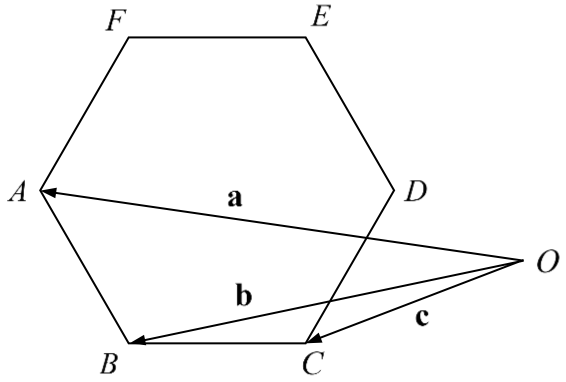
VECTORS (IN 2D)

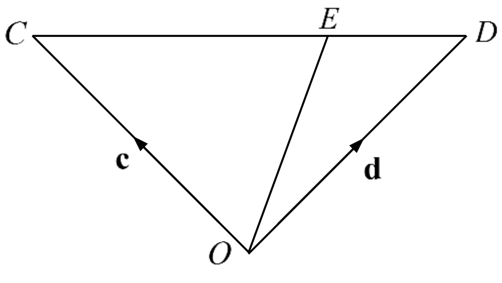
1. In the diagram, $ABCDEF$ is a regular hexagon. The vectors, $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are the position vectors of A, B and C with respect to the point O respectively. Express the following vectors in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.
   (a) $\overrightarrow{FE}$
   (b) $\overrightarrow{AD}$
   (c) $\overrightarrow{AF}$
   (d) $\overrightarrow{BF}$



[Ans: (a) $\boldsymbol{c}-\boldsymbol{b}$ (b) $2(\boldsymbol{c}-\boldsymbol{b})$ (c) $\boldsymbol{a}-2\boldsymbol{b}+\boldsymbol{c}$ (d) $2\boldsymbol{a}-3\boldsymbol{b}+\boldsymbol{c}$]

(a) $\overrightarrow{FE}=\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}=\mathbf{c}-\mathbf{b}$ (b) $\overrightarrow{AD}=2\overrightarrow{FE}=2(\mathbf{c}-\mathbf{b})$ (c) $\overrightarrow{AD}=2(\mathbf{c}-\mathbf{b})$ $\overrightarrow{OD}-\overrightarrow{OA}=2(\mathbf{c}-\mathbf{b})$ $\overrightarrow{OD}=2(\mathbf{c}-\mathbf{b})+\mathbf{a}$ $\overrightarrow{AF}=\overrightarrow{CD}=\overrightarrow{OD}-\overrightarrow{OC}$ $=[2(\mathbf{c}-\mathbf{b})+\mathbf{a}]-\mathbf{c}=\mathbf{a}-2\mathbf{b}+\mathbf{c}$	(d) $\overrightarrow{AF}=\mathbf{a}-2\mathbf{b}+\mathbf{c}$ $\overrightarrow{OF}-\overrightarrow{OA}=\mathbf{a}-2\mathbf{b}+\mathbf{c}$ $\overrightarrow{OF}=2\mathbf{a}-2\mathbf{b}+\mathbf{c}$ Alternatively $\overrightarrow{OF}=\overrightarrow{OB}+\overrightarrow{BA}+\overrightarrow{AF}$ $=\mathbf{b}+(\mathbf{a}-\mathbf{b})+(\mathbf{a}-2\mathbf{b}+\mathbf{c})$ $=2\mathbf{a}-2\mathbf{b}+\mathbf{c}$ $\overrightarrow{BF}=\overrightarrow{OF}-\overrightarrow{OB}$ $=(2\mathbf{a}-2\mathbf{b}+\mathbf{c})-\mathbf{b}$ $=2\mathbf{a}-3\mathbf{b}+\mathbf{c}$

2. In the diagram, $CE=ED=4:3$. Find $\overrightarrow{OE}$ in terms of $\mathbf{c}$ and $\mathbf{d}$.



[Ans: $\frac{3}{7}\boldsymbol{c}+\frac{4}{7}\boldsymbol{d}$]

$\overrightarrow{CD}=\overrightarrow{OD}-\overrightarrow{OC}=\mathbf{d}-\mathbf{c}$ $\overrightarrow{CE}=\frac{4}{7}\overrightarrow{CD}=\frac{4}{7}(\mathbf{d}-\mathbf{c})$	$\overrightarrow{OE}=\overrightarrow{OC}+\overrightarrow{CE}$ $=\mathbf{c}+\frac{4}{7}(\mathbf{d}-\mathbf{c})=\frac{3}{7}\mathbf{c}+\frac{4}{7}\mathbf{d}$

