{(7)(6)-(1)(-8)}\begin{pmatrix} 6 & 8 \\ -1 & 7 \end{pmatrix}$

$= \begin{pmatrix} \frac{3}{25} & \frac{4}{25} \\ -\frac{1}{50} & \frac{7}{50} \end{pmatrix}$

$8p - 7q + 11 = 0 \Rightarrow 7q - 8p = 11$

$6p + q + 7 = 0 \Rightarrow q + 6p = -7$

$\therefore \begin{pmatrix} 7 & -8 \\ 1 & 6 \end{pmatrix}\begin{pmatrix} q \\ p \end{pmatrix} = \begin{pmatrix} 11 \\ -7 \end{pmatrix}$

$\begin{pmatrix} q \\ p \end{pmatrix} = \begin{pmatrix} \frac{3}{25} & \frac{4}{25} \\ -\frac{1}{50} & \frac{7}{50} \end{pmatrix}\begin{pmatrix} 11 \\ -7 \end{pmatrix} = \begin{pmatrix} \frac{1}{5} \\ -\frac{6}{5} \end{pmatrix}$

$q = \dfrac{1}{5}, p = -\dfrac{6}{5}$

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4. (i) $\dfrac{d}{dx}\left(x^3 \ln x\right)$

$= x^3\left(\dfrac{1}{x}\right) + 3x^2 \ln x$

$= x^2 + 3x^2 \ln x$

(ii) From (i),

$\int x^2 + 3x^2 \ln x \, dx = x^3 \ln x + c_1$

$\int x^2\, dx + 3\int x^2 \ln x \, dx = x^3 \ln x + c_1$

$\dfrac{1}{3}x^3 + 3\int x^2 \ln x\, dx = x^3 \ln x + c_1$

$3\int x^2 \ln x\, dx = x^3 \ln x - \dfrac{1}{3}x^3 + c_1$

$\int x^2 \ln x\, dx = \dfrac{1}{3}x^3 \ln x - \dfrac{1}{9}x^3 + c_2$

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5. (i) Let

$\dfrac{8x - 46}{(x-5)(x+1)} = \dfrac{A}{x-5} + \dfrac{B}{x+1}$

$8x - 46 = A(x+1) + B(x-5)$

Let $x = 5$,

$8(5) - 46 = A(5+1) \Rightarrow A = -1$

Let $x = -1$,

$8(-1) - 46 = B(-1-5) \Rightarrow B = 9$

$\therefore \dfrac{8x-46}{(x-5)(x+1)} = -\dfrac{1}{x-5} + \dfrac{9}{x+1}$

(ii) $y = -(x-5)^{-1} + 9(x+1)^{-1}$

$\dfrac{dy}{dx} = (x-5)^{-2} - 9(x+1)^{-2}$

$= \dfrac{1}{(x-5)^2} - \dfrac{9}{(x+1)^2}$

When $x = 2$,

$\dfrac{dy}{dx} = \dfrac{1}{(2-5)^2} - \dfrac{9}{(2+1)^2} = -\dfrac{8}{9}$

6. (i) At B, $v = 0$

$6t - \frac{1}{2}t^2 = 0$

$t\left(6 - \frac{1}{2}t\right) = 0$

$t = 0$ or $t = 12$

$\therefore$ It took 12s to travel from A to B.

(ii) $s = \int 6t - \frac{1}{2}t^2 dt$

$= 6\left(\frac{t^2}{2}\right) - \frac{1}{2}\left(\frac{t^3}{3}\right) + c$

$= 3t^2 - \frac{1}{6}t^3 + c$

At A,

$s = 0 + 0 + c = c$

At B,

$s = 3(12)^2 - \frac{1}{6}(12)^3 + c = 144 + c$

$\therefore$ Distance $AB = (144 + c) - c = 144$

(iii)

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7. $\frac{dy}{dx} = \frac{(2 - \cos x)\cos x - \sin x(\sin x)}{(2 - \cos x)^2}$

$= \frac{2\cos x - \cos^2 x - \sin^2 x}{(2 - \cos x)^2} = \frac{2\cos x - 1}{(2 - \cos x)^2}$

For tangent of curve parallel to x-axis,

$\frac{dy}{dx} = 0$

$\frac{2\cos x - 1}{(2 - \cos x)^2} = 0$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$

