VECTORS (IN 2D)

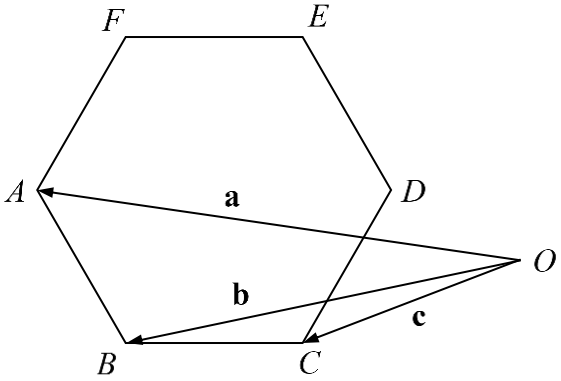
1. In the diagram, $ABCDEF$ is a regular hexagon. The vectors, $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are the position vectors of A, B and C with respect to the point O respectively. Express the following vectors in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.

   (a) $\overrightarrow{FE}$

   (b) $\overrightarrow{AD}$

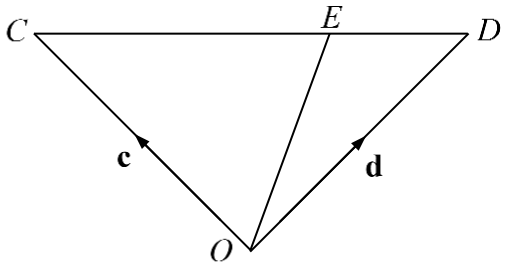
   (c) $\overrightarrow{AF}$

   (d) $\overrightarrow{BF}$



[Ans: (a) $\boldsymbol{c}-\boldsymbol{b}$ (b) $2(\boldsymbol{c}-\boldsymbol{b})$ (c) $\boldsymbol{a}-2\boldsymbol{b}+\boldsymbol{c}$ (d) $2\boldsymbol{a}-3\boldsymbol{b}+\boldsymbol{c}$]

2. In the diagram, $CE=ED=4:3$. Find $\overrightarrow{OE}$ in terms of $\mathbf{c}$ and $\mathbf{d}$.



[Ans: $\frac{3}{7}\boldsymbol{c}+\frac{4}{7}\boldsymbol{d}$]

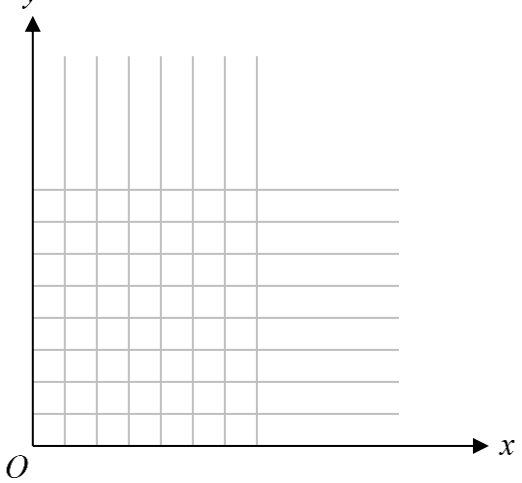
3. $OABC$ is a parallelogram where O is the origin, $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

(a) On the diagram, clearly label the points A, B and C.

(b) Express as column vector, $\overrightarrow{OB}$ and $\overrightarrow{CA}$.

[Ans: (a) label (b) $\overrightarrow{OB} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$; $\overrightarrow{CA} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$]

(a)



(b)

4. (a) If $\mathbf{p} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$, find $|\mathbf{p} + \mathbf{q}|$.

(b) The position vectors of A and B are $\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$. If $\overrightarrow{OC} = 2\mathbf{a} + 3\mathbf{b}$ and $\overrightarrow{OD} = 2\mathbf{a} - 2\mathbf{b}$, find the coordinates of C and of D.

[Ans: (a) $\sqrt{65}$ units (b) $C(0, 23)$, $D(10, -2)$]

5. It is given that $\mathbf{p} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 8 \\ m \end{pmatrix}$.

   (a) Express the following as column vectors.

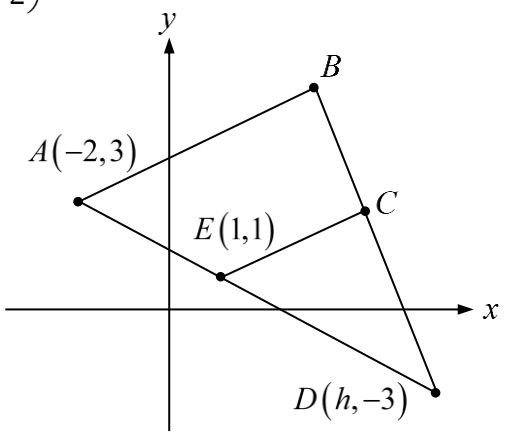
   (i) $3\mathbf{p} + \mathbf{q}$

   (ii) $\mathbf{p} - 2\mathbf{r}$

   (b) If $3\mathbf{p} + \mathbf{q}$ and $\mathbf{p} - 2\mathbf{r}$ are parallel, find the value of m.

[Ans: (a)(i) $\begin{pmatrix} 13 \\ -8 \end{pmatrix}$ (ii) $\begin{pmatrix} -13 \\ -5-2m \end{pmatrix}$ (b) $-\frac{13}{2}$]

6. In the diagram, E is the point $(1,1)$ and $\overrightarrow{CE} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$. AED and BCD are straight lines.



(a) Find

(i) $\left|\overrightarrow{CE}\right|$,

(ii) the coordinates of C.

(b) The point A is $(-2,3)$ and $\overrightarrow{AB} = \frac{3}{2}\overrightarrow{EC}$.

Find the coordinates of B.

(c) The point D is $(h,-3)$.

(i) Find in terms of h, the vector $\overrightarrow{AD}$.

(ii) Given that AED is a straight line, find h.

[Ans: (a) (i) $2\sqrt{5}$ units (ii) $C(5,3)$ (b) $B(4,6)$ (c)(i) $\begin{pmatrix} h+2 \\ -6 \end{pmatrix}$ (ii) $h = 7$]