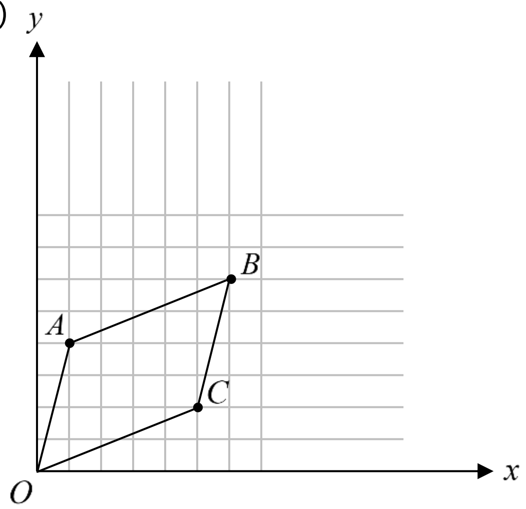
3. $OABC$ is a parallelogram where O is the origin, $\overrightarrow{OA}=\begin{pmatrix}1\\4\end{pmatrix}$ and $\overrightarrow{OC}=\begin{pmatrix}5\\2\end{pmatrix}$.

(a) On the diagram, clearly label the points A, B and C.

(b) Express as column vector, $\overrightarrow{OB}$ and $\overrightarrow{CA}$.

[Ans: (a) label (b) $\overrightarrow{OB}=\begin{pmatrix}6\\6\end{pmatrix}$; $\overrightarrow{CA}=\begin{pmatrix}-4\\2\end{pmatrix}$]

(a)



(b) $\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}=\overrightarrow{OA}+\overrightarrow{OC}$

$=\begin{pmatrix}1\\4\end{pmatrix}+\begin{pmatrix}5\\2\end{pmatrix}=\begin{pmatrix}6\\6\end{pmatrix}$

$\overrightarrow{CA}=\overrightarrow{OA}-\overrightarrow{OC}$

$=\begin{pmatrix}1\\4\end{pmatrix}-\begin{pmatrix}5\\2\end{pmatrix}=\begin{pmatrix}-4\\2\end{pmatrix}$

4. (a) If $\mathbf{p}=\begin{pmatrix}2\\3\end{pmatrix}$ and $\mathbf{q}=\begin{pmatrix}-3\\5\end{pmatrix}$, find $|\mathbf{p}+\mathbf{q}|$.

(b) The position vectors of A and B are $\mathbf{a}=\begin{pmatrix}3\\4\end{pmatrix}$ and $\mathbf{b}=\begin{pmatrix}-2\\5\end{pmatrix}$. If $\overrightarrow{OC}=2\mathbf{a}+3\mathbf{b}$ and $\overrightarrow{OD}=2\mathbf{a}-2\mathbf{b}$, find the coordinates of C and of D.

[Ans: (a) $\sqrt{65}$ units (b) $C(0,23)$, $D(10,-2)$]

(a) $\mathbf{p}+\mathbf{q}=\begin{pmatrix}2\\3\end{pmatrix}+\begin{pmatrix}-3\\5\end{pmatrix}=\begin{pmatrix}-1\\8\end{pmatrix}$

$|\mathbf{p}+\mathbf{q}|=\sqrt{(-1)^2+(8)^2}=\sqrt{65}$

(b) $\overrightarrow{OC}=2\begin{pmatrix}3\\4\end{pmatrix}+3\begin{pmatrix}-2\\5\end{pmatrix}$

$=\begin{pmatrix}6\\8\end{pmatrix}+\begin{pmatrix}-6\\15\end{pmatrix}=\begin{pmatrix}0\\23\end{pmatrix}$

$\therefore C(0,23)$

$\overrightarrow{OD}=2\begin{pmatrix}3\\4\end{pmatrix}-2\begin{pmatrix}-2\\5\end{pmatrix}$

$=\begin{pmatrix}6\\8\end{pmatrix}-\begin{pmatrix}-4\\10\end{pmatrix}=\begin{pmatrix}10\\-2\end{pmatrix}$

$\therefore D(10,-2)$

5. It is given that $\mathbf{p}=\begin{pmatrix}3\\-5\end{pmatrix}$, $\mathbf{q}=\begin{pmatrix}4\\7\end{pmatrix}$ and $\mathbf{r}=\begin{pmatrix}8\\m\end{pmatrix}$.

(a) Express the following as column vectors.

(i) $3\mathbf{p}+\mathbf{q}$

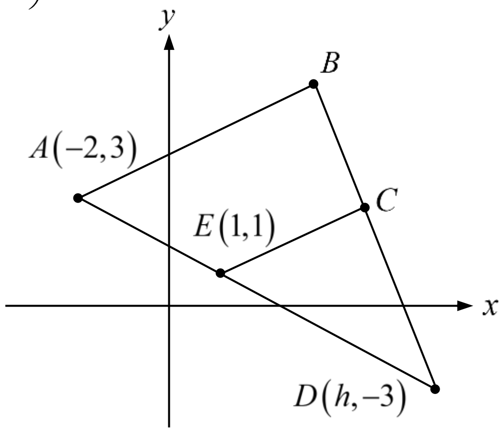
(ii) $\mathbf{p}-2\mathbf{r}$

(b) If $3\mathbf{p}+\mathbf{q}$ and $\mathbf{p}-2\mathbf{r}$ are parallel, find the value of m.