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8. (i) LHS

$= \sin 3x + \sin x$

$= 2\sin\left(\frac{3x + x}{2}\right)\cos\left(\frac{3x - x}{2}\right)$

$= 2\sin 2x \cos x$

$= 2(2\sin x \cos x)\cos x$

$= 4\sin x \cos^2 x$

(ii) $\sin 3x + \sin x = 2\cos^2 x$

$4\sin x \cos^2 x = 2\cos^2 x$

$2\sin x \cos^2 x - \cos^2 x = 0$

$\cos^2 x\,(2\sin x - 1) = 0$

$\cos x = 0$ or $\sin x = \frac{1}{2}$

$x = \frac{\pi}{2}$

Basic $\angle$

$= \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

$x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$= \pi, \frac{5\pi}{6}$

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9. Let Ann's age be A and Betty's age be B.

Given $A > B$

$A^2 - 2B^2 = 6(A - B) \quad (1)$

$A + B = 5(A - B)$

$4A = 6B \Rightarrow A = \frac{3}{2}B \quad (2)$

Sub. (2) into (1)

$A^2 - 2B^2 = 6(A - B)$

$\left(\frac{3}{2}B\right)^2 - 2B^2 = 6\left(\frac{3}{2}B - B\right)$

$\frac{9}{4}B^2 - 2B^2 = 3B$

$\frac{1}{4}B^2 = 3B$

$B = 12$

Sub. $B = 12$ into (2)

$A = \frac{3}{2}(12) = 18$

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10. (a) $a > 0$, and

Discriminant < 0

$5^2 - 4(a)(2) < 0$

$8a > 25 \Rightarrow a > 3.125$

Smallest integer $a = 4$

(b) Discriminant < 0

$b^2 - 4(-5)(-2) < 0$

$b^2 - 40 < 0$

$(b - \sqrt{40})(b + \sqrt{40}) < 0$

$-\sqrt{40} < b < \sqrt{40} \Rightarrow -6.325 < b < 6.325$

Smallest integer $b = -6$

11. (i) T_{r+1}

$= \binom{7}{r} x^{7-r} \left(\frac{k}{x}\right)^r = \binom{7}{r} k^r x^{7-2r}$

Let $7 - 2r = 3 \Rightarrow r = 2$

$T_3 = \binom{7}{2} k^2 x^3 = 21k^2x^3$

Let $7 - 2r = 1 \Rightarrow r = 3$

$T_4 = \binom{7}{3} k^3 x = 35k^3x$

$21k^2 = 35k^3 \Rightarrow k = \frac{3}{5}$

(ii) $(1-5x^2)\left(x + \frac{3}{5x}\right)^7$

$= (1 - 5x^2)\left(x^7 + 7(x^6)\left(\frac{3}{5x}\right) + \cdots\right)$

$= (1 - 5x^2)\left(x^7 + \frac{21}{5}x^5 + \cdots\right)$

$= x^7 - 21x^7 + \cdots$

$= -20x^7 + \cdots$

Coefficient of $x^7 = -20$

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12. (i) $y = kb^x$

$\lg y = \lg(kb^x)$

$\lg y = \lg k + x \lg b$

$\lg k = 1.3 \Rightarrow k = 10^{1.3} = 20$

$\lg b = \frac{1.3 - 0.8}{0 - 11} = -\frac{1}{22}$

$b = 10^{-\frac{1}{22}} = 0.90$

(ii) From (i),

$\lg y = 1.3 - \frac{1}{22}x$

When $x = 8$,

$\lg y = 1.3 - \frac{1}{22}(8)$

$\lg y = \frac{103}{110} \Rightarrow y = 10^{\frac{103}{110}} = 8.64$

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13. (i) Perimeter $= 360$

$h + 5x + 5x + h + 6x = 360$

$2h + 16x = 360$

$2h = 360 - 16x \Rightarrow h = 180 - 8x$

Height of ΔQRS

$= \sqrt{(5x)^2 - (3x)^2} = \sqrt{16x^2} = 4x$

$A = (6x)(h) + \frac{1}{2}(6x)(4x)$

$= 6x(180 - 8x) + 12x^2$

$= 1080x - 48x^2 + 12x^2$

$= 1080x - 36x^2$

(shown)

(ii) $\frac{dA}{dx} = 1080 - 72x$

When A is stationary,

$\frac{dA}{dx} = 0$

$1080 - 72x = 0 \Rightarrow x = 15$

When $x = 15$,

Stationary A

$= 1080(15) - 36(15)^2 = 8100$

(iii) $\frac{d^2A}{dx^2} = -72 < 0$

A is a maximum.