[Ans: (a)(i) $\begin{pmatrix}13\\-8\end{pmatrix}$ (ii) $\begin{pmatrix}-13\\-5-2m\end{pmatrix}$ (b) $-\frac{13}{2}$]

(a) (i) $3\mathbf{p}+\mathbf{q}$ $=3\begin{pmatrix}3\\-5\end{pmatrix}+\begin{pmatrix}4\\7\end{pmatrix}$ $=\begin{pmatrix}9\\-15\end{pmatrix}+\begin{pmatrix}4\\7\end{pmatrix}=\begin{pmatrix}13\\-8\end{pmatrix}$ (ii) $\mathbf{p}-2\mathbf{r}$ $=\begin{pmatrix}3\\-5\end{pmatrix}-2\begin{pmatrix}8\\m\end{pmatrix}$ $=\begin{pmatrix}3\\-5\end{pmatrix}-\begin{pmatrix}16\\2m\end{pmatrix}=\begin{pmatrix}-13\\-5-2m\end{pmatrix}$	(b) Let $3\mathbf{p}+\mathbf{q}=k(\mathbf{p}-2\mathbf{r})$, $k\in\mathbb{R}$ $\begin{pmatrix}13\\-8\end{pmatrix}=k\begin{pmatrix}-13\\-5-2m\end{pmatrix}=\begin{pmatrix}-13k\\k(-5-2m)\end{pmatrix}$ $-13k=13\Rightarrow k=-1$ $-(-5-2m)=-8$ $5+2m=-8$ $m=-\frac{13}{2}$

6. In the diagram, E is the point $(1,1)$ and $\overrightarrow{CE}=\begin{pmatrix}-4\\-2\end{pmatrix}$. AED and BCD are straight lines.



(a) Find

(i) $\left|\overrightarrow{CE}\right|$,

(ii) the coordinates of C.

(b) The point A is $(-2,3)$ and $\overrightarrow{AB}=\frac{3}{2}\overrightarrow{EC}$.

Find the coordinates of B.

(c) The point D is $(h,-3)$.

(i) Find in terms of h, the vector $\overrightarrow{AD}$.

(ii) Given that AED is a straight line, find h.

[Ans: (a) (i) $2\sqrt{5}$ units (ii) $C(5,3)$ (b) $B(4,6)$ (c)(i) $\begin{pmatrix}h+2\\-6\end{pmatrix}$ (ii) $h=7$]

(a) (i) $\left|\overrightarrow{CE}\right|$

$=\sqrt{(-4)^2+(-2)^2}=\sqrt{20}=2\sqrt{5}$

(ii) $\overrightarrow{CE}=\begin{pmatrix}-4\\-2\end{pmatrix}$

$\overrightarrow{OE}-\overrightarrow{OC}=\begin{pmatrix}-4\\-2\end{pmatrix}$

$\overrightarrow{OC}=\begin{pmatrix}1\\1\end{pmatrix}-\begin{pmatrix}-4\\-2\end{pmatrix}=\begin{pmatrix}5\\3\end{pmatrix}$

$\therefore C(5,3)$

(b) $\overrightarrow{AB}=\frac{3}{2}\overrightarrow{EC}$

$\overrightarrow{AB}=\frac{3}{2}\left(-\overrightarrow{CE}\right)$

$\overrightarrow{OB}-\overrightarrow{OA}=\frac{3}{2}\left[-\begin{pmatrix}-4\\-2\end{pmatrix}\right]$

$\overrightarrow{OB}=\frac{3}{2}\begin{pmatrix}4\\2\end{pmatrix}+\begin{pmatrix}-2\\3\end{pmatrix}=\begin{pmatrix}6\\3\end{pmatrix}+\begin{pmatrix}-2\\3\end{pmatrix}$

$=\begin{pmatrix}4\\6\end{pmatrix}$

$\therefore B(4,6)$

(c) (i) $\overrightarrow{AD}$

$=\overrightarrow{OD}-\overrightarrow{OA}$

$=\begin{pmatrix}h\\-3\end{pmatrix}-\begin{pmatrix}-2\\3\end{pmatrix}=\begin{pmatrix}h+2\\-6\end{pmatrix}$

(ii) $\overrightarrow{AE}$

$=\overrightarrow{OE}-\overrightarrow{OA}$

$=\begin{pmatrix}1\\1\end{pmatrix}-\begin{pmatrix}-2\\3\end{pmatrix}=\begin{pmatrix}3\\-2\end{pmatrix}$

Let $\overrightarrow{AE}=k\overrightarrow{AD}$, $k\in\mathbb{R}$

$\begin{pmatrix}3\\-2\end{pmatrix}=k\begin{pmatrix}h+2\\-6\end{pmatrix}=\begin{pmatrix}k(h+2)\\-6k\end{pmatrix}$

$-6k=-2\Rightarrow k=\frac{1}{3}$

$\frac{1}{3}(h+2)=3\Rightarrow h=7$